Algebra 2

contents

There are three lessons in this unit, **Algebra 2**.

objectives

The objectives covered in this unit are:

- Know addition and subtraction facts to 20.
- Recall multiplication facts to 10 × 10 and derive associated division facts.
- Count on and back in steps of constant size.
- Recognise and use multiples, factors and primes.
- Know and use the order of operations, including brackets.
- Solve problems and investigate in number.

Using the lesson plans in this unit

These lesson plans supplement the *Springboard 7* materials for Key Stage 3 pupils working toward level 4 in mathematics. All the lessons are examples only. There is no requirement to use them. If you decide to use the lessons, you will need to prepare overhead projector transparencies (OHTs) and occasional resource sheets for pupils to use.

The lessons consolidate work at level 3 and extend into level 4. They are suitable for a group of pupils or a whole class. Whatever the size of the group, the pupils are referred to as 'the class'.

Each lesson will support about 30 to 40 minutes of direct teaching. To help match the time to your timetable, each plan refers to 'other tasks' for pupils, based on *Springboard 7* resources. Select from these, textbook exercises or your own materials to provide practice and consolidation in the main part of a lesson and to set homework. Aim to choose tasks that vary in their level of demand, to suit pupils' knowledge, confidence and rate of progress.

Although the 'other tasks' are listed for convenience at the end of the main part of the lesson, they can be offered at any point, especially between the 'episodes' that form the main activity.

The lesson starters are of two kinds: practice starters and teaching starters. The former are opportunities to rehearse skills that will be needed later in the lesson. Teaching starters introduce an idea that is then developed in the main activity.

You will need to tell pupils what they will learn in the lesson, either in the starter or at the beginning of the main activity. Use the plenary to check pupils' learning against the lesson's objectives and to draw attention to the key points that pupils should remember.

Interactive teaching programs (ITPs)

Interactive teaching programs are interactive animated visual aids that can be used with a laptop and data projector or with an interactive whiteboard. As extra support for this unit, you may find it useful to download and use these ITPs from the website www.standards.dfes.gov.uk/numeracy:

for lesson A2.2: *Number grid*

A2.1 Order of operations and brackets

objectives

- Know addition and subtraction facts to 20.
- Recall multiplication facts to 10 × 10 and derive associated division facts.
- Know and use the order of operations, including brackets.
- Solve problems and investigate in number.

starter

Vocabulary
operation
addition
subtraction
multiplication
division

Resources
OHTs A2.1a, A2.1b
mini-whiteboards

Tell the class that mathematicians have decided an order for doing the operations of addition, subtraction, multiplication and division. Multiplication and division are always done before addition and subtraction.

Write on the board:

$4 \times 5 + 2$	Do the multiplication first.
$= 20 + 2$	Then the addition.
$= 22$	Answer: 22.
$8 - 12 \div 4$	Do the division first.
$= 8 - 3$	Then the subtraction.
$= 5$	Answer: 5.

Write three or four examples on the board, one by one. Ask pupils to copy them on their whiteboards and to underline the part they would do first. For example:

$5 + 4 \times 10$ $\quad 16 \div 4 + 5$ $\quad 12 - 9 \div 3$ $\quad 4 \times 3 - 6$

Show **OHT A2.1a**. Work with the class to find the five matching pairs: AH, BJ, CD, EF, GI.

Show **OHT A2.1b**, which gives an expression with missing operations. Say that the challenge is to find four different answers by replacing each circle with one of the four operations. For example, $7 - 6 \div 2$ would give the answer 4.

Ask pupils to work in pairs on the problem. Take feedback on the solutions, which are $7 + 6 - 2 = 11$, $7 \times 6 - 2 = 40$, $7 + 6 \times 2 = 19$, $7 + 6 \div 2 = 10$.

main activity

Vocabulary
digit
operation
brackets

Resources
OHT A2.1c, including photocopies
mini-whiteboards

Say that sometimes we want to do addition and subtraction first. We use brackets to show this. Write on the board:

$5 \times (4 + 3)$	Do the brackets first.
$= 5 \times 7$	Then the multiplication.
$= 35$	Answer: 35.
$(4 + 6) \div (5 - 3)$	Do the brackets first.
$= 10 \div 2$	Then the division.
$= 5$	Answer: 5.

Write three or four examples on the board, one by one. For example:

$(12 - 4) \times 3$ $15 \div (7 - 4)$ $(8 - 4) \times (1 + 3)$

Ask pupils:

Q What is the answer? Explain how you worked it out.

Write a problem on the board. Ask pupils to discuss in pairs what the answer is.

Jack has 8 paperbacks. Rupee has 2 paperbacks.
Each paperback cost £5.
What was the total cost of the paperbacks?

Q The answer is £50. Did you get that? How did you work it out?

Invite a pair to explain their method to the class.

Q Did anyone do it a different way?

Draw out that one method is to work out the cost of Jack's books (£5 × 8) and the cost of Rupee's books (£5 × 2), and then to add the two amounts. This calculation can be recorded as:

$(5 \times 8) + (5 \times 2) = 40 + 10 = 50$

A second method is first to add the number of Jack's books to the number of Rupee's books, and then to find the total cost of all the books at £5 per book. This calculation would be recorded as:

$5 \times (8 + 2) = 5 \times 10 = 50$

Explain that the brackets are needed to show that you must first work out what is inside the brackets. Stress that the brackets make a difference and demonstrate this by writing on the board and working out $3 \times (4 + 5)$ and $(3 \times 4) + 5$. Repeat with another example: $12 - (3 \div 3)$ and $(12 - 3) \div 3$.

Write randomly on the board the four digits 1, 2, 3 and 4, and the four operation signs +, –, × and ÷.

Tell pupils that you want them to work in pairs. They are to use any of the four digits 1, 2, 3 and 4, with any of the four operations addition, subtraction, multiplication or division, to make numbers. Show them a couple of examples:

$1 + 2 + 3 - 4 = 2$ $21 + 4 = 25$

Give pupils one or two minutes to get the idea, then draw the class together. Give pupils a worksheet photocopied from **OHT A2.1c**. Tell them that you now want them to make each of the numbers 1 to 20. Say that you are going to add some rules.

- All four digits must be used.
- A digit must not be repeated.
- The digits can be used in any order.
- Any operation can be used – pupils do not have to use all four operations.
- An operation can be repeated.
- Brackets should be used where they are needed.

Remind the class that digits can be combined to make two-digit numbers. Say that this is a joint effort from the whole class, and to start with everyone should try to make the number 9. Allow a minute or two for pairs to work on this. (If a pair finds a solution very quickly, suggest that they look for another one.) Then take feedback. Establish that there may be different ways to make the numbers: for example, 9 can be made as:

$41 - 32$ $(4 + 3 + 2) \times 1$ $14 - (3 + 2)$

Stress the difference that the brackets make with the third example.

$14 - (3 + 2) = 9$, but $14 - 3 + 2 = 13$

Show **OHT A2.1c**, and write one of the pupils' solutions against the number 9. Say that when they have found a way to make a number, they should record it on their worksheet in a similar way.

Allow the pairs to work on making the numbers for about five minutes, then draw the class together. Ask:

Q What numbers have you found so far?

Check their suggestions and fill in as many numbers as possible. Ask them to use the remaining time to try to make the numbers that are still missing. While they are doing this, circulate and prompt by asking:

Q If you put the brackets in a different place, would you get a different answer?

other tasks

Springboard 7
Unit 15

Unit 15 section 5: Brackets

plenary

Resources
OHT A2.1d
mini-whiteboards

Write these calculations on the board:

$2 + 7 \times 3 = 27$ $2 + 7 \times 3 = 23$

Tell pupils that the answers are correct but that the brackets are missing. Ask them to work in pairs to put in the brackets. Discuss their solutions. Repeat with another example, such as:

$8 \div 4 - 2 = 0$ $8 \div 4 - 2 = 4$

Finish by working through the test question on **OHT A2.1d**.

Remember

- Always work out the calculation in the brackets first.
- If there are no brackets, multiply and divide before you add and subtract.

A2.2 Multiples

objectives

- Count on and back in steps of constant size.
- Recall multiplication facts to 10 × 10 and derive associated division facts.
- Recognise multiples.
- Solve problems and investigate in number.

starter

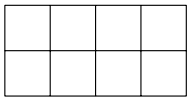
Vocabulary
product

Resources
two dice
mini-whiteboards

Tell the class that they are going to play a bingo game that will help them to remember multiplication and division facts. Write these 20 numbers in a list across the board (or prepare them on an OHT).

9, 12, 15, 16, 18, 20, 21, 24, 25, 28, 30, 32, 35, 36, 40, 42, 48, 49, 56, 64

Ask pupils to sketch a 4 by 2 grid on their whiteboards.



Ask them to choose any numbers from the list and write one number in each box of their grid. Each number must be different.

Say that you are going to roll two dice but you will add 2 to each number rolled, so that the smallest number will be 3 and the largest will be 8. You will then call out the two numbers. If the product of the two numbers is on a player's board, the player can cross out the number. The first player to get a row of four crossed out numbers calls out 'Bingo!' and wins the game.

Play the game once or twice. Keep a record of the products for checking purposes. As the winner reads out their line of numbers for checking, ask, for example:

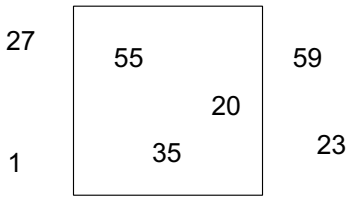
Q Fifty-six was the product of which two numbers?

main activity

Vocabulary
even
multiple

Resources
OHT A2.2a and A2.2b
copies of OHT A2.2a
dice
counters in two colours
OHP 'counters', e.g. bolts/washers in two shapes, about 1 cm in diameter
0 to 9 digit cards
ITP *Number grid* (optional)

Draw a large square box on the board. Ask pupils to suggest some numbers below 60. If they are multiples of 5, write them in the box. If not, write them outside.



Once there are at least three numbers in the box, ask:

Q What is my rule for putting numbers in the box?

Continue asking for numbers and placing them in or out of the box until pupils recognise the rule. Repeat this activity with multiples of 10, multiples of 2, and multiples of 11. This time invite pupils who think that they know what the rule is to come to the board and write another number in the box.

Use pupils' explanations to remind them of and refine their ideas of a multiple.

- Multiples of 2 are numbers that divide exactly by 2. They include the numbers in the two times table.
- The rule to generate the sequence of multiples of 2 is 'add 2'.
- Multiples of a number will appear as equal jumps along a number line.

Use **OHT A2.2a**, a dice and 'counters'. Play a multiples game with the whole class divided into two teams. The rules are:

- Take turns to roll the dice (1 counts as 7).
- You then have a choice. Suppose you roll 5. You can:
  - either cover a multiple of 5 with one of your counters;
  - or remove one of the other player's counters from a multiple of 5.
- The winner is the first to get three counters in a line, horizontally, vertically or diagonally.

After each roll of the dice, ask:

Q What multiples of your dice number can you see on the board?

Q If you cover a number, what would be the best one to choose? Why?

Q If you remove one of the other team's counters, which would be the best one to choose? Why?

Allow time for groups within a team to discuss the questions.

Play the game once, then discuss strategies with the whole class. For example:

Q Is it better to cover a number in the corner of the board or a number in the centre? Why?

Q Is it better to remove a counter at the centre or a counter in a corner? Why?

Q Which numbers are easier to cover? Why?
Which are more difficult to cover? Why?

Q Why is it better to try to cluster the numbers you are covering?

You could, if you wish, give out a copy of **OHT A2.2a**, a dice and counters to groups of four pupils and get the groups to play the game.

Ask pupils to work in pairs. Each pair should use one pack of digit cards from 0 to 9. Ask the pairs to use all ten cards to create five different numbers, each of which is a multiple of 3. The numbers can have any number of digits. (There are lots of possibilities. One example is 12, 30, 45, 69, 78.)

Then ask them to make five multiples of 7. (Again, there are various possibilities. One example is 7, 21, 49, 56, 308.)

A further challenge, if there is time, is to make five multiples of 18 (e.g. 18, 36, 54, 72, 90).

You may wish to support this activity by using the ITP *Number grid*, downloaded from www.standards.dfes.gov.uk/numeracy.

Count in multiples of 9 to 90. Ask:

Q **What is the next multiple of nine?** (99) **And the next?** (108)
How did you work it out? (add nine to the previous multiple)

Work through the test questions on **OHT A2.2b**.

other tasks

Springboard 7
Unit 9

Unit 9 section 2: Multiples of numbers

plenary

Vocabulary
palindrome
reverse

Resources
mini-whiteboards
OHP calculator

Q **What happens when you reverse multiples? Do all multiples of 3 stay multiples of 3 when you reverse them?**

Ask the class to suggest a few multiples of 3. Confirm that they will still be multiples of 3 when reversed. If necessary, use an OHP calculator to check.

Q **Can you tell me an even number that stays even when you reverse it?** (e.g. 46, 82, 208)

Point out that if numbers like 02 are allowed, multiples of 10 also fit the rule.

Q **Do all even numbers stay even when you reverse them?** (no – for example, when 16 is reversed it becomes 61, which is odd)

Q **Can you suggest a multiple of 5 that stays a multiple of five when you reverse it?** (5, 55, 515, 525, …)

Q **What happens when you reverse multiples of 9? Do they stay as multiples of 9?**

For larger multiples, use an OHP calculator to confirm suggestions.

Remember

- Multiples of 7 are numbers that divide exactly by 7. They include the numbers in the seven times table.
- The rule to generate the sequence of multiples of 7 from 0 is ‘add 7’.

A2.3 Factors and primes

objectives

- Recall multiplication facts to 10 × 10 and derive associated division facts.
- Recognise and use factors and primes to 20.
- Solve problems and investigate in number.

starter

Vocabulary
addition
multiplication

Resources
watch with a second hand
mini-whiteboards

Ask pupils to draw a 4 by 4 addition grid in their exercise books or on whiteboards, and to write the numbers from 2 to 9 in any order along two sides.

+	4	8	5	3
6				
2				
9				
7				

Ask them to fill in their grid as quickly as they can when you say 'Go!' Allow a minute or two and then say 'Stop!' Ask them to count how many squares of the grid they were able to complete in the time.

Repeat with a multiplication grid.

×	9	6	3	8
2				
7				
5				
4				

main activity

Vocabulary
factor
prime factor
divisible

Resources
OHT A2.3a
calculators
OHP calculator

Tell the class that 12 can be divided exactly by 3. We say that 3 is a *factor* of 12, and that 12 is a *multiple* of 3.

Q Are there any other factors of 12? What other numbers will divide exactly into 12?

Draw out that other factors of 12 are 1, 2, 4, 6 and 12. Explain that factors always occur in pairs. The factor pairs for 12 are 1 × 12, 2 × 6 and 3 × 4.

Use **OHT A2.3a**. In the column for 12, shade all the factors of 12: 1, 2, 3, 4, 6 and 12. Then shade all the factors of 6 (1, 2, 3 and 6).

12												
11												
10												
9												
8												
7												
6												
5												
4												
3												
2												
1												
	1	2	3	4	5	6	7	8	9	10	11	12

Q What do you notice about the factors of 6? (they are also factors of 12)

Shade all the factors for 3 (1 and 3).

Q What do you notice about the factors of 3? (they are also factors of 6 and of 12)

Now complete the chart with the class by shading all the factors of every other number from 1 to 12.

Q What factor do all the numbers have in common? (1)

Q Which numbers have 2 as a factor? (the even numbers)

Q Which numbers have only one pair of factors? (2, 3, 5, 7 and 11)

Explain to the class that these numbers are called *prime numbers*. A prime number has only two factors: 1 and itself. 1 is not a prime number.

Q Can you think of some more prime numbers less than 20? (13, 17, 19)

Refer again to **OHT A2.3a**.

Q Which numbers have an odd number of different factors? (1, 4 and 9)
What is special about these numbers? (they are square numbers)

Q What are the next two square numbers after 12? (16, 25)

Ask pupils in pairs to test 16 and 25 to find out the number of different factors (1, 2, 4, 8 and 16, and 1, 5 and 25 respectively). Confirm that these two square numbers also have an odd number of different factors.

Ask the class to help you to list on the board the prime numbers less than 20:

2, 3, 5, 7, 11, 13, 17, 19

Write on the board:

$\square \times \square \times \square = 231$

Tell the class that each box contains a different prime number. Explain that each of these prime numbers is a factor of 231, so it divides exactly into 231.

Q Could one of the prime factors be 2?

Establish that, since 231 is odd, 2 is not a factor. Ask the class to use their calculators to investigate the possibilities.

Systematic working through the list should help pupils to discover that 3 is one of the prime factors. Dividing 231 by 3 gives 77, which pupils should recognise as the product of 7 and 11. Confirm on a calculator that $3 \times 7 \times 11 = 231$.

Refer again to **OHT A2.3a**. Write on the board:

$156 \div 12 = 13$

Get pupils to check this with their calculators. Say that this means that 156 divides exactly by 12; 12 is a factor of 156, and 156 is a multiple of 12.

Q Is 2 a factor of 156?

Establish that 156 is even, so 2 is a factor of 156.

Q Is 3 a factor of 156?

Confirm with calculators that 156 ÷ 3 = 52, so 3 is a factor of 156.

Say that, so far, we know that 1, 2, 3 and 12 are factors of 156. Point to the column for 12 on OHT A2.3a. Ask:

Q Are the other factors of 12 also factors of 156?

Ask pupils to use their calculators to confirm that 4 and 6 are also factors of 156. Stress that what they have discovered is that, since 12 is a factor of 156, all the factors of 12 are also factors of 156.

other tasks

Springboard 7
Unit 9

Unit 9 section 4: Factors

plenary

Resources
mini-whiteboards
calculators

Tell the class that you are thinking of a number. Say:

Q I multiply my number by 15 and the product is 210. What is my number?

Remind them that *product* refers to multiplication. Ask pupils to write on their whiteboards a number sentence or equation that represents the question. Collect responses and write on the board:

□ × 15 = 210

Q We know that 15 is one factor of 210. How could we find its partner?

Draw out that 210 is a multiple of 15. It is also a multiple of the unknown number. Establish that □ × 15 = 210 means that □ = 210 ÷ 15.

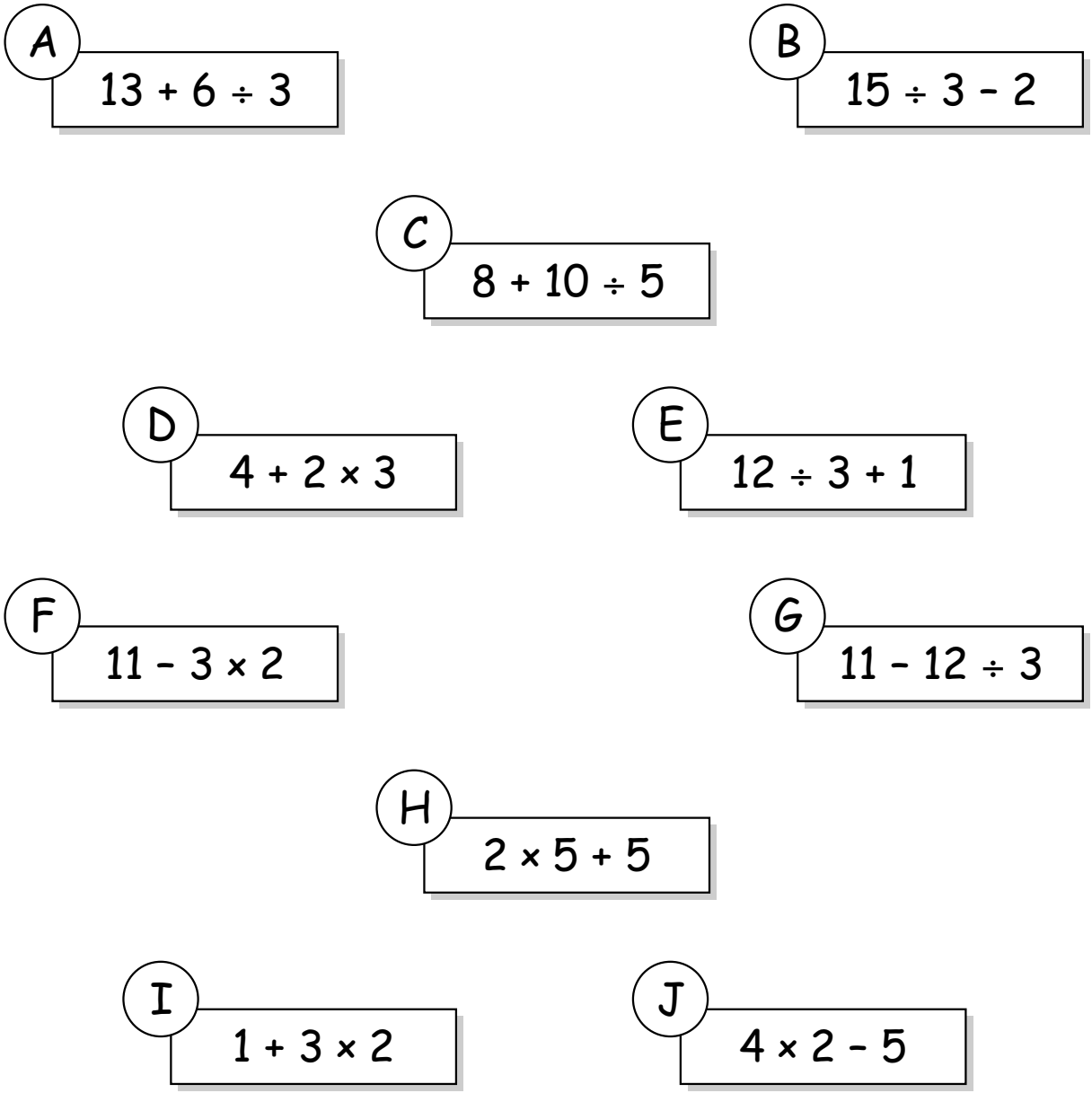
Ask pupils to use their calculators to find the answer. Check the answer by confirming that 14 × 15 = 210.

Remember

- A number that divides exactly into another number is called a factor of that number. The number 10 has four factors, 1, 2, 5 and 10.
- Factors occur in pairs. The factor pairs for 10 are 1 × 10 and 2 × 5.
- Prime numbers have only two factors, themselves and 1. Examples of prime numbers are 2, 3, 11 and 19. 1 is not a prime number.

OHT A2.1a

Make five matching pairs.



OHT A2.1b

Replace each circle with any one of +, −, ×, ÷.

Make answers of 11, 40, 19, 10.

7 ◯ 6 ◯ 2 = 11

7 ◯ 6 ◯ 2 = 40

7 ◯ 6 ◯ 2 = 19

7 ◯ 6 ◯ 2 = 10

You must not change the order of the numbers.

OHT A2.1c

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

OHT A2.1d

Write the answers.

$(4 + 2) \times 3 =$ .....

$4 + (2 \times 3) =$ .....

$(2 + 4) \times (6 + 3 + 1) =$ .....

Copy the calculation.

Put in brackets to make the answer **50**.

$4 \quad + \quad 5 \quad + \quad 1 \quad \times \quad 5 \quad = \quad 50$

Copy the calculation again.

Now put in brackets to make the answer **34**.

$4 \quad + \quad 5 \quad + \quad 1 \quad \times \quad 5 \quad = \quad 34$

OHT A2.2a

20	63	6	21	40
9	48	5	49	25
16	35	60	36	18
27	42	15	24	7
70	28	8	12	30

Rules

- Take turns to roll the dice (1 counts as 7).
- You then have a choice. Suppose you roll 5. You can:
  - either cover a multiple of 5 with one of your counters;
  - or remove one of the other player's counters from a multiple of 5.
- The winner is the first to get three counters in a line, horizontally, vertically or diagonally.

OHT A2.2b

Circle the three numbers on the grid which divide by 5 with no remainder.

84	85	86
91	92	93
98	99	100
105	106	107

Circle two numbers on the grid which can be divided by 9 with a remainder of 1.

97	98	99
107	108	109
117	118	119

Complete this three-digit number so that it is a multiple of 9.

2		

OHT A2.3a

12											
11											
10											
9											
8											
7											
6											
5											
4											
3											
2											
1											
	1	2	3	4	5	6	7	8	9	10	11	12