Unit 3

Developing objective led lessons in mathematics

Following the training in the generic unit *Objective led lessons*, it is important to consider how the key messages of the training apply to mathematics. As part of the whole-school focus on Assessment for learning, this subject development material is intended to help you consider the key messages of the training unit and identify any areas requiring development in your department.

The following is a brief summary of the training unit.

Objectives

- To define what is meant by learning objectives and learning outcomes.
- To demonstrate the purpose and importance of sharing learning objectives with pupils.
- To provide strategies for sharing learning objectives with pupils.

Key messages

- Effective learning takes place when learners understand what they are trying to achieve. Sharing objectives with pupils ensures they are aware of what they are learning and why. Sharing the learning objectives gives a clear focus for the teacher and the pupil to review progress in their learning within the lesson.
- What the teacher intends the pupils to learn is called the learning objective, and how achievement will be demonstrated by pupils is called the learning outcome.
- In stating the learning objective in a lesson, it is common practice to summarise the content of previous lessons and outline how it links to future lessons. A learning objective should be set in a learning context and help connect current learning with longer-term purposes, e.g. objectives of a unit of work, end of unit assessments or pupil targets.
- Learning objectives and intended learning outcomes should be the principal focus in planning, and appropriate activities should be chosen to support them.
- Using stems (*to know*, *to be able to*, etc.) helps to ensure that learning objectives focus on learning rather than on the supporting activities.

The following material builds on the tasks outlined in the 'Ready for more?' section of the *Objective led lessons* training unit and it is intended for all those who teach mathematics.

Reviewing existing practice in objective led lessons

The table below provides a tool for a department to self-review current practice and to help identify an appropriate starting point.

As a department, agree and highlight the statements below that best reflect the practice of the whole department. At the bottom of each column is a reference to the tasks that will support your current practice and provide the appropriate material to develop from this point.

Having completed this review you should read 'Making effective use of the subject development material' on the next page.

	Focusing	**Developing**	**Establishing**	**Enhancing**
Teachers	The subject leader has identified where: • planning is mainly task rather than learning objectives focused • learning objectives and learning outcomes are not routinely shared with pupils before beginning tasks • feedback does not relate directly to learning objectives and learning outcomes. There is no agreed whole-school or departmental approach to sharing objectives in lessons.	Some departmental planning focuses on learning objectives. There is limited exemplification of the learning outcomes. Sometimes there is a lack of distinction between the task and learning objective. Teachers are beginning to share learning objectives and learning outcomes with pupils prior to carrying out the task. Some teachers are explaining the longer-term purposes of the learning. Teacher feedback sometimes relates to learning objectives, though this is not consistent across the department or school.	Departmental planning usually focuses on learning objectives and intended learning outcomes linked to standards in each subject. This approach is becoming consistent across the school. The sharing of learning objectives, intended learning outcomes and the bigger picture with pupils is becoming routine practice within departments and across the school. Teachers' feedback typically relates directly to the learning objectives.	Learning objectives and outcomes are an integral feature of all departmental planning across the school. All teachers respond to the impact these are having on standards in each subject. Objectives and intended outcomes are routinely shared, discussed and understood by pupils in all lessons. Review of learning in relation to objectives is a routine part of lessons and its outcomes inform future planning. Teachers regularly involve pupils in establishing success criteria and actively involve them in determining their progress, through peer and self assessment.
Pupils	The subject leader has identified: • the lessons in which pupils are not able to explain what they are trying to learn and the purpose of the task.	Most pupils, in most lessons, understand what they are trying to learn and can explain this with limited use of subject-specific language. Some pupils understand how they can show success, but others are unclear about what is expected of them. Some pupils understand the longer-term purpose (big picture) of what they are learning.	With some prompting, all pupils are able to explain clearly what they are trying to learn, how well they are doing and what they need to do to improve. Pupils are increasingly confident in discussing the progress they are making against the learning objectives with each other and with their teacher. Pupils, when supported, are able to recognise and improve their achievements against predetermined criteria and some are beginning to contribute to determining the criteria.	All pupils understand what they are trying to achieve and why, and routinely review their progress against the learning objectives for the lesson. Pupils are aware of a range of possible learning outcomes and are able to determine and improve their achievements in relation to success criteria. Pupils are able to identify independently their achievements against criteria they have collaboratively agreed.
	Start with Task 3A	Start with Task 3A	Start with Task 3B	Start with Task 3C

Making effective use of the subject development material

The tasks you have been referred to are intended to support the development or extension of objective led lessons in mathematics and provide guidance on how to embed this into regular practice in mathematics lessons.

The results of the self-review will have suggested the appropriate task(s) that will support your department's development needs.

To make best use of the supporting material the following sequence will be helpful.

1 Read the task and the supporting exemplification.

This describes how a department has approached the task and worked through each of its stages. It is given as an *example* of how the task might be addressed. It is not intended that you follow this approach, which is given as a guide to the process that will support improvements in your subject.

2 Identify what the department did and the impact it had on pupils.

Discuss as a team the example provided and establish the key areas that helped to develop this practice and the impact it had on pupils. It will be helpful to identify the changes in teachers' practice and how these impacted on pupils' learning.

3 Agree and plan the actions that will develop your practice.

As a department, agree how you intend to approach this task. Clarify what you are focusing on and why. The example given will act as a guide, but be specific about which classes, which lessons and which aspects of the curriculum will be your points of focus.

4 Identify when and how you will evaluate its impact on pupils.

The purpose of focusing on this is to improve pupils' achievement and attainment in mathematics. You will need to be clear on what has helped pupils to learn more effectively in your subject. Part of this will be how your practice has adapted to allow this. You should jointly identify what has worked well and which areas require further attention.

5 Having evaluated these strategies, consider what steps are required to embed this practice.

You will need to undertake an honest evaluation of what you have tried and the impact it has had on your teaching and on pupils' learning. One outcome might be that you need to spend longer on improving this area or you may be in a position to consider the next task.

Other departments in the school will have been focusing on this area and you should find out about the progress they have made.

You may find that some teachers in the department will require further time to develop and consolidate new practice, while others will be ready to progress further through the tasks in this area (while continuing to support their colleagues). Practice across a department will need to be consolidated before focusing on a new area of Assessment for learning.

The subject development tasks

Task 3A
Ensure there is a clear focus in your planning on what you intend pupils to learn (the learning objectives) and the evidence to demonstrate that pupils have achieved this (the learning outcomes). Over the next four weeks introduce and focus lessons with learning objectives, e.g. by displaying them, discussing them, asking questions related to them and structuring plenaries around them. Evaluate the strategies you have used in relation to learning objectives and assess their impact on pupils' motivation and learning. Use the outcomes of your evaluation to further improve your use of learning objectives.
Task 3B
Having established the practice of sharing objectives with pupils in lessons, question pupils during the lessons to check that: • they understand the learning objectives • they can explain how they will know when they have achieved them. Plan the use of questions and plenaries to focus on learning objectives and recognising learning outcomes. Involve pupils actively in this.
Task 3C
Having planned and shared the learning objectives with pupils, focus your feedback on these objectives. Ensure that your feedback focuses on what pupils have done successfully, where they could improve and how they could improve. (Further guidance is given in Unit 4, *Oral and written feedback* and in the related subject development materials.)

The following pages provide exemplification of each task.

Task 3A

Ensure there is a clear focus in your planning on what you intend pupils to learn (the learning objectives) and the evidence to demonstrate that pupils have achieved this (the learning outcomes).

Over the next four weeks, introduce and focus lessons with learning objectives, e.g. by displaying them, discussing them, asking questions related to them and structuring plenaries around them.

Evaluate the strategies you have used in relation to learning objectives and assess their impact on pupils' motivation and learning.

Use the outcomes of your evaluation to further improve your use of learning objectives.

Context

Following the whole-school training on Assessment for learning, the mathematics department decided to strengthen their practice in sharing learning objectives with pupils. Prior to the training, all mathematics teachers recorded the learning objectives on the whiteboard at the beginning of the lesson, and some teachers asked pupils to write them in their exercise books. The whole-school training started teachers questioning the impact this was having.

Process

The head of department led an initial discussion in a department meeting, using the lesson plan 'Proportion or not?' (see **appendix 3A.1**) from the *Enhancing proportional reasoning* materials. The department focused on the learning objective.

- Identify the necessary information to solve a problem (by recognising problems involving direct proportion).

Having looked in detail at the lesson plan, the department wrote the learning objective in pupil speak.

Teaching objective	Learning objective
Identify the necessary information to solve a problem (by recognising problems involving direct proportion).	To be able to: • recognise when two sets of variables are in direct proportion • select the information needed to solve a problem.

The head of department then posed the question: 'In this lesson, how does the teacher ensure the pupils are clear about what they are learning?'

The department discussed this and identified two key questions:
- 'Are the two sets in direct proportion?'
- 'How do you know?'

These questions were used to help pupils focus on the learning objective throughout the lesson, including the plenary.

At this stage, the teachers decided to focus on Year 7 groups. In pairs, they discussed the next unit in the scheme of work, which was on equations, formulae and identities. They worked on the learning objectives for the unit, first identifying the learning outcomes and re-writing the objective in pupil speak, then devising two or three questions to help focus pupils on the learning. The questions were discussed by the whole team and written into the unit plan, so teachers could start to use them over the next few weeks.

Evaluation

In the next department meeting, the team discussed what they and the pupils had gained from the focus on learning objectives.

- Teachers felt more focused during each part of the lesson.
- The stronger focus on objectives sharpened the use of questioning.
- Pupils could more easily recognise and talk about what they had learned.

The department agreed to work on the units to be taught in the spring term. Units were allocated to pairs of teachers who repeated the process they had collectively completed for the algebra unit.

Task 3B

Having established the practice of sharing objectives with pupils in lessons, question pupils during the lessons to check that:
- **they understand the learning objectives**
- **they can explain how they will know when they have achieved them.**

Plan the use of questions and plenaries to focus on learning objectives and recognising learning outcomes. Involve pupils actively in this.

Context

Following the whole-school training on Assessment for learning, the mathematics department decided to strengthen their practice in sharing learning objectives with pupils. Prior to the training, all mathematics teachers recorded the learning objectives on the whiteboard at the beginning of the lesson, and some teachers asked pupils to write them in their exercise books. The whole-school training started teachers questioning the impact this was having.

Process

Following on from the focus on engaging pupils with the learning objectives, the department then decided to focus on using questioning to help pupils assess their understanding. As before, the head of department used the lesson plan 'Proportion or not?' (see **appendix 3A.1**), but this time with the associated resource sheets (see **appendix 3B.1**). The following questions were used to stimulate discussion.

- In the lesson plan, which of the questions are likely to help pupils understand the objective?
- What evidence will the pupils have that they have achieved the objective?

The teachers identified several questions that were significant in helping pupils get a full understanding of the objective. These included the following.

- Are the two sets in direct proportion? How do you know?
- Can you name another pair of numbers we could add to the set (which share the same relationship)?
- How could you describe the relationship between the sets?

The teachers also thought the approach of providing data sets and situations for pupils to classify as either 'in proportion' or 'not in proportion' (**appendix 3B.1**) helped pupils to judge whether they had achieved the objective.

After this discussion, the department watched the video sequence from the *Enhancing proportional reasoning* materials. This extract shows Ali teaching her Year 8 class the stages of the problem-solving cycle in the context of proportional reasoning. They focused mainly on the plenary which illustrated how Ali:

- used questioning to assess whether pupils had understood the learning objective
- constructed a diagram of the problem-solving cycle to help pupils assess their understanding.

Following these activities, the department decided to continue with their focus on devising questions specifically linked to the learning objectives for other Year 7 units of work. They also decided to explore further the teaching approach of getting pupils to classify examples for a given criterion (in this case either 'in proportion' or 'not in proportion') and justify their answers.

Evaluation

A month later, the department met again to evaluate the effect of these changes on pupils' responses.

Task 3C

Having planned and shared the learning objectives with pupils, focus your feedback on these objectives.

Ensure that your feedback focuses on what pupils have done successfully, where they could improve and how they could improve.

(Further guidance is given in Unit 4, *Oral and written feedback* and in the related subject development materials.)

Context

Following the whole-school training on Assessment for learning, the mathematics department decided to strengthen their practice in sharing learning objectives with pupils. Prior to the training, all mathematics teachers recorded the learning objectives on the whiteboard at the beginning of the lesson, and some teachers asked pupils to write them in their exercise books. The whole-school training started teachers questioning the impact this was having.

Process

In the final stages of this department's work on developing objective led lessons, they decided to focus on providing focused written feedback to pupils' work. In the week prior to the department meeting, two teachers had used the Year 9 Booster lesson 'Lines and angles' (Mathematics, lesson 8). They brought to the meeting the work they had marked (see **appendix 3C.1** for two examples).

The learning objective for the lesson, along with the version in pupil speak and the identified learning outcomes, are given in the table below.

Teaching objective	Learning objectives	Learning outcomes
Solve geometrical problems using side and angle properties, explaining reasoning with diagrams and texts. (Year 8)	To be able to: • calculate angles in triangles and other polygons by using your knowledge of angle properties of parallel lines, triangles and polygons • justify and explain your answers by labelling diagrams and describing in writing the properties you have used.	**All** of you will know and use the angle properties of parallel lines, triangles and polygons to calculate angles in given diagrams. **Most** of you will be able to justify and explain your reasoning by labelling the diagrams and describing clearly in writing the angle properties used.

Pairs of teachers looked closely at the written feedback given in the work. They were asked to refer to the learning objectives and select examples of written feedback they felt to be most useful.

The following points were made about the two examples.

- **Example 1**: The written feedback focuses on the learning objectives and identifies where the pupil has been successful. The question posed by the teacher has the potential to help the pupil complete the task, assuming time is allocated for this.

- **Example 2**: The written feedback confirms accuracy, but does not help the pupil to see an efficient way to record the properties they have used. It might have been helpful to provide one example for the pupil and, again, give the pupil time to complete the task.

The teachers decided to concentrate on providing focused written feedback for Year 9 classes over the next half term.

Evaluation

The department planned to review progress at a subsequent meeting by looking for the impact of improved written feedback on pupils' work. They agreed to explore this further (see Unit 4, Module 4.2 *Written feedback*).

Following the developments in your subject as a result of completing these tasks, you should evaluate its impact on teaching in the subject and how pupils have responded, particularly in relation to their standards of attainment. The following questions may help to structure this.

- How has teaching been adapted to the key messages of objective led lessons?
- How has sharing learning objectives with pupils impacted on pupils' learning?
- How has an explicit focus on learning objectives ensured that plenaries are more focused?
- What more do we need to do to be more effective in objective led lessons?

Subject-specific references

Referenced strategy materials

Interacting with mathematics in Key Stage 3: Enhancing proportional reasoning (DfES 0093/2003). These materials can be found at www.standards.dfes.gov.uk/keystage3 by selecting 'mathematics' and then 'mathematics publications'.

Video: *Interacting with mathematics in Key Stage 3: Enhancing proportional reasoning*. This was included with the school pack for these materials (DfES 0093/2003)

QCA materials

Using assessment to raise standards in mathematics, Section 2: Using effective questioning techniques (QCA, www.qca.org.uk)

Lesson plan: Proportion or not?

A lesson plan from phase 2 of Year 8 multiplicative relationships

Objectives

- Consolidate understanding of the relationship between ratio and proportion.
- **Identify the necessary information to solve a problem** (by recognising problems involving direct proportion).

Starter

Vocabulary
corresponding pairs
direct proportion
relationship

Resources
Data sets selected from resource sheet PR1
Calculators

Proportion or not?

On the OHP display a pair of data sets which are in direct proportion. (See the resource sheet PR1, 'Proportion or not? Data sets'.)

Q Are the two sets in direct proportion?

Q How do you know?

Explore a range of different strategies to draw out different relationships between the numbers.

Repeat using different pairs of sets, some in direct proportion and some not.

When the pairs are in direct proportion:

Q Can you name another pair of numbers we could add to the set (which share the same relationship)?

Q How could you describe the relationship between the sets? (corresponding pairs)

Q If *x* is added as member of the first set, which *y* goes with it in the second? For example:

2	7
8	28
20	70
10	?

Draw out (2 | 8) : (7 | 28). Emphasise the greater flexibility of the multiplicative relationship (20 : 2) : (70 : 2).

Solving problems

Describe/illustrate a proportion problem. For example:

Main activity

Vocabulary
As starter

Resources
Collection of real data sets from resource sheet PR2
Collection of problems from resource sheet PR3

Q A bottle of diet cola indicates that 100 ml contains 0.4 kcal of energy. How much energy would 200 ml contain?

Construct a table of values:

Cola (ml)	Energy content (kcal)
100	0.4
200	0.8

Add other 'easy' figures (e.g. 150 ml of cola and 1 litre of cola) then less obvious figures (e.g. a typical glass of 180 ml).

Q How can we work out the amount of energy?

Q Are the two sets of values in direct proportion?

Q Why/how do we know?

Relate the answers to the table of numbers and also to the situation (i.e. the uniform nature of cola); draw out the use of 'for every'. Ensure this is well established with the class.

If time permits, use this question to confirm pupils' understanding of the situation:

Q Can you calculate or estimate how much cola would provide 5 kcal?

Point out that the volume of cola and the amount of energy are two values which can vary and are connected in some way. This is typical of many mathematical problems. To solve them, it is important to know how the variables are connected.

Distribute a selection of data sets from resource sheet PR2, 'Proportion or not? Contextualised data sets' (or similar). Say that you want pupils to work in pairs to identify quickly which sets of variables are in direct proportion. They should record their reasons briefly.

Mini-plenary

After a few minutes, select two or three of the examples and ask selected pairs to share their thinking with the class. Address any misconceptions revealed.

Now distribute a selection of problems from resource sheet PR3, 'Proportion or not? Problems' (or similar). Ask pupils to classify them according to whether the variables are in direct proportion or not and to solve those they can.

Simplification

Initially, restrict situations and problems to those with more obvious relationships.

Extension

Think about a length of elastic tied to a fixed hook with a light pan suspended on the other end. Ask pupils to consider the length of the elastic as different weights are added to the pan. Will the length be proportional to the mass? Will the extension be proportional to the mass? Ask them to design an experiment to test their theories.

Plenary

Resources

Collection of situations from resource sheet PR4

Address any issues which you have identified while pupils were working on the problems.

Select two or three situations from resource sheet PR4, 'Proportion or not? Situations' (or elsewhere), and present them on an OHT.

Q What are the variables?

Q Are they in direct proportion?

Q How can you justify your answer?

Remember

- You might know that two sets of numbers are in direct proportion because you are familiar with the context and know how one variable relates to the other.
- You might observe that two sets of numbers are in proportion by looking at a table of values and noting the pattern of entries.
- Both of these points can be checked by ensuring that a constant multiplier connects every pair of values.

Resource sheets for lesson plan shown in Appendix 3A.1

Resource sheet PR1

Proportion or not? Data sets

These examples can be used to raise the question ‘Are these data sets in direct proportion?’

A	1	3	**B**	20	5
	2	6		28	7
	3	9		44	11
	7	21		4	1
				84	21
C	3	4	**D**	10	15
	7	8		12	20
	14	15		14	25
				16	30
E	3	10	**F**	77	11
	4	13		21	3
	5	16		672	96
	6	17			
G	411	611	**H**	3	9
	457	657		5	25
	429	629		8	64
				10	100
I	42	4	**J**	14.2	65.32
	84	8		6.9	31.74
	105	10		321	1476.6
	252	24		55.55	255.53
	357	34			

These sets are in direct proportion: A, B, F, I, J.

Resource sheet PR2

Proportion or not? Contextualised data sets

These examples can be used to raise the question ‘Which sets of variables are in direct proportion?’ by considering the sets of data in context.

Playgroup

The following table is designed to show staff at a playgroup how many adults are needed to look after different sized groups of pupils.

Number of adult staff	Maximum number of children
2	8
3	16
4	24
5	32

Although there is a constant difference in each column, the figures are not in direct proportion. The relationship could be described by $c = 8(a - 1)$.

Phone bill

A mobile phone bill shows the following details.

Calls to other networks

Duration (min:sec)	Cost (pence)
2:35	77.5
7:12	216
3:04	92
12:55	387.5
10:10	305
1:44	52

The cost is directly proportional to the duration (30p/min) but the times may need to be converted to seconds to make the relationship clear.

Belt prices

A clothing website allows customers to pay in pounds sterling (£) or euros (€)

These are the prices for four different belts:

£6.99	or	€11
£13.99	or	€22
£15.99	or	€25
£25	or	€39

The two prices are very nearly in direct proportion. A conversion rate of £1 = €1.56 has been used, but the euro prices have been rounded to the nearest whole number.

Clicko kits

Clicko building kits come in five sizes. Their components are listed below.

Kit	Base plates	Long rods	Short rods	L-joints	H-joints
Beginner	1	10	30	20	15
Designer	1	16	48	32	24
Advanced	1	24	72	48	36
Expert	2	42	126	84	63
Supreme	2	60	180	120	90

The number of base plates is not in proportion to the numbers of other components. However, the others are provided in the ratio 2 : 6 : 4 : 3.

Photographs

A photographic shop offers to reprint enlargements of photographs according to the following table.

Size of print	Cost
10″ by 8″ (25 cm by 20 cm)	£5.00
14″ by 10″ (36 cm by 25 cm)	£9.00
18″ by 12″ (46 cm by 30 cm)	£13.80

The size data are 'real'. Several comparisons are possible. The ratio of the height to width of the different prints is not consistent (either measured in inches or centimetres). The conversion rate from inches to cm is nearly consistent at 1″ = 2.5 cm. The price is directly proportional to the area of the prints given in cm^2.

Cooling coffee

In a science experiment, the temperature of a cup of coffee is measured over half an hour. The results are tabulated.

Elapsed time in minutes	Temperature in °C
0	98
5	73
10	56
15	42
20	34
25	30
30	28

The temperature is not directly proportional to the elapsed time.

Wire

A factory sells spools of wire cable by weight. The labels show the length of wire on each spool.

Weight (kg)	Length (m)
0.75	57
1.5	114
3.5	266
5	380

The weight and length are in direct proportion. The 'size' of the wire can be expressed in different units, for example 76 m/kg.

Beethoven's symphonies

A boxed set of Beethoven's nine symphonies provides the following information.

Symphony number	Duration of recording in minutes
1	36
2	30
3	48
4	31
5	29
6	33
7	32
8	24
9	64

Clearly, these figures show no arithmetic relationship.

Water drum

A large concrete drum holds water for cattle on an Australian farm. The farmer measures the depth of the water and uses this table to estimate its volume.

Depth of water	Volume
0.9 m	150 gallons
1.2 m	200 gallons
1.5 m	250 gallons
2.4 m	400 gallons

The depth and volume measures are in direct proportion.

Resource sheet PR3

Proportion or not? Problems

These closed questions require a decision about whether the variables are in direct proportion.

1 2.5 litres of paint are sufficient to cover 80 square metres. How much paint do I need to cover 250 square metres?

2 A seaside harbour has a tide marker showing the depth of water inside the harbour. At midnight the depth is 4.2 m. At 2:00 am it is 4.9 m. What will the depth be at midday?

3 A garage sells diesel fuel at 73.9p per litre. How much can I buy for £20?

4 Henry the Eighth had six wives. How many wives did Henry the Fourth have?

5 My recipe for 9 scones uses 200 grams of flour. How much flour will I need for 24 scones? The nine scones need 8 minutes in a hot oven. How long will I need to cook 24?

6 A gardener has a lawn which is 15 m by 12 m. She decides to feed it with fertiliser applied at 1.5 grams per square metre. How much fertiliser does she need?

7 A sprinter can run 100 m in 11.2 seconds. How long will it take the sprinter to run 250 m?

8 A shop buys cans of soft drink in boxes of 24 for £1.99 per box. They sell the cans at 39p each. Is their total profit proportional to:

(a) the number of boxes they buy;

(b) the number of cans they sell;

(c) the money they take?

9 When Robyn was 1 year old she weighed 11 kg. When she was 2 years old she weighed 14 kg. How much did she weigh when she was 4 years old?

10 A 750 g box of cornflakes costs £2.19. How much does a 1 kg box cost?

Resource sheet PR4

Proportion or not? Situations

These examples can be used to raise the question ‘Are the variables in direct proportion?’ by considering the situation rather than sets of data.

True or false?

1 In the different countries of the world, the number of cars on the road is directly proportional to the population.

2 The weight of flour in a sack is directly proportional to the volume of flour.

3 The monthly electricity bill is directly proportional to the amount of electricity used.

4 The time an audio tape plays for is directly proportional to the length of tape.

5 The temperature of a saucepan of soup is directly proportional to the time it has been on the stove.

6 The cost of an article of clothing is proportional to how long it will last.

7 The time taken to read a maths problem and the time taken to solve it are in direct proportion.

8 The cost of a train journey is directly proportional to the distance travelled.

When could we reasonably assume the following to be true and when false?

9 The time taken to drive a journey is directly proportional to the distance covered.

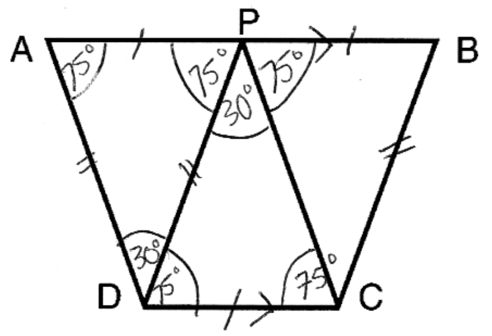
10 The amount of money a waitress earns is directly proportional to the number of hours she works.

11 The cost of a phone call is proportional to the length of the call.

12 The amount of wallpaper I have to buy is directly proportional to the area of the walls I want to cover.

13 The time taken to read a book is directly proportional to the number of pages in the book.

Example 1: Isosceles trapezium



ABCD is an isosceles trapezium with AB parallel to DC.

How might this fact help you to find the other two angles?

P is the midpoint of AB, and AP = CD, AD = DP.

∠DAP = 75°.

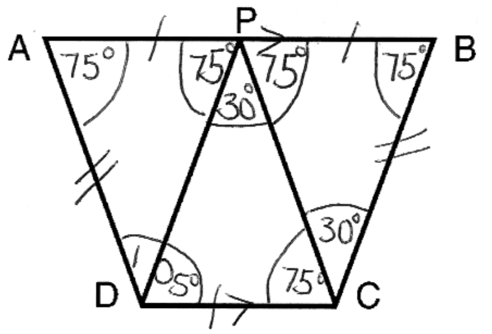
Calculate the sizes of the other angles.

∠APD = 75° (ISOSCELES TRIANGLE)
∠PDC = 75°
∠PDA = 30° (ANGLES IN A TRIANGLE ADD UP TO 180°)
∠PCD = 75° (OPPOSITE ANGLES OF A PARALLELOGRAM ARE EQUAL)

∠DPC = 30°

You have used angle and side properties well to calculate and justify most of the angles.

Example 2: Isosceles trapezium



ABCD is an isosceles trapezium with AB parallel to DC.

P is the midpoint of AB, and AP = CD, AD = DP.

∠DAP = 75°.

Calculate the sizes of the other angles.

The angles are correct, but you need to provide reasons for your answers.

