



Guidance

Curriculum and Standards

Bridging plans: from Key Stage 3 to Key Stage 4

Mathematics

Heads of mathematics departments

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Key Stage 3 Strategy publications referred to in this booklet are available at the following website address:

www.standards.dfes.gov.uk/keystage3/publications

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The websites referred to in these materials existed at the time of going to print. Tutors should check all website references carefully to see if they have changed and substitute other references where appropriate.

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Introduction

Background to the series

This booklet is designed to help mathematics departments plan for effective transition from Key Stage 3 to Key Stage 4 by creating teaching and learning plans that bridge the key stages. In the literal sense, a bridge has two ends. Bridging plans will be effective only if teachers give thought to how the work at the end of Year 9 can be linked with the beginning of Year 10.

From a pupil's perspective, moving from Key Stage 3 to Key Stage 4 may be less marked than other key stage transitions because it does not usually involve a change of school. However, apart from the completion of the National Curriculum tests, there are other milestones. Perhaps for the first time in their lives, pupils make choices about the subjects they will study and, to some extent, the paths they will follow. They also become aware of how GCSE courses will be different, for example in having an element of assessment by coursework.

Many mathematics departments avoid a sense of anti-climax at the end of Year 9 by offering pupils opportunities that complement or extend the work they have been doing previously. For example, some departments do more extended investigational work in order to deepen pupils' mathematical understanding; others make use of computer facilities in order to improve pupils' ICT skills and enable them to experience mathematical concepts through a different medium.

The aim of these materials is to suggest additional ways in which you could help pupils make a confident start to Key Stage 4, particularly by making best use of the time in Year 9 after the National Curriculum tests in May. This is not just about starting GCSE courses early: it is about stimulating pupils' interest and keeping them engaged. It is also about creating a sense of moving on, with an expectation of increasing maturity and independence as a learner. The aim should be to develop bridging plans that can be incorporated into a scheme of work and, with appropriate revision, used from year to year.

How to use this booklet

A suggested sequence of steps is to:

- Read the booklet and reflect on the suggestions it contains – you might want to encourage other colleagues to do so as well.
- At a meeting of the department:
 - highlight some general points from the booklet;
 - review what you currently do during the post-test period in Year 9 and the first few weeks of Year 10;
 - consider possibilities for development, perhaps using the summary of suggestions on page 7 as a starting point;
 - agree on the changes you want to make, possibly delegating detailed development to a smaller group.
- Allow time to review the implementation of your plan and make changes for future years.

Developing independent learners

Increasing numbers of departments recognise the need to think about pupils, not only in terms of their capabilities in the subject, but in terms of their learning skills. Maturing towards independence is gradual; pupils do not suddenly change as they move from one key stage to another. However, the transition between Key Stage 3 and Key Stage 4 provides an opportunity to refocus attention, of both teachers and pupils, and to set up activities that could have a positive impact on pupils' learning skills.

Expectations for the end of Key Stage 4

Pupils who are effective learners have the skills to learn on their own. They can be relied on to work independently, even for long periods. Southampton LEA worked with teachers to identify statements that describe effective learners at different stages of their development. These statements identify the learning skills that pupils need to be taught. The following statements are for the end of Key Stage 4.

By age 16 effective learners:

- are well organised and plan their work confidently, balancing priorities;
- show independence in solving problems, selecting the most effective strategy with confidence and seeking help when needed;
- gather information efficiently and take notes in a variety of ways, selecting the method to suit the purpose;
- can reorganise their work and present it with a clear sense of audience;
- are effective team members and can recognise the different roles needed to complete a task and will often take on that role to ensure completion;
- search for a purpose for learning and will challenge and question to ensure that what they are learning is appropriate;
- explore how this new learning fits with existing knowledge and accommodate any changes to their overall 'map';
- assess their own work and can identify areas for improvement and seek help to clarify how they can improve.

Taking this list as a goal, think about some of your current Year 9 pupils. What steps do they need to take towards becoming independent learners?

Strategies for developing pupils' learning skills

Some pupils, or even whole classes, lack the necessary skills to work independently of their teachers. For example, in most lessons these pupils are not organised: they give up when they meet a challenge and opt out of discussion and group work. Such pupils require particular attention, but the need to develop learning skills is not confined to these groups. As they grow older, *all* pupils need to acquire a greater perception of themselves as learners.

Research shows that pupils can be taught to become more independent in their work and thus become more effective learners. It often pays to start small, by concentrating on a particular learning skill, such as organising information.

- Model for pupils how the particular skill is carried out.
- Select tasks carefully to match your goals and to ensure that pupils experience success.
- Give good examples and make clear what are the criteria of success.
- Monitor individuals and the whole class and deal with difficulties.
- Provide positive oral and written feedback, not just marks and grades.

As you begin to foster new habits in pupils, gradually increase expectations.

- Set challenging tasks for the whole class, building in necessary support.
- As a whole class, work collaboratively through the stages of solving a problem, gradually reducing the support you provide.
- Focus particularly on understanding problems and planning the solutions.
- Include short spells of carefully structured paired or small-group work.
- Expect pupils to share, comment on and evaluate each other's work.
- Develop thinking skills by raising questions about ways of working and encouraging reflection on strategies for learning.

Note: This section draws on *Teaching and Learning in Secondary Schools: Pilot, Unit 8: Developing effective learners* (DfES 0348/2003).

Mathematics: from Key Stage 3 to Key Stage 4

Following the Key Stage 3 national tests, it is quite common for departments to think of starting the GCSE course early. But what does this feel like for pupils? Since mathematics is a compulsory subject, pupils may not have a sense of taking responsibility for their choices in mathematics as they do with optional subjects. How can you make best use of the time available to promote the features of mathematics in Key Stage 4 that make it feel exciting, new or challenging for pupils?

By the end of Key Stage 3 pupils have encountered the strands of number, algebra, shape, space and measures and handling data. In Key Stage 4 they will:

- take more responsibility for improving their own learning and performance;
- develop their problem-solving skills to encompass a wider range of applications, within and across the strands;
- develop their communication and reasoning skills to a higher level;
- engage in more independent in-depth work, such as coursework investigations and statistical enquiry.

For teachers, there will be new perspectives to consider when targeting pupils' development within the GCSE tiering structure, for example:

- introducing new topics to pupils in the higher tier;
- seeking new ways to re-engage foundation-tier pupils with mathematics that they may have encountered before, helping them to recover their understanding and make more connections.

The period at the end of Year 9 and the beginning of Year 10 offers an opportunity to develop pupils' key skills. Teachers will want to decide which skills and contexts are most appropriate for their pupils and to consider which projects could help them to approach their chosen route through Key Stage 4.

Selecting bridging projects for development

The table on the next page summarises four possible bridging projects in mathematics. Several deliberately build on existing Strategy materials. You will want to select ideas carefully, taking account of what you have done in Year 9 and the time available. Consider whether you already incorporate bridging activities in your current scheme of work. If so, is there value in reviewing what you currently do? If you do not yet use bridging projects, is there value in developing one or more of the projects outlined here? You might find it useful to copy the summary table and use it as a starting point for discussion with colleagues in your department.

The last section of the booklet gives details of each project and lists necessary resources so that you can scan the suggested projects to clarify what they are offering and check whether you have, or can obtain, any materials needed.

Mathematics bridging projects

Project	Title	Time required	Summary	Page
1	Pupil self-assessment	Year 9: 1 hour Year 10: 30 minutes	<p>Pupils should be encouraged to take increasing responsibility for their own progress as learners. This means knowing what they do well and what they could do better.</p> <p>This project adopts an interesting and illuminating approach to involving pupils in analysing their Key Stage 3 test papers and assessing strengths in number, algebra, shape, space and measures and handling data.</p> <p>A brief record from this work could be the start of a personal progress folder, to be developed throughout Key Stage 4.</p>	10
2	Making connections – proportional reasoning	Year 9: 2 hours Year 10: 30 minutes	<p>This project involves teachers and pupils reflecting together on links across the strands of mathematics.</p> <p>It takes advantage of the fact that the Key Stage 3 PoS has been tested recently and should be at the front of pupils' minds.</p> <p>Making connections sheds new light on the coherence of mathematical concepts and may reveal areas of misconception. It should encourage pupils to consider big ideas in mathematics, which will help them to make sense of Key Stage 4.</p>	13
3	Projects in handling data	Year 9: 6 hours Year 10: 2–3 hours	<p>Completing a coursework project on handling data is now a requirement of GCSE courses. To be successful, pupils need to address all stages of the handling data cycle and to produce more extended writing, describing the decision-making process and explaining their reasoning.</p> <p>Through carefully guided work in small groups, with the teacher modelling each stage of the process, this project prepares pupils for independent work in Key Stage 4.</p>	16
4	Developing geometrical reasoning	Year 9: 6 hours Year 10: 6 hours	<p>There is an increased emphasis at GCSE level on geometrical reasoning, with pupils expected to be able to write clear reasons for the steps in an argument.</p> <p>This project suggests how to help them meet the challenges involved, at the level appropriate to the needs of different groups. It builds on a Year 9 unit in the <i>Interacting with mathematics in Key Stage 3</i> series, with ideas for developing into Key Stage 4.</p>	21

Implementing your bridging plans

Working with your department

Assuming you are following the guidance on page 3, 'How to use this booklet', and on page 6, 'Selecting bridging projects for development', there are a number of other issues to consider when implementing your plans.

Actively following up in Year 10 the developments initiated in Year 9 should ensure continuity and progression – the essential purpose of your bridging plans. Whether your bridging projects are located mainly towards the end of Year 9, split evenly between Years 9 and 10, or mainly at the beginning of Year 10, there needs to be an explicit link between the key stages, so that pupils can see that they are developing what they have begun.

Discuss with colleagues how you will overcome potential obstacles to this process. For example, there may be significant changes in pupil groupings and staff allocations between Year 9 and Year 10.

- How can you ensure that these changes will not inhibit what you do and that all pupils gain full benefit from your bridging course plans?
- Do your plans involve all pupils in the year group?
- What records or notes will need to be kept by pupils and teachers?
- How will you round off the work in Year 9 and re-engage with it in Year 10?

Detailed planning, which might be delegated to a smaller group of colleagues, might raise various questions to consider.

- What adaptations will you need to make to your scheme of work in order to incorporate the selected projects?
- What new material or adaptations to existing material do you propose to include?
- How will you ensure that teachers are briefed and that resources are prepared in time?

Networking within your school

Effective implementation of plans needs to involve your school's senior leadership team. Support from the leadership team might include, for example:

- coordinating bridging plans across subjects;
- fostering development of pupils' learning skills as a whole-school focus through transition to Key Stage 4.

You might seek the assistance of the leadership team in making arrangements such as:

- teacher release to develop ideas;
- timetable changes towards the end of term;
- increased access to computer facilities.

You might also want to liaise directly with another subject department:

- to work on a joint initiative, for example a handling data project with the geography department;
- to seek support, for example by negotiating with the PE department to collect data in a form suitable for detailed analysis by mathematics classes at a later date.

Networking with other schools and the LEA

There are considerable advantages to setting up or linking into local development groups to pool ideas and perhaps develop shared materials. This is an effective way of making best use of local capacity. Possibilities to consider are:

- linking with one or more local departments to initiate a joint development;
- linking into LEA facilities and networks by working with your Key Stage 3 consultant and keeping them informed of developments.

Project 1: Pupil self-assessment

Learning outcomes

In this project, pupils should become more aware of the strands of mathematics and their relative strengths in each strand. From the simple profile that pupils build up, they can review topics to be taught in the following term and assess where they may need to pay particular attention or seek more support. The project develops the key skill **Improving own learning and performance**.

Structure of the project

Teachers/technician: classifying questions and preparing spreadsheet	After the Key Stage 3 tests, teachers complete a simple analysis of questions on each paper, classifying them according to the strand that they principally address: number, algebra, shape, space and measures or handling data. A technician could set up the spreadsheet, perhaps adapting one used the previous year.	2 hours to prepare
Year 9 pupils: entering data	On the return of the papers, pupils complete their individual entries in the spreadsheet. They enter a response based on their own judgement and marks on the test paper.	About 5 minutes per pupil
Year 9 pupils	On completion of data entry, each pupil is provided with a print-off of their sheet. The teacher and class draw out common issues identifying the topics in which everyone needs to improve. Pupils then individually highlight particular points for their own attention, which will be entered in a personal progress folder.	1 hour
Year 10 pupils	At the beginning of the autumn term, the Year 10 teacher discusses an outline of units of work for the term. Using their personal progress folder, pupils highlight units of work that are especially important for them.	30 minutes

Background to the project

Pupils are curious to see some of the detail of how their test papers have been marked. Teachers know the value of reflecting on learning, test preparation and test performance. This project aims to avoid the drearier aspects of test analysis and get pupils involved in a lively and stimulating way, making the experience feel a little different and giving it a fairly light touch. The project, as developed in Thurrock LEA:

- uses a simple computer spreadsheet for pupils to record their successes;
- keeps the question analysis simple, not going into too much detail;
- ensures that pupils can see benefits to their work in Key Stage 4.

Teachers who used the 'Test Visualiser Toolkit' commented:

'Pupils felt part of the process. They made choices and thought that their views were regarded as important. They were keen to update the record as they felt they made progress later in the term.'

'It made me think about how I taught certain topics. I've made changes for this year and already I'm being more effective.'

'It really showed which strands we needed to focus on and which units we needed to revisit.'

Managing the project in Year 9

When classifying questions, it will be simpler if no question is double classified and the 'using and applying' strand is not identified. A spreadsheet is then easy to prepare and complete. An example can be found on the Thurrock LEA website (under Resources). It requires pupils to judge whether they confidently scored half marks or more on each question. In fact, they may choose to discount marks allocated if they know that their answer was a guess. Each question confidently answered is recorded as a smiley face, colour-coded according to the mathematical strand addressed. The completed sheet gives an immediate visual cue to the types of question where pupils' knowledge is secure and those where it is not. Pupils respond positively to this task, which is quickly completed.

1			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
2	Possible mark		2	2	4	2	2	4	3	5	5	4	3	4	4	3	3	4	3	3
3	Names																			
4			☺	☺	☺	☺	☺	☺	☺	☺	☺	☺	☺	☺	☺	☺	☺	☺	☺	☺
5																				
6	BILLIE	BLACKMAN	☺	☺	☺	☺	☺		☺					☺		☺				
7	MICHELLE	BELLER	☺		☺	☺			☺	☺			☺						☺	
8	MANJEET	BHULLAR			☺		☺		☺	☺	☺				☺					
9	ROBBIE	CARD	☺	☺	☺	☺	☺	☺	☺	☺	☺			☺	☺			☺	☺	☺
10	ADAM	D'LANG			☺		☺	☺	☺	☺			☺	☺						

The lesson based on the test analysis should first identify topics in which everyone needs to improve. When considering their own performance, pupils should be encouraged to appreciate their successes before addressing development points, using prompts such as the following:

Points for pupil reflection	Implications for learning
Is this a topic I really do not understand?	Where do I need to recover my understanding in Year 10?
Did I forget important facts during the test even though I understand the topic?	In what ways could I have revised more effectively?
Did I make a careless mistake in the test even though I understand the topic and know all the important facts?	What aspects of my test technique could I improve?

This lesson could be the start of a personal progress folder for each pupil. In it they should make specific personal notes of the implications for their learning.

There are also points for you to consider as a teacher:

Points for teacher reflection	Implications for teaching
Which topics do a substantial number of pupils not fully understand?	Which Year 10 units need to be emphasised in order to address emerging difficulties for these pupils? Which Year 9 units need redevelopment in order to prevent similar difficulties emerging next year?
Are a substantial number claiming to have problems in remembering important facts for the test?	What revision could be included in starters of lessons, leading up to the next assessment? What actions would support more thorough revision for Year 10 tests?
Are a substantial number of pupils revealing poor test technique?	Where could past examination questions be included in plenaries of Year 10 lessons? What whole-school opportunities could be used to focus on test technique?

DfES software, *Pupil Achievement Tracker*, is a more sophisticated tool for teacher analysis of test performance, further supported by the QCA Standards Report. As an adjunct to this project, in the autumn term you might carry out further analysis and revisit the implications for teaching.

Developing the project in Year 10

It is important to return to pupils' progress folders at the beginning of Year 10. The following tasks need not take more than half an hour.

- Refer pupils to their individual profiles, completed at the end of the previous term, and remind them of what they did.
- Provide brief details of teaching units for the first term of Year 10.
- Talk through the list with the class, making sure everyone understands what aspects of mathematics will be covered.
- Provide an opportunity for pupils to identify and make notes on topics where they need to pay particular attention or seek more support.

Personal progress folders could be developed by pupils during Key Stage 4 for analysing performance, setting targets in particular topics, devising and recording challenging revision questions and planning techniques for revision.

Resources

- Pupils' Key Stage 3 mathematics test papers
- A simple spreadsheet such as the Visualiser tool from Thurrock LEA: www.igfl.org/lift

Project 2: Making connections – proportional reasoning

Learning outcomes

By engaging with these activities pupils should become more aware of their mathematical thinking. They are given a chance to ‘think out loud’, articulating the connections they make. The activities encourage them to consider the validity of the links they make and, when challenged, to reconsider connections. This project develops the key skills of **Communication** and **Working with others**.

Structure of the project

Teachers: selecting an activity	As a department, teachers work together on the card classification exercise from the detailed lesson plans in <i>Thinking proportionally</i> in Year 8 (see resource list for details). They then make decisions on adaptations and preparations needed to run the lessons with the current Year 9.	1 hour to prepare
Year 9 pupils	Pupils work together on the card classification exercise about ratio and proportionality. This is used as the basis for reflecting on the connections they have made.	2 hours
Year 10 pupils	In an early lesson, pupils consider a summary description of topics to be covered in the autumn term of Year 10. They are reminded of discussions the previous term and asked to speculate about where they will be likely to apply their proportional reasoning.	30 minutes

Background to the project

Proportional reasoning is a major theme in the *Interacting with mathematics in Key Stage 3* series (see resources list), illustrating progression in the teaching and learning of a key aspect of mathematics through Key Stage 3. It is a rich area of mathematics for making links because:

- there is a web of connections between the concepts and notations associated with fractions, decimals, percentages, ratio and proportion;
- the concept of proportionality is applicable not only in number, but also in algebra, geometry and handling data.

This project is based on the Year 8–9 transition lessons *Thinking proportionally* (Year 8), designed originally to assess pupils’ progress in understanding proportionality towards the end of Year 8. With a slight shift of focus, attention can be concentrated on developing pupils’ awareness of the connections described above.

The lessons are based around a challenging card classification exercise. Classification tasks are particularly useful for developing pupils’ thinking skills. Sorting and classifying is a natural process by which we make sense of things.

Reasoning is involved when justifying categories that are chosen or links that are made. When pupils do this for themselves, or in collaboration with others, concepts and ideas become more meaningful and are therefore more likely to be understood and remembered.

Some comments from teachers on the proportional reasoning mini-packs:


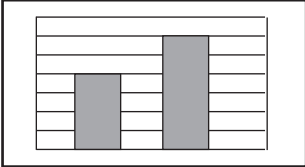
'The lessons stimulated a lot of discussion and thinking and promoted real understanding. My pupils have a vastly improved understanding of the effects of multiplying by numbers greater than or less than 1.'

'The approach really did help pupils to grasp the idea of proportionality. Once they properly understand the concept they can apply it to so many areas of mathematics and I am much more aware of bringing these ideas into other units.'

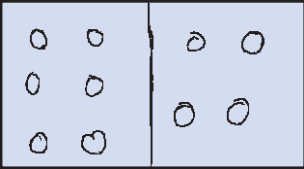
'The Year 9 proportional reasoning mini-pack definitely helped us with the contents of the Year 8 mini-pack. The progression became much clearer, and then the Year 7 pack has clarified this even more. Pupils too have enjoyed approaching the work in this way.'

Managing the project in Year 9

If you have not used the lesson plans before then the first task is to print them from the web. Arrange to have sets of cards duplicated and cut up (you may decide to laminate them). To understand the richness of this task it is a good idea to try it out. Engaging members of the department with the classification task can provoke much debate! By selecting appropriate cards, the task can be adapted to provide a suitable challenge for different teaching groups.

$\frac{2}{3}$			<p>A shop has a special offer on chocolate bars: 'Buy two, get one free!'</p>
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If you have used the lesson plans with pupils when they were in Year 8, it is possible to extend the task as described in the 'extension' section of the lesson plan. Asking pupils to create their own cards to fit specific groupings of cards is challenging and develops creative thinking.

	$3x + x : 3x(2)$	<table border="1" style="border-collapse: collapse; text-align: center;"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>20</td> <td>30</td> </tr> <tr> <td>1</td> <td>1.5</td> </tr> <tr> <td>$\frac{2}{3}$</td> <td>1</td> </tr> <tr> <td>...</td> <td>...</td> </tr> </tbody> </table>	x	y	20	30	1	1.5	$\frac{2}{3}$	1	<p>A person buys a chocolate bar at £1.50. Another person buys a chocolate bar at £2.25</p>
x	y												
20	30												
1	1.5												
$\frac{2}{3}$	1												
...	...												

Mathematical issues arising from the discussions might include:

- pupils' perception of 'part : whole' or 'part : part' interpretations of ratio. Pupils should justify their links. The 'picture' cards may help to illustrate their thinking if they struggle to explain in words;
- pupils using additive thinking, rather than the multiplicative relationships illustrated on the cards.

Developing the project in Year 10

The follow-up in Year 10 is important, but need not take up more than half an hour. If you are developing the pupil self-assessment project, this would fit in very well as a sequel. Having discussed the strands of mathematics, then consider where ideas of proportionality might arise in different topics.

Plan to incorporate the follow-up in a lesson during the first week of the autumn term in Year 10.

- Remind pupils of the card classification task and discussion about ratio and proportion towards the end of the previous term. Emphasise the connections made, particularly between different forms of mathematical notation and to applications within different strands of mathematics.
- Provide pupils with a list of topics to be covered in the autumn term of Year 10 – a little more than just topic headings, but not too much detail. Talk through the list, clarifying points as appropriate.
- Encourage the class to speculate. Which topics are likely to involve proportional thinking? Why? Can they be more specific about how proportion might be involved?
- Allow time for pupils to make brief notes of their ideas in their personal progress folders, if they have them. During the term, invite pupils to draw attention to any links they make.

Resources

Key Stage 3 National Strategy publications:

These publications are also available from the website: www.standards.dfes.gov.uk

- *Framework for teaching mathematics: Years 7, 8 and 9* (DfES 0020/2001)
- *Interacting with mathematics in Key Stage 3 mini-packs:*
 - Year 7 fractions and ratio (DfES 0093/2003)
 - Year 8 multiplicative relationships (DfES 0220/2002)
 - Year 9 proportional reasoning (DfES 0588/2003)
- *Interacting with mathematics in Key Stage 3: Enhancing the teaching of proportional reasoning* (DfES 0093/2003)
- *Thinking proportionally* Year 7–8 and Year 8–9 transition lessons (available only from the website)

Project 3: Projects in handling data

Learning outcomes

The focus of this project is the handling data cycle, with particular reference to the skills of decision-making at each stage. It seeks to give pupils experience of producing a high-quality written report as part of a small-group project. It develops the key skills of **Communication** and **Working with others**.

Structure of the project

Year 9 pupils: producing a report	Pupils are systematically guided through the stages of preparing a handling data report. The report seeks to answer a 'big question'. Pupils are engaged in collaborative research in small groups to produce a combined report.	6 hours
Teachers: preparing a 'mystery' for Year 10	Following the Year 9 work, teachers meet and choose reports to use in Year 10. This may be only one or two reports duplicated, cut up, shuffled and placed in envelopes for pairs of pupils to work with. In addition, and based on the same data, teachers produce: <ul style="list-style-type: none">■ data that is processed in a way that does not help to answer the question;■ a couple of spurious interpretations of graphs or charts.	2 hours to prepare
Year 10 pupils: solving the 'mystery' and assessing quality	Pairs of pupils are presented with the challenge of collating and making sense of one of the selected 'mystery' reports. Finally, pupils are guided in using GCSE-type criteria to make suggestions for improving the report.	2–3 hours

Background to the project

The project builds on approaches to handling data developed in two previous sets of training and resources in the *Interacting with mathematics in Key Stage 3* series:

- *Year 8 Handling data*: a mini-pack made available through training from summer 2002. Note in particular:
 - the explicit attention given to the particular challenges at each stage of the handling data cycle (poster included in the pack);
 - the extensive data library on CD-ROM;
 - a specific activity (M3.5 Reports) involving sorting and ordering component parts of two statistical reports.
- *Securing progression in handling data*: an element of core training and supporting resources available from spring 2004. Note in particular:
 - a focus on specifying a problem, planning and collecting data: starting handling data activities in the classroom; supporting pupils' planning; using secondary data, including the Y8 CD-ROM; designing questions;

- a focus on interpreting data and explaining reasoning, including a small-group activity on drafting and revising written explanations (prompts for Year 9: Revising explanations).

Some comments from teachers on the *Year 8 handling data* unit:

'The task (first lesson of the unit) is one I would normally give to Year 10. I was impressed by the thought processes that it engendered in pupils. If we follow this approach through it will have a positive impact on statistical work in Key Stage 4 and at sixth-form level.'

'Use of the Year 8 handling data mini-pack is an excellent preparation for GCSE data handling coursework, especially the emphasis on interpretation within the cycle.'

Managing the project in Year 9

In this project, pupils work in small groups, with the teacher modelling the processes they need to go through at each stage of the handling data cycle. To enable whole-class discussion throughout the project:

- choose one context or source of data within which all groups will work;
- allow only a restricted range of questions to be considered.

For example, you might choose an accessible and relevant website, such as CensusAtSchool. Or you could liaise earlier in the year or key stage with another department to collect a quantity of relevant data, for example weather statistics in the geography department or athletics data in the PE department.

At each stage of the handling data cycle, pupils should be supported in making and justifying decisions about what they are going to do before they embark on the detailed work. Allow time for pupils to express their own ideas, make firm choices, justify their choices and organise tasks for their small group to complete. Here is a suitable plan to follow:



Generate ideas: In groups, make a free choice, supported by a clear description of what makes a good choice. Make brief notes of decisions and reasons to inform whole-class discussion.



Make choices: Discuss choices as a class and make a selection from a small number of alternative strategies or ideas. Groups discuss and begin to compose the next step of the plan.



Give reasons: In groups, make a final decision and agree a group statement describing the tasks, giving reasons for the decision. This statement is part of the report.



Complete tasks: Delegate roles and implement the steps decided. For each stage of the handling data cycle the following questions and structure will help to guide pupils' decisions.

Specifying the problem and planning

- What is the question you are aiming to answer or problem you are aiming to investigate?
- How can the question/problem be tackled?
- Where and how might you find the data?
- How much data might you need and what is the best way of getting it?
- Can you foresee any problems with the plan and do you have any strategies to cope with these?



Ideas: Think of a question/problem.



Choices: Make a choice from a small number of questions/problems.



Reasons: Write down the reasons for your choice of problem/question. This is the first part of your report.



Tasks: Write a short plan for various stages of the problem. Agree the best wording. This is the second part of your report.

Collecting, processing and representing data

- How will you collect data so that it is relevant to the question/problem?
- How will you present and organise it so that it is clear?
- Which diagrams will you use so that they help answer the question?
- What calculations will you carry out and how will these help with the problem?



Ideas: Decide how to collect and organise the data.



Choices: Make a choice from a small number of sources and possible ways of organising.



Ideas: Decide on the kinds of diagram that would be useful.



Choices: Make a choice from a small number of diagrams.



Ideas: Decide on the calculations that would be useful.



Choices: Make a choice from a small number of calculations.



Reasons: Write down reasons for your choices of collection, diagram and calculation and outline what you are going to do. This is the third part of your report – make sure you agree on wording.



Tasks: Decide who is going to do what and begin the task of collecting, processing and representing the data.

Interpreting and discussing results

- How will you describe the patterns you notice in the data?
- How will you use the diagrams and calculations to help address your original question/problem?
- How sure are you that your conclusions are accurate?
- How good has your strategy been?



Ideas: Decide how you will use the diagrams and calculations to help address the question/problem.



Choices: Make a choice from a few different ideas on how to use the diagrams and calculations.



Reasons: Write down the reasons for choosing the points you plan to draw out of the data. This is the part of the report where you pull together the diagrams and calculations to be sure you are addressing the problem.



Tasks: Decide who is going to do what and begin the task of interpreting and discussing the data. Make sure you agree a final statement that reflects on how well you have addressed the question/problem.

Developing the project in Year 10

Prepare one or two selected pupil reports, following the guidance set out in the table on page 16. With highlighted examples added by you, the minimum set should include cards with:

- a 'big question' or statement;
- an outline plan with explanation;
- a plan for data collection and organisation with reasons for that choice;
- the data collected and organised as planned;
- the data organised differently from the plan with no reasons given;
- a plan for diagrams to be used with reasons for that choice;
- the diagrams as planned;
- other diagrams to those planned, accurate but irrelevant;
- a plan for calculations to be used with reasons for that choice;
- the planned calculations;
- some unplanned, accurate but irrelevant calculations;
- interpretive statements linking the data, diagrams and calculations;
- statements that cannot be supported by the data, diagrams or calculations shown;
- conclusions relating to the initial problem/question;
- accurate but irrelevant conclusions;
- reflective comments on the limitations and successes of the report;
- ideas for improvement.

Remind pupils of the report they constructed in Year 9. Recap on the stages of the handling data cycle, emphasising that decision-making and justification were a key part of the way they worked. Remind pupils that it was crucial that each part of the report was relevant to the question or problem.

Organise pupils into pairs and distribute the cards. The pupils should draw together those cards that address the question/problem. They should then order them into a coherent report.

Once the report is agreed pupils should consider how it could be improved. One way of reviewing it is to use the questions (above) under each stage of the handling data cycle. Alternatively, prepare a *simplified* form of the GCSE marking criteria and ask them how the report could have gained more marks.

Resources

- GCSE handling data coursework guide and marking criteria from examination boards:

AQA: www.aqa.org.uk

Edexcel: www.edexcel.org.uk

OCR: www.ocr.org.uk

- CensusAtSchool project:

www.censusatschool.ntu.ac.uk

- Strategy publications:

- *Framework for teaching mathematics: Years 7, 8 and 9* (DfES 0020/2001)
- *Interacting with mathematics in Key Stage 3: Year 8 handling data mini-pack* (DfES 0220/2002)
- *Interacting with mathematics in Key Stage 3: Securing progression in handling data* (DfES 0656/2003)
- *Literacy in mathematics* (DfES 0035/2004)

Project 4: Developing geometrical reasoning

Learning outcomes

This project develops pupils' ability to visualise and refine the language needed to describe their images. Pupils are also introduced to deductive argument and taught how to write short logically ordered explanations of their reasoning. It develops the key skills of **Problem solving** and **Communication**.

Structure of the project

Core groups	Year 9	Pupils begin the geometrical reasoning unit, with some support to ensure that the pitch of the work is appropriate.	6 hours
	Year 10	Pupils revisit ideas in the geometrical reasoning unit and are expected to work independently or in pairs to produce personal records of the proofs required. Visualisations are more complex, perhaps extending into 3-D. Support structures are reduced or removed, for example angle fact cards are no longer used.	6 hours
Extension groups	Year 9	Pupils work on the full geometrical reasoning unit.	6 hours
	Year 10	Pupils revisit the approach used in the geometrical reasoning unit with some harder examples chosen from the problem bank and an expectation of more formally accurate written proofs. Then they extend to some new visualisations and build-ups, leading to proofs of the circle theorems encountered on the GCSE higher paper.	6 hours

Background to the project

This project outlines a differentiated approach to developing pupils' skills in geometrical reasoning, considering the needs of pupils working below or just at level 6 (core groups) and those working comfortably within or beyond level 6 in Year 9 (extension groups). It is based on the *Year 9 geometrical reasoning mini-pack* from the *Interacting with mathematics in Key Stage 3* series, made available through training from spring 2003.

Many teachers are already using ideas stimulated by the mini-pack with pupils in Years 7 and 8. In particular, visualisations as a way of developing language and argument are easily adaptable. The Framework Yearly teaching programmes and Supplement of examples are useful to ensure that expectations are adjusted appropriately. Although the detail is not described here, a similar adaptation would be possible for lower-attaining groups in Years 9 and 10 (see Buckinghamshire LEA website, under Resources).

Particular features of the geometrical reasoning unit to note are the use of:

- visualisation and reconstruction exercises, to develop awareness of spatial relationships and associated language;
- 'build-ups', simple line diagrams in the form of overlays, to illustrate how 'new' facts can be established from 'given' facts;
- careful scaffolding of problem solving, where 'given' facts are stated as justification in logically ordered explanations and proofs.

Some comments from teachers on the Year 9 geometrical reasoning unit:

'After we had worked on the visualisations I was amazed at how much more easily my pupils could see through the complexities of a diagram to the heart of the geometrical problem.'

'The Y9 geometrical reasoning mini-pack used after the tests was a very good way to consolidate skills and then to extend them to "proof" for GCSE. It was great to see Key Stage 3 pupils so involved in considering proof!'

A comment from a Year 9 pupil:

'Now I find it much easier to see what the problem is about and have a better idea of what it means by "Give your reasons".'

Managing the project in Year 9

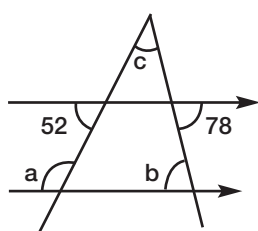
Located towards the end of Year 9, the geometrical reasoning unit demands a level of thinking that is appropriately challenging for pupils in the **extension group**. However, it is envisaged that as standards rise over time, some of the content will eventually be placed earlier: some schools are already incorporating visualisations and simple 'build-ups' in earlier years.

The section of the geometrical reasoning mini-pack on differentiation (pages 5 and 6) describes ways of making the activities in the unit more accessible to pupils in the **core group**. Mathematics consultants and teachers in Buckinghamshire LEA found the following adaptations particularly helpful:

- take extra time over the visualisations;
- provide pupils with drawing tasks to practise using the language and notation developed, for example:

Draw a triangle ABC. Extend the side AB beyond B and mark the exterior angle at B. Draw a straight line BD that bisects the exterior angle at B. You are given that the line BD is parallel to AC. Mark up the facts you are given.

- give pupils practice in finding unknown angles on simple diagrams:



- support by providing angle facts on card for pupils to use as they solve simple problems, for example:

Angle fact 1

Angles on a straight line add to 180°

This helps pupils to be more aware of the properties they can consider in their reasoning;

- model the proofs and work in detail on language and notation. Expect pupils to appreciate the proofs and contribute to the recording.

Developing the project in Year 10

Core groups should revisit ideas in the geometrical reasoning unit in Year 10. Following the suggestions set out in the table on page 21 ensures appropriate progression. In particular, modelling each stage of the proof for the whole class should be gradually phased out. Pupils should make more decisions about the steps required and what given facts are needed. They should work more independently and assess one another's written proofs in order to improve the rigour of their work.

Extension groups might first recap on the geometrical reasoning unit by solving some of the harder problems and writing rigorous proofs. You might then consider some new material that has developed out of the unit. Prompted by the positive response of Year 9 pupils to the visualisations and build-ups, some teachers in Cambridgeshire LEA worked with their LEA mathematics team to extend the ideas. The materials they produced (see resource list) take the same form as the activities in phase 1 of the geometrical reasoning mini-pack and are designed to prove some of the circle theorems pupils meet at GCSE on the higher paper.

Here are some examples, to give a flavour of the materials:

Year 10 visualisations

The visualisations are designed to be used as starters to generate thinking about the geometrical properties of circles. It is best if pupils are encouraged to close their eyes during these activities, as it tends to remove distraction and assist visualisation of the images. In the reconstruction of these visualisations pupils should be encouraged to talk about their images without using hands or pencil and paper.

Visualisation example: tangents to circles

Imagine a circle. Imagine a radius of that circle. Now imagine a line that intersects the radius at right angles and that intersects the circumference of the circle in two places. How many right angles can you see? *Very slowly* slide the line away from the centre of the circle, always keeping it perpendicular to the radius. Stop sliding your line when it gets to the end of the radius. How many times does it intersect the circle now? How many right angles can you see now?

Year 10 build-ups

It is suggested that teachers and pupils work through the build-ups in order. When a circle theorem is proved using the build-ups, it can be added to the list of givens that pupils already have. Only three givens (shown below) are needed to start these build-ups and all should be familiar to pupils from geometrical work in Years 7 to 9. This may be a good opportunity to remind pupils of other givens that have been proved in the Year 9 mini-pack (e.g. corresponding angles, alternate angles, vertically opposite angles).

Givens:

- angles on a straight line sum to 180° ;
- angles in a triangle sum to 180° ;
- angles opposite the equal sides of an isosceles triangle are equal.

Build-up 1: the angle at the centre is double the angle at the circumference

(Figures are illustrated here as thumbnails only and figure 5 is not shown. Note that in each case the centre of the circle is marked with a small x.)

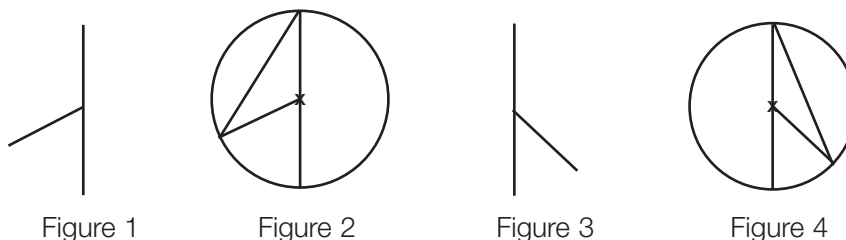


Figure 1. Mark the acute angle shown (p).

Figure 2. Slide it over figure 1. Use the givens about the isosceles triangle to mark the angles in the triangle ($180 - p$, $\frac{1}{2}p$, $\frac{1}{2}p$). Remove figures 1 and 2.

Figure 3. Mark the acute angle shown (q).

Figure 4. Slide it over figure 3. Use the givens about the isosceles triangle to mark the angles in the triangle ($180 - q$, $\frac{1}{2}q$, $\frac{1}{2}q$).

Slide figure 2 over figure 4. Can replace with a freeze frame using figure 5. Show that angle at the centre is $p + q$, whereas angle at the circumference is $\frac{1}{2}(p + q)$. So the angle at the centre is double the angle at the circumference.

The Cambridgeshire materials include descriptions of build-ups 2 and 3, for angles in the same segment and angles in a semi-circle, and a set of fifteen build-up diagrams, of a suitable size for copying and making into overlays.

Resources

- Key Stage 3 Strategy publications:
 - *Framework for teaching mathematics: Years 7, 8 and 9* (DfES 0020/2001)
 - *Interacting with mathematics in Key Stage 3: Year 9 geometrical reasoning mini-pack* (DfES 0588/2002)
- Cambridgeshire LEA geometry materials (www.ccceducation.net)
- Buckinghamshire LEA geometry materials (www.bucksgfl.org.uk)

Bridging plans: from Key Stage 3 to Key Stage 4

- Do you have specific plans for lessons that bridge between the end of Key Stage 3 and the beginning of Key Stage 4?
- Do you plan units of work that give Year 9 pupils a fresh learning experience after the national tests in May?
- Do your pupils have a sense of a new beginning when they start Key Stage 4 courses?
- Do you develop pupils' expectations for future learning in your subject?
- Do you take the opportunity to enhance pupils' skills as independent learners?

Giving attention to these questions can ensure that Year 9 pupils receive a fresh stimulus at the end of Key Stage 3 and that their work links effectively with the challenges of Key Stage 4.

This booklet is one of an initial set of three, focused on the core subjects of English, mathematics and science. The aim is to capture and make best use of time at the end of Year 9 and into Year 10. Each booklet:

- suggests a common approach to fostering pupils' development as independent learners;
- identifies aspects of the chosen subject that might be developed to improve transition from Key Stage 3 to Key Stage 4.

The maximum benefit will be found where senior managers lead and support these developments across the school.

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