Unit 5

The Framework for teaching mathematics and the approach to calculation

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Objectives

- To introduce teachers to the *Framework for teaching mathematics: Years 7, 8 and 9*

- To exemplify oral and mental work in mathematics

- To consider the approach to calculation

Suggested use and organisation

- All schools; whole-school training day or staff meetings.

- This unit is particularly suitable for teachers of subjects that make use of calculation skills. Unit 6 considers the use of calculators.

- This unit brings out information that will support the departmental elements of unit 9.

- Participants should be organised into groups of up to six people; where possible there should be a member of the mathematics department with each group.

- The unit is also suitable for a single department working with a member of the mathematics department.

Resources

- OHTs 5.0–5.5

- Handout 1.4 or 4.3 (teachers should bring their own copies)

- *Framework for teaching mathematics: Years 7, 8 and 9* (one copy per group)

- Photocopies of the teaching programmes for Years 7, 8 and 9 (Framework section 3, pages 4–11; one set per participant)

- *Framework for teaching mathematics from Reception to Year 6* (reference copy)

Session outline   75 minutes

The Framework for teaching mathematics Giving a brief overview of the Framework	Talk, group discussion	10 minutes
Oral and mental work Modelling an oral and mental starter which develops mental imagery	Activity	10 minutes
The approach to calculation Outlining the development of calculation skills from mental through to written methods	Talk, group activities	40 minutes
The teaching programme Studying expectations in the Framework supplement of examples	Groups	10 minutes
Conclusion Reviewing the session and identifying implications at departmental and whole-school level	Talk, questions	5 minutes

The Framework for teaching mathematics   10 minutes

Show **OHT 5.0** and outline the session's objectives.

OHT 5.0

Objectives

- To introduce teachers to the *Framework for teaching mathematics: Years 7, 8 and 9*

- To exemplify oral and mental work in mathematics

- To consider the approach to calculation

This session provides teachers of all subjects with a brief overview of the *Framework for teaching mathematics: Years 7, 8 and 9*.

Using **OHT 5.1**, quickly outline the contents of the Framework, and mention briefly the purpose of each section. There are notes describing each section on **page 4** of the Guide to the Framework.

OHT 5.1

Contents of the Framework for teaching mathematics

- Guide to the Framework

- Key objectives

- Yearly teaching programmes

- Supplement of examples: Years 7, 8 and 9

- Vocabulary checklist

Refer everyone to the photocopies of the yearly teaching programmes for Years 7, 8 and 9 and display **OHT 5.2** to illustrate the structure of the programmes. The yearly teaching programmes are broken down into teaching objectives for each of Years 7, 8 and 9. Where appropriate, explain that some pupils may be working

from objectives from earlier years while some higher attaining pupils may be working on those from later years. Point out that the yearly teaching programmes for Years 5 and 6 are included in the Framework, which also contains extension material for higher attaining pupils.

OHT 5.2

Strands of the yearly teaching programmes

- Using and applying mathematics to solve problems
- Numbers and the number system
- Calculations
- Algebra
- Shape, space and measures
- Handling data

Allow about 5 minutes for participants to read through part of the yearly teaching programmes. You might suggest that everyone studies the progression in measures or handling data, since these areas of mathematics are used in several other subjects. Alternatively, you could allocate specific strands to different groups. Note that later this unit focuses on the calculations strand.

Each group should have a copy of the *Framework for teaching mathematics: Years 7, 8 and 9* available in order to refer to the supplement of examples. The member of the mathematics department attached to each group is to:

- find appropriate illustrations in the supplement of examples;

- explain unfamiliar terms;

- clarify the timings of particular topics in the mathematics scheme of work.

Oral and mental work   10 minutes

One of the key features of the Key Stage 3 Framework for teaching mathematics is an emphasis on oral and mental work, usually conducted with the whole class and involving the active participation of all pupils.

Show **OHT 5.3**, which lists some of the aspects of mathematics developed through oral and mental work.

OHT 5.3

Oral and mental work

- Develop and explain mental calculation strategies, including figuring out new facts from known facts
- Apply calculation skills in algebra, for example to substitute numbers in expressions or to solve simple equations
- Develop estimation skills
- Practise the interpretation of data
- Develop mental imagery of shapes, movements and constructions
- Rehearse the use of mathematical vocabulary and develop oral communication skills
- Develop the ability to generalise, reason and prove

Introduce an oral and mental activity designed to develop pupils' powers of mental imagery. This approach could be adapted and used in other subjects.

Mental imagery

Ask participants to close their eyes. Say:

> Imagine a large white rectangle on the table in front of you.
>
> Now imagine two smaller black right-angled triangles inside the white rectangle.
>
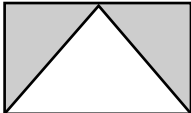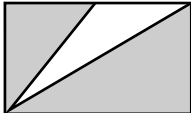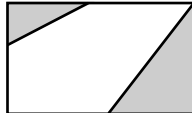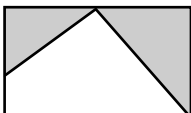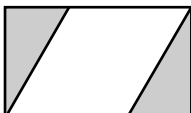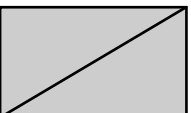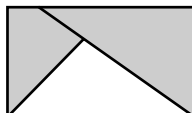> Slide one of the triangles so that its right angle fits into one corner of your rectangle.
>
> Slide the second triangle to fit into one of the other corners of your rectangle.
>
> Now, without saying anything, open your eyes and sketch the white shape that is left.
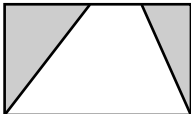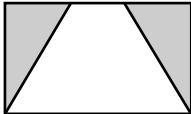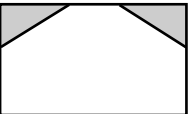
After a minute or so, ask for volunteers to describe their shapes. Then show **OHT 5.4** containing some, but by no means all, of the possible answers.

OHT 5.4

Some of the possible visualisations

isosceles triangle   scalene triangle   irregular pentagon   irregular pentagon

trapezium   trapezium   irregular hexagon   irregular hexagon

quadrilateral   parallelogram (or rhombus)   no white shape   overlapped triangles

In mathematics, this could be the starting point of an investigation looking at classifying all possible solutions.

Encourage teachers to consider what opportunities they have for similar mental imagery activities in their own subjects.

The approach to calculation   **40 minutes**

Introduce this section by explaining that this is one of the areas where the emphasis of the mathematics curriculum and expectations of pupils have changed in recent years. Since pupils need to do calculations in almost all areas of the curriculum, it is important that pupils are supported by a common approach from staff.

Mental calculation

Explain that an ability to *calculate mentally* lies at the heart of much of mathematics. The Framework for teaching mathematics stresses the importance of mental calculation methods, encouraging pupils to ask themselves 'Can I do this in my head?' before they select any other calculation method.

In Key Stages 1 and 2, pupils are encouraged to build up their understanding of the number system alongside their ability to calculate mentally. Learning in each of these areas is mutually supportive and they are taught hand-in-hand. Pupils explore and use a range of calculating strategies.

Show these four additions on a **flipchart**:

$$42 + 35 \qquad 45 + 46 \qquad 6 + 78 \qquad 37 + 199$$

Ask teachers to calculate the answers in their heads. Then ask them to think about the methods they used. Discuss the different approaches.

Repeat for these four subtractions:

$$67 - 32 \qquad 54 - 28 \qquad 60 - 19 \qquad 2005 - 1996$$

Where appropriate, show other methods commonly used by pupils if these have not arisen from the discussion.

Draw participants' attention to the following:

*   there is no single correct method for adding or subtracting (or any other calculation);

*   the strategy selected will depend not only on the operation but also on:
    *   the actual numbers involved;
    *   the pupil's awareness of (and confidence with) a range of possible strategies;

*   some methods are more efficient or more generally applicable than others.

Show **OHT 5.5** and encourage participants to suggest examples of each of the skills.

Skills of mental calculation

- Remember number facts and recall them without hesitation

- Use known facts to figure out new facts

- Draw on a repertoire of mental strategies to work out calculations, with some thinking time

- Understand and use the relationships between operations to work out answers and check results

- Approximate calculations to judge whether or not an answer is about the right size

- Apply skills to solve numerical problems

Some possibilities are given below.

- What known facts might you use to help you work out 250 – 123? Perhaps that 125 is half of 250.

- What mental strategies might you use to calculate the following?

    326 – 81   223 × 4   2.5% of £3000

   Remember to stress the importance of allowing time to think.

- To illustrate understanding and using the relationships between operations to work out answers and check results, suggest that to work out 900 ÷ 15 you might use the fact that 6 × 150 = 900.

- What approximate calculations might you use to judge whether or not an answer to 'a quarter of 57.9' is about the right size?

- Solve numerical problems such as: 'How many CDs at £3.99 each can I buy with £25?' or: 'Roughly how long will it take me to go 50 miles at 30 mph?'

Mention that an expectation of the National Numeracy Strategy is that the majority of pupils, at age 11, will know basic addition and subtraction facts along with multiplication tables and the associated division facts.

Transition to written calculations

Explain that mental calculation does not preclude the use of jottings: for example, 'the empty number line' is a useful practical (and mental) tool to support addition and subtraction.

Demonstrate 1032 – 786 using an empty number line, for example:
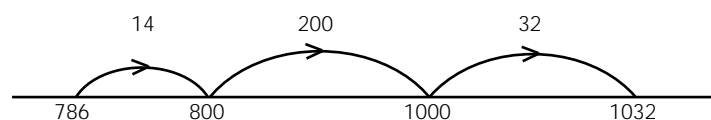
So 1032 – 786 = 14 + 200 + 32 = 246. Explain that the method is very flexible – different pupils might choose different sized 'steps' to suit their thinking – but widely applicable.

Say that many pupils at the beginning of Key Stage 3 may prefer 'expanded' written methods for multiplication and division. Typically these include the grid method for multiplication and 'chunking' for division. Add references to any others which are commonly used in your school.

Illustrate these approaches by working through a couple of examples.

Grid method for multiplication

58 × 34

×	30	4	
50	1500	200	1700
8	240	32	272
			1972

So 58 × 34 = 1972

Chunking method for division

387 ÷ 24

```
    387
−   240        10 × 24
    147
−    96         4 × 24
     51
−    48         2 × 24
      3
```

Answer: 16 R 3 or 16 $^3/_{24}$ or 16 $^1/_8$

The role of standard written methods

Standard written methods are reliable and efficient procedures for calculating which, once grasped, can be used in many different contexts. But they are of no use to someone who applies them inaccurately and who cannot judge whether the answer is reasonable. For each operation, a standard written method will be taught to most pupils, then refined and practised. Pupils are expected to be able to explain their chosen method and apply it reliably.

Point out that in helping pupils who may be unsure of a calculation or are getting answers wrong, it is usually profitable to ask them to explain the method they are using. This will often reveal whether they understand it (in which case it is likely to be a small error which is easily corrected) or they are trying to use a method they do not understand (in which case it is better that they revert to a method they are confident about).

The teaching programme   10 minutes

Allow colleagues to spend time looking through the Framework supplement of examples for the calculations strand on **pages 82–111**. This shows the expected progression across Key Stage 3, but point out that:

- some pupils may be working at levels outside this;

- the sequence of objectives does not dictate the order of teaching.

Suggest that colleagues who teach subjects which regularly require pupils to do calculations may wish to return to study these examples in more detail after the session.

Conclusion   5 minutes

If you are working with one or a few departments, suggest they consider the implications of the unit for their subject in departmental groups and record them on their list of points for action on **handout 1.4** or **4.3**, 'Priorities for cross-curricular numeracy'.

If you are working with the whole school, ask groups to spend a few minutes identifying the main messages from the session for the whole school.

Take feedback, noting particularly any issues to be followed up by the mathematics department or any coordinating group.

Conclude by reminding teachers of the main points:

- Oral and mental mathematics includes much more than rapid recall of number facts and mental tests.

- The *Framework for teaching mathematics from Reception to Year 6* details a clear progression in calculation skills from mental skills through jottings to written methods and the use of calculators.

- Pupils should be encouraged to approach a calculation by first asking themselves what is the most appropriate method: mental, written or using a calculator.

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Key Stage 3 *National Strategy*

Contents of the Framework for teaching mathematics

- Guide to the Framework

- Key objectives

- Yearly teaching programmes

- Supplement of examples: Years 7, 8 and 9

- Vocabulary checklist

Key Stage 3 *National Strategy*

OHT 5.2

Strands of the yearly teaching programmes

- Using and applying mathematics to solve problems

- Numbers and the number system

- Calculations

- Algebra

- Shape, space and measures

- Handling data

Key Stage 3 *National Strategy*

OHT 5.3

Oral and mental work

- Develop and explain mental calculation strategies, including figuring out new facts from known facts

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- Develop estimation skills

- Practise the interpretation of data

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Key Stage 3 *National Strategy*
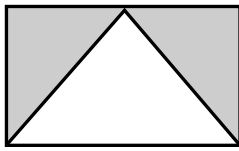
Some of the possible visualisations

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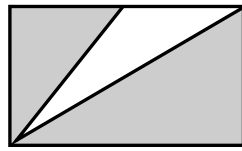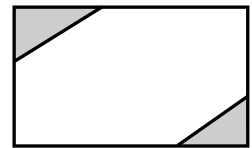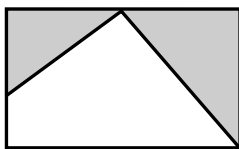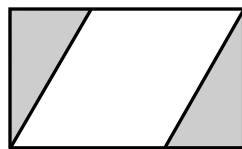
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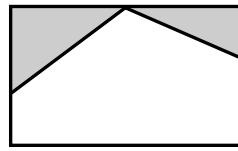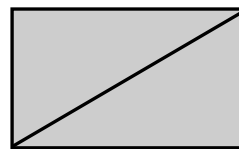
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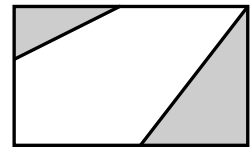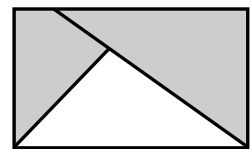
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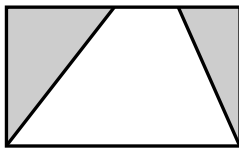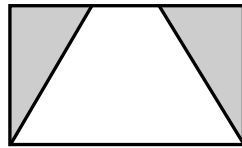
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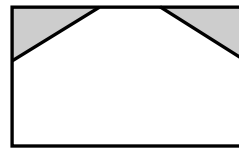
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Key Stage 3 *National Strategy*

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- Understand and use the relationships between operations to work out answers and check results

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Key Stage 3 *National Strategy*