Algebra 3

contents

There are three lessons in this unit, **Algebra 3**.

objectives

The objectives covered in this unit are:

- Recall multiplication facts to 10 × 10 and derive associated division facts.
- Understand and use inverse operations.
- Count on and back in steps of constant size.
- Recognise multiples and use tests of divisibility.
- Generate terms of a simple sequence, including from practical contexts.
- Recognise the first few triangular numbers.
- Solve problems and investigate in number and algebra.

Using the lesson plans in this unit

These lesson plans supplement the *Springboard 7* materials for Key Stage 3 pupils working toward level 4 in mathematics. All the lessons are examples only. There is no requirement to use them. If you decide to use the lessons, you will need to prepare overhead projector transparencies (OHTs) and occasional resource sheets for pupils to use.

The lessons consolidate work at level 3 and extend into level 4. They are suitable for a group of pupils or a whole class. Whatever the size of the group, the pupils are referred to as 'the class'.

Each lesson will support about 30 to 40 minutes of direct teaching. To help match the time to your timetable, each plan refers to 'other tasks' for pupils, based on *Springboard 7* resources. Select from these, textbook exercises or your own materials to provide practice and consolidation in the main part of a lesson and to set homework. Aim to choose tasks that vary in their level of demand, to suit pupils' knowledge, confidence and rate of progress.

Although the 'other tasks' are listed for convenience at the end of the main part of the lesson, they can be offered at any point, especially between the 'episodes' that form the main activity.

The lesson starters are of two kinds: practice starters and teaching starters. The former are opportunities to rehearse skills that will be needed later in the lesson. Teaching starters introduce an idea that is then developed in the main activity.

You will need to tell pupils what they will learn in the lesson, either in the starter or at the beginning of the main activity. Use the plenary to check pupils' learning against the lesson's objectives and to draw attention to the key points that pupils should remember.

A3.1 Inverse functions

objectives

- Recall multiplication facts to 10 × 10 and derive associated division facts.
- Understand and use inverse operations.

starter

Vocabulary
divided by

Resources
set of digit cards 2 to 9
mini-whiteboards

Write on the board 48 ÷ 6. Read this with the class: forty-eight divided by six. Say that they can think of this as multiplication. Ask:

Q How many sixes make 48?

Confirm by counting in sixes along a number line: 6, 12, 18, … Complete on the board: 6 × 8 = 48 and 48 ÷ 6 = 8.

Write on the board 45 ÷ 5. Read it together: forty-five divided by five, or how many fives in forty-five? Ask:

Q Which multiplication table do we need to think of to find the answer?
(the five times table)

Choose a pupil to record on the board: 5 × 9 = 45 and 45 ÷ 5 = 9.

Repeat with other 'awkward' division facts, relating them to multiplication: for example, 36 ÷ 9, 32 ÷ 8, 63 ÷ 7, 24 ÷ 6, 35 ÷ 7, 72 ÷ 9.

Take a set of digit cards 2 to 9 and shuffle them. Invite a pupil to the front of the class to choose two of them, showing you but without showing the class. The pupil multiplies the two numbers mentally, says the answer and shows the class one of the cards, keeping the second card hidden. The rest of the class then write on their whiteboards what they think the other number is.

Set the two cards aside and repeat another three times.

main activity

Vocabulary
inverse
function machine

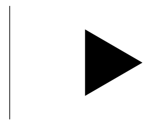
Resources
mini-whiteboards
non-permanent OHP pen
OHT A3.1a

Tell pupils that an inverse gets you back to where you started. It is an action or operation to reverse what you have done. The inverse of getting dressed is getting undressed. Ask pupils to suggest what the inverse would be of actions such as:

- opening the gate;
- switching on the kettle;
- turning 90° clockwise;
- getting on the bus.

Ask the class to suggest more actions and their inverses (e.g. walking upstairs, running backwards, turning the light on).

Draw a line and a simple shape on the board:

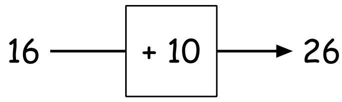


Q What would be the inverse of reflecting this shape in this line?

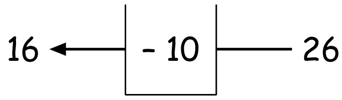
Q What would be the inverse of sliding the shape 20 cm to the right?

Ask the class to use the operation signs and to show you on their whiteboards what would be the inverse of: multiply by 4; add 8; divide by 6; subtract 3.

Remind pupils that they have previously used function machines to represent ‘add 10’. Draw this diagram on the board.

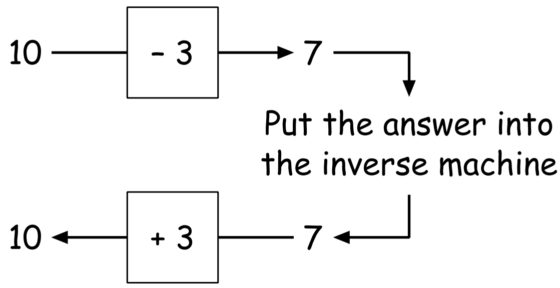


Explain that if you put 16 into an ‘add 10’ machine, you get the answer 26. Show the inverse by drawing the machine working in reverse. Put 26 into a ‘subtract 10’ machine and you get the answer 16. You have returned to where you started.



One at a time, draw on the board a few examples, such as:

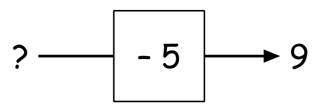
Ask pupils to draw on their whiteboards the inverse function machines. Check that each inverse works by drawing a diagram and feeding in a number such as 10.



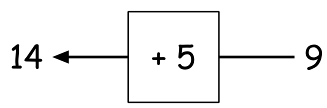
Tell pupils that you are thinking of a number. You subtract 5, and the answer is 9.

Q What’s the number? (14)

Say that you will show them how to use inverse function machines to help them to find answers to questions like this. Represent the question with a function machine.

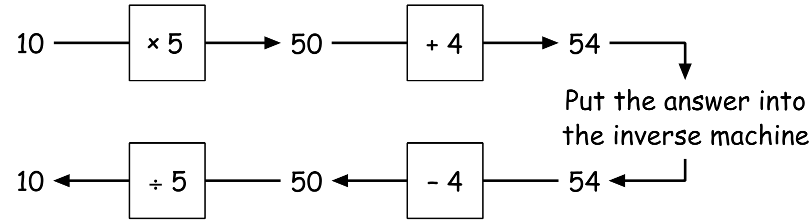


Underneath, draw the inverse function machine.



Ask some similar questions, drawing on different operations each time.

Show **OHT A3.1a**. Explain that a function machine can have two steps. Write, for example, × 5 and + 4 in the upper machine. Say that the inverse machine returns you to where you started. Write ÷ 5 and – 4 in the lower machine. Check by feeding in a number such as 10 to the first machine. Talk through each step and show how the inverse returns the answer of 54 to the original input of 10.

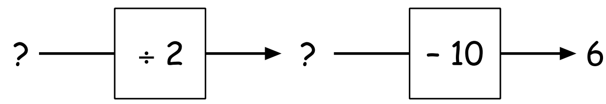


Feed through a different number, again to confirm that the inverse returns you to where you started. Repeat with a different machine (for example, – 6 and × 8), again feeding through a couple of numbers to check.

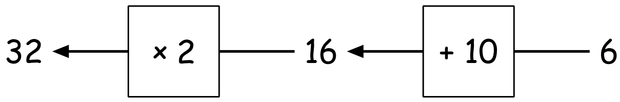
Tell pupils that you are thinking of a number. You divide by 2, and subtract 10. The answer is 6.

Q What is the number? How can we use inverse function machines to help solve the problem?

Represent the question with a function machine.



Put the answer of 6 into the inverse function machine.



Ask some similar questions, varying the operations.

other tasks

There are no exercises on inverse functions in the *Springboard 7* folder. Choose suitable tasks from textbooks or other resource materials, or devise your own.

plenary

Resources
calculators

Tell pupils that you are thinking of a number.

Q I multiply my number by 37 and the product is 518. What's my number?

Explain that the problem can be represented by ☐ × 37 = 518, or by a function machine. Ask pupils to use their calculators to find the answer. Repeat with ☐ ÷ 45 = 23.

Remember

- The inverse returns you to where you started.
- The inverse can be represented by drawing the function machine 'backwards', replacing each operation with its inverse.
- A function machine can have more than one step.
- Function machines can help to solve 'I am thinking of a number' problems and equations like ☐ ÷ 45 = 23.

A3.2 Tests of divisibility

objectives

- Recall multiplication facts to 10 × 10 and derive associated division facts.
- Recognise multiples and use tests of divisibility.
- Solve problems and investigate in number and algebra.

starter

Vocabulary
product
sum

Write four single-digit numbers on the board: for example, 3, 4, 5, 8. Tell the class that you want to make two pairs of numbers. Multiply the numbers in each pair, then add the two products together. Record on the board:

3 4 5 8

3 × 5 → 15
4 × 8 → 32
15 + 32 → 47

Q Is 47 the biggest answer we can get? Can we pair the numbers to get a larger answer?

Ask pupils to discuss this in pairs, trying out different combinations of numbers. They should find that pairing 3, 4 and 5, 8 results in a larger answer: 52. Ask them to choose four more single-digit numbers and to make the biggest possible answer.

Q Can you find any rules for how to pair the four numbers to give you the largest possible result?

Allow time for further investigation, then discuss findings. Draw out that pairing the largest two numbers and the smallest two numbers gives the largest result. Ask:

Q What if we add the numbers in each pair, and then multiply the two sums? How should we pair the numbers to get the biggest result?

After further investigation, draw out that the largest result in this case is produced by pairing the middle two numbers, and the largest and the smallest number.

main activity

Vocabulary
multiple
factor
divisible
digit sum
remainder

Resources
OHT A3.2a

Explain that if you multiply each counting number by 7 you get the table below.

1 × 7	2 × 7	3 × 7	4 × 7	5 × 7	6 × 7	7 × 7
7	14	21	28	35	42	49

Remind pupils that 7, 14, 21, … are the multiples of 7, and that 7 is a factor of every multiple of 7. The complete set of factors of 21 is 1, 3, 7 and 21.

Show **OHT A3.2a**, a set of ‘Who am I?’ puzzles.

Show pupils how to solve the first problem by listing numbers systematically in columns. For example, list in column A all the multiples of 4 less than 20. In column B, add 3 to each number to create numbers that have a remainder of 3 when divided by 4. In column C, list the multiples of 5 less than 20. In column D, add 4 to each of these numbers to give numbers that have a remainder of 4 when divided by 5. Now look for the number that is common to both columns B and D.

A	B
4	7
8	11
12	15
16	19

C	D
5	9
10	14
15	19

Ask pupils to work in pairs to solve the other three puzzles. Remind them about working systematically. When time is up, ask pairs of pupils to present their solutions to the class, drawing out that questions 2, 3 and 4 each have two solutions.

Write 456 on the board. Choose a pupil to read out the number in words.

Q Is this number odd or even? (even)

Point out that the last digit, 6, is even, so the whole number is even. Explain that an even number is *divisible* by 2, which means that it divides exactly by 2 with no remainder. A number that is divisible by 2 is also a multiple of 2.

Say a few numbers and ask pupils to put up a hand if the number is divisible by 2: for example, one hundred and thirty-four, five hundred and seventy-three, one thousand and eight.

Point to 456 again.

Q Is this number divisible by 3?

Tell the class that there is a quick way to find out by adding up all the digits. Write 4 + 5 + 6 = on the board and ask for the answer. Point to the answer 15, explaining that this is the sum of the digits. Because the digit sum is a multiple of 3, the whole number 456 is divisible by 3.

Try testing a few numbers for divisibility by 3, this time writing them on the board: for example, 234, 113, 559.

Point again to 456.

Q Is this number divisible by 4?

Say that there is an easy way to find out. We know that is even so it divides exactly by 2. We look at the answer after we have halved it.

Find half of 456 by writing 400 + 50 + 6 and finding half of each part. Add together 200 + 25 + 3 to get then answer of 228. Point out that because this answer is even, 456 can be divided exactly by 2 and then exactly by 2 again. It is therefore divisible by 4.

Try testing a few numbers for divisibility by 4, writing them on the board: for example, 128, 146, 504.

Point again to 456.

Q Is this number divisible by 5 or by 10?

Confirm that it is not, since it does not end in 5 or 0.

Q Is it divisible by 6?

Explain that all multiples of 6 divide exactly by 2 and also by 3. We know that 456 divides exactly by 2 because it is even. We also know that it divides exactly by 3 because the sum of its digits is a multiple of 3. So 456 is divisible by 6.

other tasks

Springboard 7
Unit 9

Unit 9 section 3: Testing for divisibility

plenary

Resources
OHT A3.2b
mini-whiteboards
OHP calculator

Show **OHT A3.2b**, a set of division calculations. Go through the calculations one by one. Ask a pupil to read out the first calculation: nine hundred and forty-one divided by two.

Q When you divide 941 by 2, is there a remainder? Yes or no?

Ask pupils to write 'Y' for yes or 'N' for no on their whiteboards. Ask:

Q How do you know without doing the calculation? (941 is an odd number since its last digit is 1, so it is not divisible by 2)

Confirm by checking on the OHP calculator.

Repeat with the other calculations.

B Yes. 356 does not end in 5 or 0, so it is not divisible by 5 and there is a remainder.

C Yes. The sum of the digits of 1220 is 5, which is not a multiple of 3, so 1220 is not divisible by 3 and there is a remainder.

D Yes. Any number that is divisible by 8 must be even. As 783 is odd, it cannot be divisible by 8 and there is a remainder.

E No. 580 is divisible by 4, since half of 580 is 290, which is even. There is no remainder.

F Yes. If 226 is divisible by 6, it must be divisible by 3 and by 2. It is divisible by 2, since it is even. The sum of its digits is 10, which is not a multiple of 3, so it cannot be divisible by 3. There is a remainder.

Remember

- An odd number cannot be divisible by an even number.
- A number is divisible by 3 if the sum of its digits is a multiple of 3.
- A number is divisible by 4 if half of it is even.
- A number is divisible by 5 if its last digit is 5 or 0.
- A number is divisible by 6 if it is even and is also divisible by 3.

A3.3 More sequences

objectives

- Count on and back in steps of constant size.
- Recognise the first few triangular numbers.
- Generate terms of a simple sequence, including from practical contexts.
- Solve problems and investigate in number and algebra.

starter

Vocabulary
steps
halfway
interval

Resources
OHT A3.3a
counting stick

Use a counting stick. Tell the class that one end is 4, and you want them to count in steps of 3. Count together along the stick to 34. Point to the midpoint of the stick.

Q **What is this number?** (19) **How do you know?**

Establish that it is halfway between 4 and 34. Each interval or step is worth 3, so that five intervals or steps are worth 15.

Q **What is the next number?** (22) **How do you know?** (it is 3 more)

Q **What is the number before the middle number?** (16) **How do you know?**

Tell the class that one end is still 4, but that this time you want them to count in steps of 9.

Q **What is the quick way to add on 9?** (add 10 and subtract 1)

Q **What will be the last number on the stick? How do you know?**

Establish that it will be 94, since the starting number is 4 and there are ten intervals or steps, each worth 9. Write on the board $4 + (9 \times 10)$.

Q **What number is halfway between 4 and 94?**

Invite a pupil to explain why 49 is halfway between 4 and 94.

Show **OHT A3.3a**, with five number lines. Invite pupils to the projector to fill in the missing numbers on the lines. As they do so, ask them to explain their reasoning. As each line is completed, ask the class to complete the sentence: 'The numbers on this line go up in steps of …'

main activity

Vocabulary
sequence
term

Resources
OHT A3.3b
Resource A3.3c
mini-whiteboards

Write the sequence 11, 21, 31, 41 on the board. Ask:

Q **How does this sequence continue?**

Confirm that the next few terms are 51, 61, 71, and the rule is 'add 10'.

Q **Will there ever be a multiple of 10 in this sequence? Explain why.**

Establish that each term is a multiple of 10, plus 1.

Repeat with the sequence 2, 7, 12, 17. Ask for the next few terms then ask:

Q **Will there ever be a multiple of 5 in this sequence? Explain why.**

Establish that each term is 2 more than a multiple of 5, and that the rule is 'add 5'.

Show the first table on **OHT A3.3b**. Tell the class that the rule is 'add 4'. Invite a pupil to enter the next four terms (9, 13, 17, 21) along the top row of the table, with the help of the class.

Q How will the pattern continue? (25, 29, 33, 37)

Q What do you notice about all the numbers in the sequence? (they are all odd numbers)

Q Will 66 be in the sequence? Explain why or why not. (no – it's even)

Draw attention to the pattern of dots. Ask:

Q What do you notice about this pattern of dots? Can you describe the pattern?

Establish that each pattern has one dot, plus rows of four dots. Complete the bottom row of the table: 1 + (4 × 2), 1 + (4 × 3), 1 + (4 × 4), 1 + (4 × 5).

Q How would this row continue?
1 + (4 × 6), 1 + (4 × 7), 1 + (4 × 8), 1 + (4 × 9)

Work out these expressions and establish that they match the extended sequence of 25, 29, 33, 37.

Q Will 101 be in the sequence? How do you know?

Ask pupils to discuss this question in pairs, then take feedback on their decisions and reasons.

Q How many rows of four dots will there be in the pattern that represents 101?

Use the pattern to explain that the number of rows of four dots will be (101 – 1) ÷ 4 = 25. Check by calculating 1 + (25 × 4) = 101. As 101 can be represented by a pattern with one dot, plus 25 rows of four dots, it is a term in the sequence.

Q Will 51 be in the sequence? How do you know?

Use a similar method to establish that 51 – 1 = 50, and that 50 is not a multiple of 4, so 51 is not in the sequence.

On the lower table on **OHT A3.3b**, write 3 in the first box of the top row. Enter a rule of 'add 5'. Point to the sixth box in the top row and ask:

Q How can we work out this term without completing all the boxes in between?

Ask pupils to discuss this question in pairs for a minute or two. Take feedback on their strategies and establish that the sixth term will be 3 plus 5 rows of 5 dots. This can be written as 3 + (5 × 5) = 28.

Give out copies of **Resource A3.3c**. Refer to the first question. Ask pupils to use their whiteboards and to draw the next pattern in the sequence. Ask:

Q What is the same as the previous pattern? What is different?

Repeat with the second question.

Ask pupils to complete the two questions in pairs. Take feedback on their answers, inviting pupils to explain to the rest of the class the strategies that they used.

other tasks

Springboard 7
Unit 9

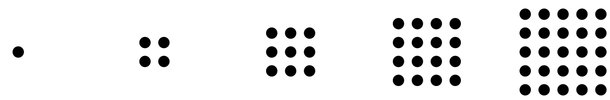
There are no relevant exercises on sequences involving spatial patterns in the *Springboard 7* folder. Choose suitable tasks or activities from textbooks or other resource materials, or devise your own. Other exercises on sequences are:

Unit 9 section 2: Multiples

3 Counting on and back in 6s, 7s, 8s and 9s — page 308

plenary

Draw on the board this pattern of dots.



1 x 1 2 x 2 3 x 3 4 x 4 5 x 5

Q How would you describe this sequence? (square numbers)

Q What would be the tenth term in the sequence? (10 × 10 = 100)
The hundredth term in the sequence? (100 × 100 = 10 000)

Write on the board and complete with the class:

1	= 1
1 + 3	= 4
1 + 3 + 5	= …
1 + 3 + 5 + 7	= …
1 + 3 + 5 + 7 + …	= …

Q Can you describe the numbers we are adding? (odd numbers)

Q What special numbers are the answers? (square numbers)

Establish that adding the first three odd numbers produces the third square number, and so on.

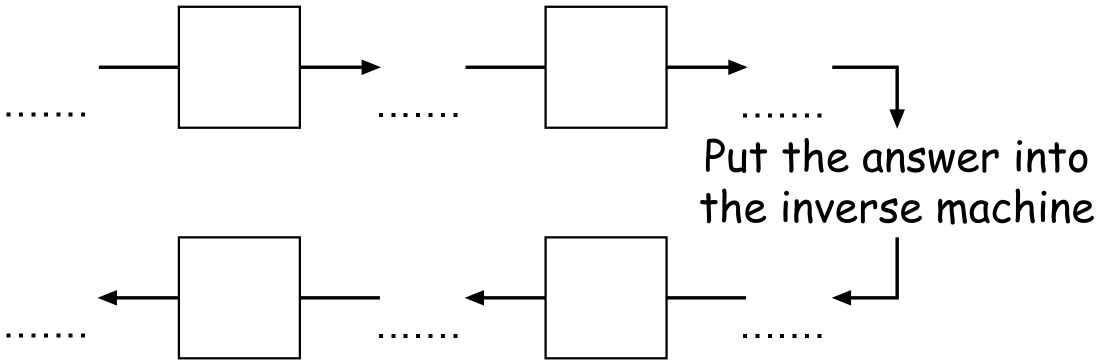
Q What is the answer to 1 + 3 + 5 + 7 + 9 + 11?

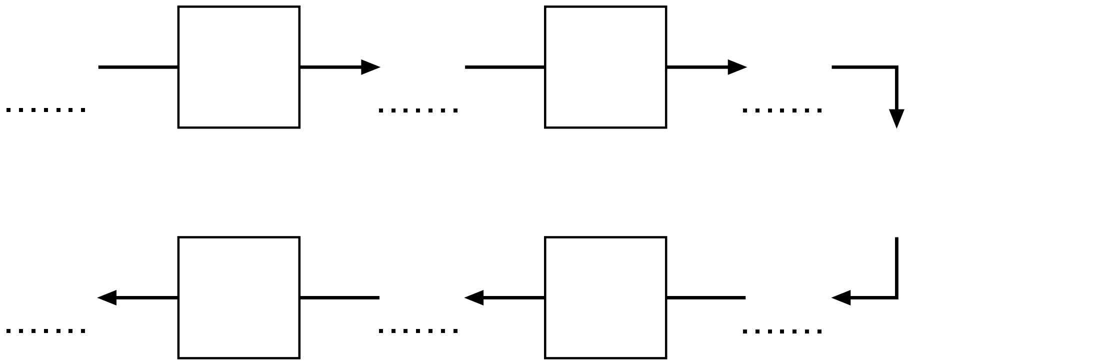
Q What is the sum of the first ten odd numbers?

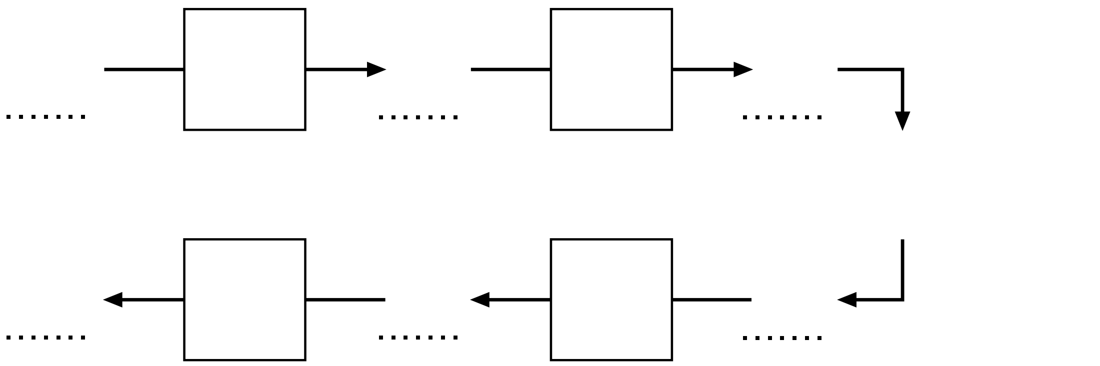
Remember

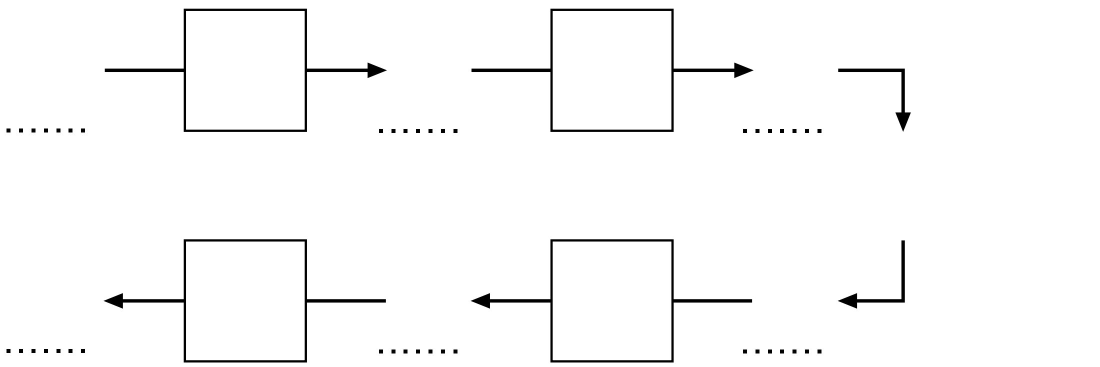
- Decide if the rule means that you add or subtract a number each time.
- If the sequences can be represented by a pattern, use the pattern to help work out terms.
- To decide whether a given number is a term in a sequence, work out the gap between the first number and the given number. Then check whether the gap is a multiple of the number in the rule.

OHT A3.1a









Who am I?

1 I am less than 20.

If you divide me by 4, the reminder is 3.
If you divide me by 5, the remainder is 4.

2 I am an odd multiple of 9.

The product of my two digits is also a multiple of 9.

3 I am a two-digit number.

I am 2 more than a multiple of 9, and 1 less than a multiple of 7.

4 I am a square number.

The sum of my two digits is one of my factors.

Do these calculations have a remainder?

A
$941 \div 2$

B
$356 \div 5$

C
$1220 \div 3$

D
$783 \div 8$

E
$580 \div 4$

F
$226 \div 6$

Fill in the missing numbers on these number lines.

350 .... 450 500

45 .... 60 ....

-20 .... 20 30

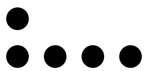
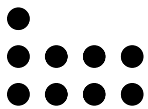
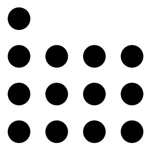
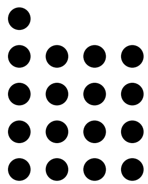
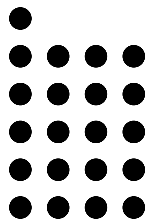
-7 .... 1 5 9 17

7.5 7.6 7.7 7.8 ....

For each line, finish this sentence:

The numbers on this line go up in steps of ....

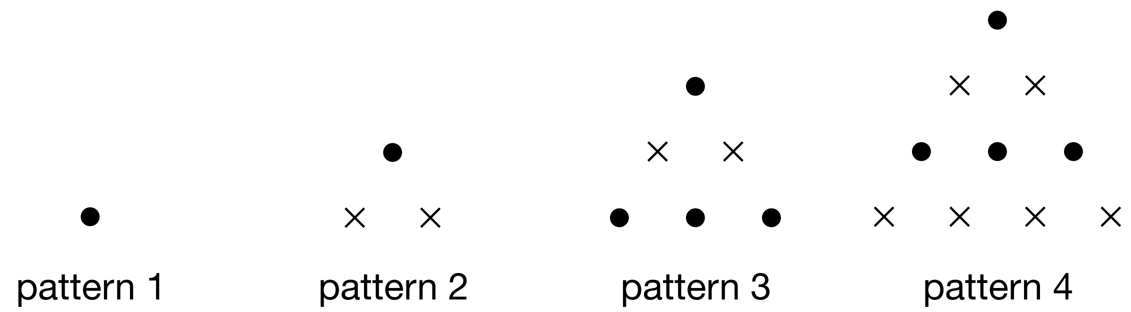
The rule is add 4.

1	5				
					
1	$1 + (4 \times 1)$				

The rule is ....

Resource A3.3c

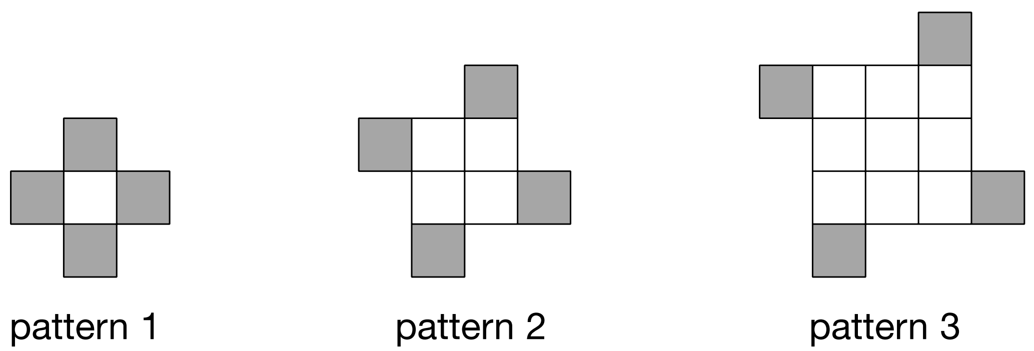
This series of patterns grows in a regular way.



How many dots would be in pattern 5? ....

How many crosses would be in pattern 5? ....

This is a series of patterns with white and grey tiles.



How many white tiles and grey tiles will there be:

in pattern 8?

.... white tiles and .... grey tiles

in pattern 16?

.... white tiles and .... grey tiles