Shape, space and measures 4

contents

There are three lessons in this unit, **Shape, space and measures 4**.

objectives

The objectives covered in this unit are:

- Visualise and describe 2-D shapes.
- Understand and use the language associated with reflections and rotations.
- Recognise and visualise the transformation and symmetry of 2-D shapes:
  - reflection in given mirror lines, and line symmetry;
  - rotation about a given point, and rotation symmetry.
- Solve problems and investigate in shape, space and measures.
- Explain reasoning with diagrams and text.

Using the lesson plans in this unit

These lesson plans supplement the *Springboard 7* materials for Key Stage 3 pupils working toward level 4 in mathematics. All the lessons are examples only. There is no requirement to use them. If you decide to use the lessons, you will need to prepare overhead projector transparencies (OHTs) and occasional resource sheets for pupils to use.

The lessons consolidate work at level 3 and extend into level 4. They are suitable for a group of pupils or a whole class. Whatever the size of the group, the pupils are referred to as 'the class'.

Each lesson will support about 30 to 40 minutes of direct teaching. To help match the time to your timetable, each plan refers to 'other tasks' for pupils, based on *Springboard 7* resources. Select from these, textbook exercises or your own materials to provide practice and consolidation in the main part of a lesson and to set homework. Aim to choose tasks that vary in their level of demand, to suit pupils' knowledge, confidence and rate of progress.

Although the 'other tasks' are listed for convenience at the end of the main part of the lesson, they can be offered at any point, especially between the 'episodes' that form the main activity.

The lesson starters are of two kinds: practice starters and teaching starters. The former are opportunities to rehearse skills that will be needed later in the lesson. Teaching starters introduce an idea that is then developed in the main activity.

You will need to tell pupils what they will learn in the lesson, either in the starter or at the beginning of the main activity. Use the plenary to check pupils' learning against the lesson's objectives and to draw attention to the key points that pupils should remember.

Interactive teaching programs (ITPs)

Interactive teaching programs are interactive animated visual aids that can be used with a laptop and data projector or with an interactive whiteboard. As extra support for this unit, you may find it useful to download and use these ITPs from the website www.standards.dfes.gov.uk/numeracy:

for lesson S4.1 *Polygon*

for lesson S4.2: *Symmetry*

Acknowledgement

The main activity in S4.1 is based on a lesson provided for the Key Stage 3 Strategy by The Mathematical Association and the Association of Teachers of Mathematics. For this lesson, you will need a computer suite with a data projector and Logo software.

S4.1 Rotation and rotation symmetry

objectives

- Visualise and describe 2-D shapes.
- Understand and use the language associated with rotations.
- Recognise and visualise rotation about a given point, and rotation symmetry.
- Solve problems and investigate in shape, space and measures.
- Explain reasoning with diagrams and text.

starter

Vocabulary
rotate
rotation symmetry
order of rotation symmetry
centre of rotation
right angle

Resources
shapes made from thin card
drawing pin
ITP *Polygon* (optional)

Hold up a rectangle made from thin card.

Q What shape is this?

Pin the centre of the rectangle to the board and draw round its boundary. Explain that the drawing is the starting position. Tell the class that the pin is the centre. Demonstrate giving the rectangle one whole turn about the centre, pausing at each quarter turn: a quarter turn, a half turn, a three-quarter turn, one whole turn. Explain that this is called *rotating* the rectangle, and that the pin is the *centre of rotation*. Say that you will rotate the shape for one whole turn again. This time you want the class to count how many times it looks the same as its starting position.

Demonstrate that the rectangle will twice fit into its starting position. Tell pupils that a shape that fits into its starting position more than once during a whole turn has *rotation symmetry*. The number of times that it fits into its starting position is called its *order of rotation symmetry*. The *centre of rotation* is the point about which it rotates. Write on the board:

rectangle order of rotation symmetry 2

Repeat with other shapes (e.g. square, isosceles triangle, parallelogram, trapezium, regular hexagon), demonstrating which of them do and which don't have rotation symmetry.

Alternatively, you could use the ITP *Polygon,* downloaded from the website www.standards.dfes.gov.uk/numeracy, to demonstrate the rotation of shapes.

main activity

Vocabulary
procedure

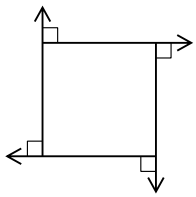
Resources
Logo
computer suite with whole-class projection
procedures for FLAG and RTRIANGLE set up in shared area of network (see lesson)
Resource S4.1a

Using a computer with a data projector, create a Logo procedure:

```
TO SQUARE
    REPEAT 4 [FD 100 RT 90]
END
```

Q Why do these instructions produce a square?

Draw out that the resulting shape will have four sides, each equal to 100 units. Draw a diagram on the board to explain that at each corner, the angle of turn is 90º. The angle inside the shape must be 90º, since angles on a straight line add up to 180º. A shape with four equal sides and four angles of 90º is a square.

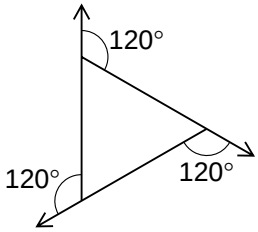


Now create a procedure to draw an equilateral triangle.

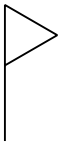
```
TO ETRIANGLE
    REPEAT 3 [FD 50 RT 120]
END
```

Q Why do these instructions produce an equilateral triangle?

Draw out that the resulting shape will have three sides, each equal to 50 units. Invite a pupil to the board to draw a diagram to explain why, at each corner, the angle of turn is 120º and the angle inside the shape is 60º. A shape with three equal sides and three angles of 60º is an equilateral triangle.



Create a Logo procedure to draw a flag.



```
TO FLAG
    FD 60
    REPEAT 3 [FD 50 RT 120]
    BK 60
END
```

Now enter: RT 90 FLAG. Ask:

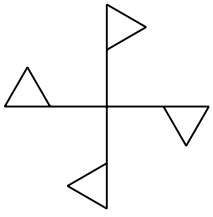
Q Can you explain what is happening?

Clear the screen, and type:

```
REPEAT 4 [RT 90 FLAG]
```

Before pressing return, ask the class:

Q Can you predict what will happen?



Q How many times did the flag shape rotate? What is the order of rotation symmetry?

Show pupils how to access the two procedures FLAG (see above) and RTRIANGLE (see below) from the shared area of the network.

```
TO RTRIANGLE
    FD 60 RT 135
    FD SQRT 7200 RT 135
    FD 60 RT 135
END
```

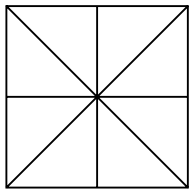
Give out copies of **Resource S4.1a**. Ask pupils to work in pairs to complete the two problems.

Bring the whole class together to share solutions to the problems.

Q What properties of the completed shapes can you identify? Which sides are equal? Which angles are equal? Are any sides parallel? Are any sides perpendicular?

Q What is the order of rotation symmetry?

Q How could we use the solution to the second problem to draw this shape using Logo?



other tasks

Springboard 7
Unit 14

Unit 14 section 4: Drawing angles
Star challenge 6: Rotating patterns page 461

You may wish to provide some further examples of rotation and rotation symmetry, including computer activities.

plenary

Resources
Resource S4.1b
tracing paper

Give out copies of **Resource S4.1b** and tracing paper. Ask pupils to complete the questions on the sheet. Encourage them to use the tracing paper to check the rotation of each shape.

Remember

- A shape has rotation symmetry if it fits on top of itself more than once in one whole turn.
- The order of rotation symmetry is the number of times that the shape fits on top of itself in one whole turn. This must be 2 or more.
- Shapes that fit on themselves only once in one whole turn have no rotation symmetry.
- The centre of rotation is the point about which the shape turns.

S4.2 Reflection and line symmetry

objectives

- Visualise and describe 2-D shapes.
- Understand and use the language associated with reflections.
- Recognise and visualise reflection in given mirror lines, and line symmetry.
- Solve problems and investigate in shape, space and measures.
- Explain reasoning with diagrams and text.

starter

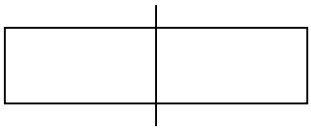
Vocabulary
line of symmetry
mirror line
reflect
congruent

Resources
paper shapes: square, rectangle, kite, parallelogram, isosceles triangle, equilateral triangle

Give each pupil a paper rectangle. Draw a rectangle on the board.

Q What shape is this?

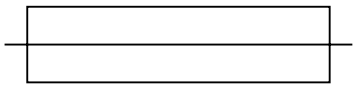
Draw a line of symmetry. Explain that this is called a *line of symmetry* and that it is sometimes referred to as a *mirror line*.



The line of symmetry divides the rectangle into two equal halves. Explain that, if the rectangle is folded along the line of symmetry, one half reflects exactly on to the other half. Demonstrate by folding a paper rectangle and ask pupils to do the same.

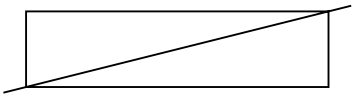
Q Does your rectangle have another line of symmetry?

Draw a second line of symmetry on the rectangle on the board.



Ask pupils to check that again the two halves are identical by opening out their rectangles and folding them along the second line of symmetry.

Draw a third line on the rectangle on the board.



Q Is this a line of symmetry?

Demonstrate that it isn't by folding the paper rectangle again. The line divides the rectangle into two identical triangles but it is not a line of symmetry since one half will not fold exactly on top of the other. Stress that the rectangle has just two lines of symmetry.

Repeat with a square, drawing four lines of symmetry, one at a time, and testing each line by folding a paper square. Stress that the square has four lines of symmetry. Repeat with other shapes, such as a kite (one line), parallelogram (no lines), isosceles triangle (one line), equilateral triangle (three lines).

main activity

Vocabulary
reflection

Resources
ITP *Symmetry* (optional)
Resource S4.2a
tracing paper
mirrors

Draw a straight line on the board. Tell the class that it is a line of symmetry or mirror line. Draw a simple shape on one side of the line. Invite a pupil to draw the *reflection* of the shape on the other side of the line. Repeat for different shapes.

Alternatively, you could use the ITP *Symmetry,* downloaded from the website www.standards.dfes.gov.uk/numeracy. Set up different shapes and ask pupils to complete the reflection in the mirror line before revealing the reflection. Make sure that some of the shapes touch the line while others do not. Use the option to vary the angle of the line. Ask pupils to respond with thumbs up or down as you ask:

Q Is this the correct reflection of the shape in the mirror line?

For either the diagram on the board, or the diagram drawn using the ITP, indicate a point on the original shape. Call this point A and ask:

Q What is the perpendicular distance from A to the mirror line?
Q What point in the reflection corresponds to this point?

Call this point A′, and ask:

Q What is the perpendicular distance from A′ to the mirror line?

Establish that the perpendicular distance of each point (A and A′) to the mirror line is the same.

Repeat for different points.

Give out copies of **Resource S4.2a** and tracing paper. Have some mirrors available. Ask pupils to complete the questions on the sheet. Get pupils to check their answers by using a mirror and placing it along the mirror line.

other tasks

Springboard 7
Unit 14

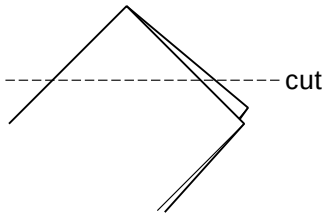
Unit 14 section 1: Line symmetry and reflection

plenary

Vocabulary
congruent

Resources
thin paper squares
scissors for each pupil

Ask each pupil to take a thin paper square and to fold it in half to make a straight line. Now fold the line along itself to make a right angle. Ask pupils to cut off a piece by making a straight cut across the right angle (the corner with two folds).



Before they open the piece they have cut off, ask:

Q What shape will your piece be when it is unfolded?

Q How many sides will the shape have? Are these sides the same length? Explain how you know.

Establish that the shape must be a square or a rhombus, because it will have four equal sides, which are folded exactly one on top of the other.

Q What size will the angles be?

Establish that, if the cut across the folded paper was made at 45º, then each angle at the corner of the shape must be double 45º, or a right angle. Otherwise, the shape will be a rhombus.

Unfold the shape.

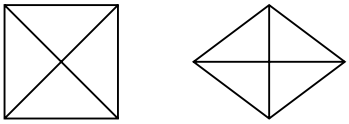
Q Does your shape have symmetry? How many lines of symmetry does it have? (square: 4; rhombus: 2)

Q What is the order of rotation symmetry of your shape? (square: 4; rhombus: 2)

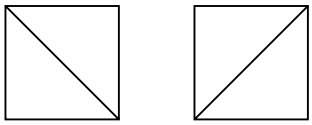
Explain that when shapes are identical, we say that they are *congruent*.

Q How many congruent triangles can you see?

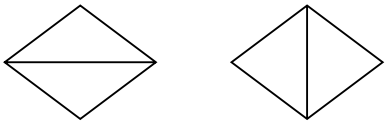
Establish that in each of the square and the rhombus there are four small right-angled congruent triangles, each equal to one quarter of the area of the whole shape.



The square has four more congruent right-angled triangles, each equal in area to half of the square, formed by folding the shape along its diagonals.



When a rhombus is folded along a diagonal, a pair of congruent isosceles triangles is formed, equal in area to half of the rhombus.



Remember

- A line of symmetry divides a shape into two identical parts. Each part is a reflection of the other. A line of symmetry is sometimes called a mirror line.
- If you fold a shape along a line of symmetry, each part fits exactly on top of the other.
- Some shapes have more than one line of symmetry. For example, a square has four lines of symmetry.
- Each point in a reflected image is the same perpendicular distance from the mirror line as the corresponding point in the original shape.
- When shapes are identical, we say that they are congruent.

S4.3 Problem solving

objectives

- Recognise and visualise the transformation and symmetry of 2-D shapes:
  - reflection in given mirror lines, and line symmetry;
  - rotation about a given point, and rotation symmetry.
- Solve problems and investigate in shape, space and measures.
- Explain reasoning with diagrams and text.

starter

Vocabulary
reflection
mirror image
mirror line
rotation
order of rotation
symmetry

Resources
OHT S4.3a
squared paper

Show **OHT S4.3a**. Tell the class that the straight lines represent mirrors. Invite different pupils to draw the reflections of the letters in the mirror lines. Stress that each point on a letter must be the same distance from the mirror line as the corresponding point in its reflection.

Point out that the letter A looks the same when reflected in a vertical mirror line. Ask pupils to work in pairs. Give each pupil a piece of squared paper.

Q Which capital letters have a mirror image that looks the same as the original letter when it is reflected in a vertical mirror line?

Encourage the pairs to visualise and to discuss their ideas before drawing the letters on paper. Allow a few minutes for the pairs to investigate, then establish that the letters with this property are A, H, I, M, O, T, U, V, W, X and Y, and that each of these letters has a vertical line of symmetry.

Now draw on the board a large letter F, and its rotation through 180°. Explain that the letter F does not look the same when rotated through 180°.

Q Which capital letters look the same as the original letter when they are rotated through 180º about their centre point?

Again encourage visualisation and discussion before pencil and paper are used. Establish that the letters with this property are H, I, N, O, S, X and Z. Point out that, other than O and X, each of these letters has rotation symmetry of order 2. Remind the class that the order of rotation symmetry must be finite and greater than 1, so that if O is a perfect circle it does not count. (If the arms of X are at right angles, it then has rotation symmetry of order 4.)

main activity

Vocabulary
line of symmetry
rotation symmetry

Resources
2 cm squared paper
scissors
OHT S4.3b
blank OHT
three thin card shapes
(see lesson notes)

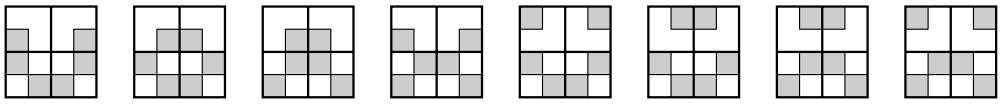
Give each pupil a piece of 2 cm squared paper and scissors. Show **OHT S4.3b**.

Ask pupils to copy the four tiles on to their squared paper, using the top part, and to cut out each tile. They should then put the four tiles together to make a larger tile that has a horizontal or vertical line of symmetry. They should copy the new tile on their squared paper and mark the lines of symmetry.

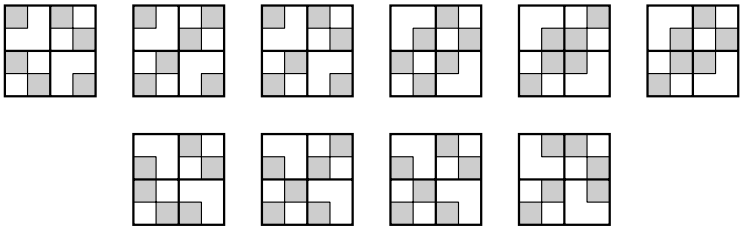
Invite some pairs to draw one of their larger tiles on **OHT S4.3b**.

Q How many different tiles can you make with a horizontal or vertical line of symmetry?

Stress the need to work systematically to discover the different possibilities, leaving two tiles in position and swapping over the other two.

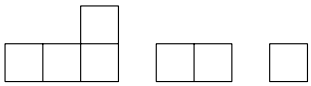


Q How many different tiles can you make with a diagonal line of symmetry?



Q Do any of your tiles have rotation symmetry? (tiles with 2 diagonal lines of symmetry) **What is the order of rotation symmetry?** (2)

Use thin card to make three shapes based on 2 cm squares:



Place the three shapes on a blank OHT.

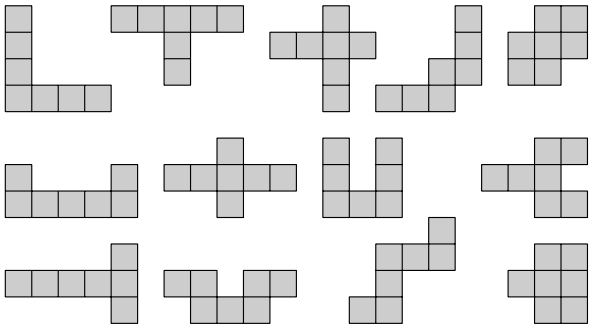
Ask pupils to work in pairs. Give each pupil another piece of 2 cm squared paper and scissors. Ask them to cut out the three shapes from the top of their squared paper. They should then combine the three shapes, making the small squares touch edge to edge, to make a symmetrical shape. They should draw each different symmetrical shape that they make on their paper.

Give pupils time to investigate, then invite different pairs to the OHT to assemble their symmetrical shape from the three pieces. Draw round the composite shape and ask the pair to identify the line of symmetry.

Q Has anyone found a different shape?

If pupils suggest a shape that it is a reflection or rotation of a previous shape, discuss with the class whether this counts as ‘different’.

Here are some of the symmetrical shapes that can be made.



other tasks

Springboard 7
Unit 8

Unit 8 section 2: Triangles and coordinates

You may wish to provide some further examples of transformations.

plenary

Resources
OHT S4.3b

Refer back to some of the symmetrical shapes made in the earlier investigations, for example on **OHT S4.3b**. Indicate a line of symmetry and say:

Q Explain how you know that this is a line of symmetry.

Look for answers that include:

- the shape could be folded along the line so that one half fits exactly on the other half;
- the line divides the shape into two identical pieces;
- each point on one half is the same distance from the line as the corresponding point on the other half.

Indicate a shape with rotation symmetry and say:

Q Explain how you know that this shape has rotation symmetry.

Look for answers that include:

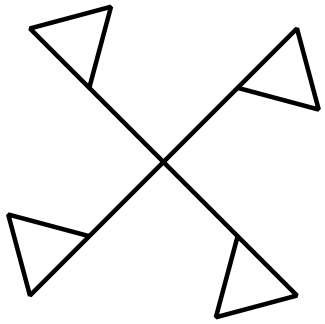
- if the shape is rotated about its centre it will fit on itself in two different ways.

Remember

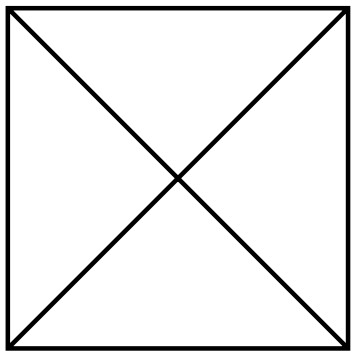
- A line of symmetry divides a shape into two identical parts. Each part is a reflection of the other.
- If you fold a shape along a line of symmetry, each part fits exactly on top of the other.
- Each point in a reflected image is the same distance from the mirror line as the corresponding point in the original shape.
- A shape has rotation symmetry if it fits on top of itself more than once in one whole turn.
- The order of rotation symmetry is the number of times that the shape fits on top of itself in one whole turn. This must be 2 or more.

Resource S4.1a

Use the procedure FLAG to create this diagram.



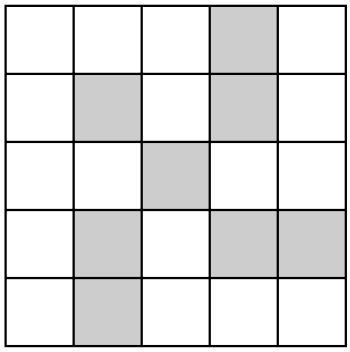
Use the procedure RTRIANGLE to create this diagram.



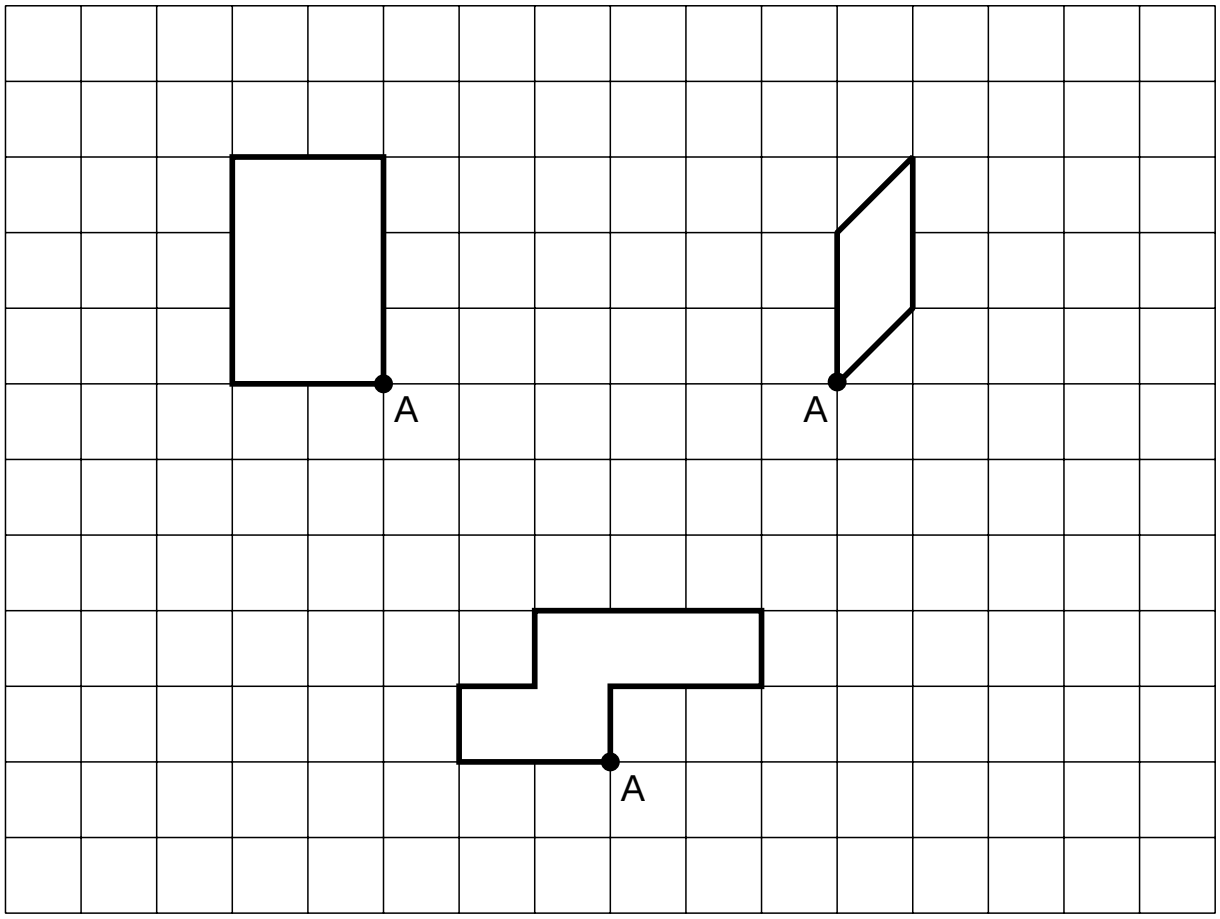
Resource S4.1b

You may use tracing paper for these questions.

Shade in one more square so that this design has rotation symmetry of order 4.



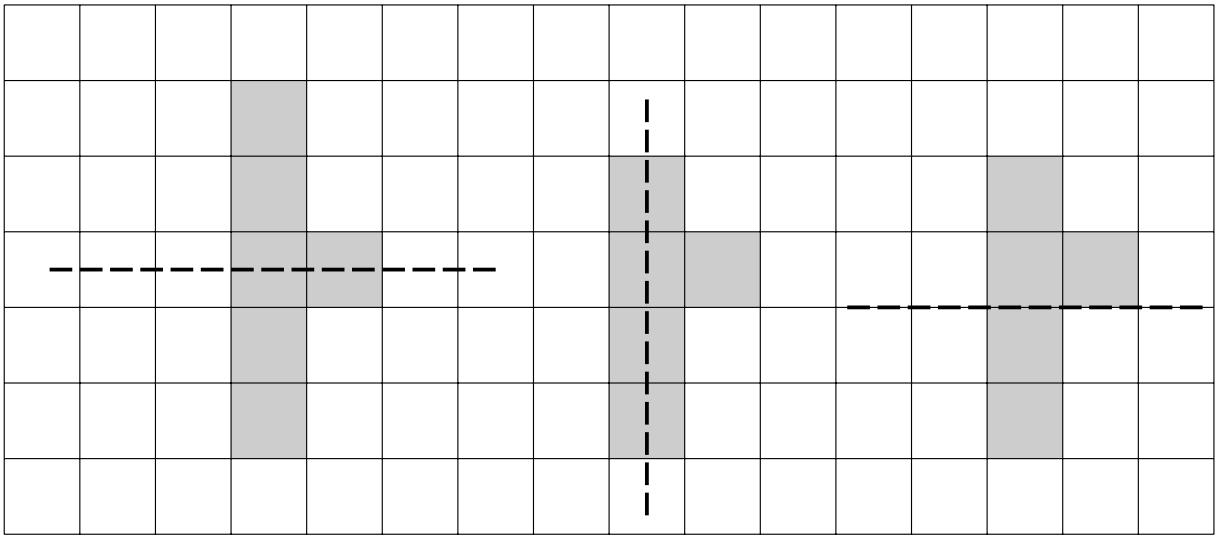
Each of these shapes is rotated 90° clockwise about point A.
Draw each shape in its new position.



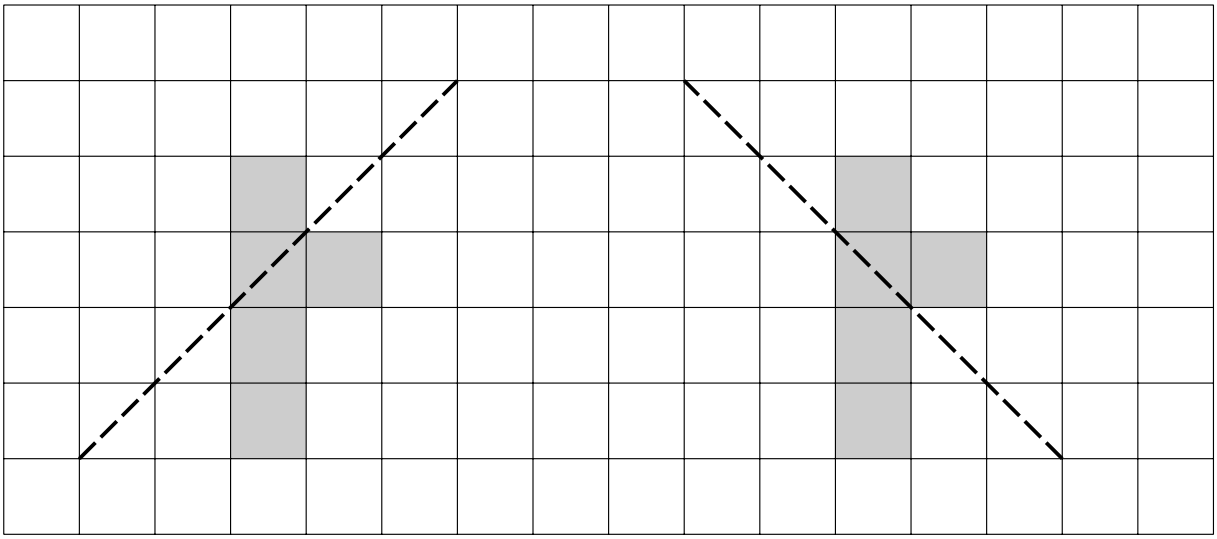
Resource S4.2a

You may use a mirror or tracing paper to help you.

On each diagram, shade in one more square to make a shape which has the dashed line as a line of symmetry.



On each diagram, shade in two more squares to make a shape which has the dashed line as a line of symmetry.



OHT S4.3a

M
A
T
H
S

M A T H S

M
A
T
H
S

OHT S4.3b