Algebra 1

contents

There are three lessons in this unit, **Algebra 1**.

objectives

The objectives covered in this unit are:

- Know addition and subtraction facts to 20.
- Recall multiplication facts to 10 × 10 and derive associated division facts.
- Count on and back in steps of constant size.
- Recognise and use multiples.
- Make general statements about odd and even numbers.
- Recognise squares to at least 10 × 10.
- Express simple functions in words.
- Generate and describe terms of a simple sequence, including sequences from practical contexts.
- Solve problems and investigate in number.

Using the lesson plans in this unit

These lesson plans supplement the *Springboard 7* materials for Key Stage 3 pupils working toward level 4 in mathematics. All the lessons are examples only. There is no requirement to use them. If you decide to use the lessons, you will need to prepare overhead projector transparencies (OHTs) and occasional resource sheets for pupils to use.

The lessons consolidate work at level 3 and extend into level 4. They are suitable for a group of pupils or a whole class. Whatever the size of the group, the pupils are referred to as 'the class'.

Each lesson will support about 30 to 40 minutes of direct teaching. To help match the time to your timetable, each plan refers to 'other tasks' for pupils, based on *Springboard 7* resources. Select from these, textbook exercises or your own materials to provide practice and consolidation in the main part of a lesson and to set homework. Aim to choose tasks that vary in their level of demand, to suit pupils' knowledge, confidence and rate of progress.

Although the 'other tasks' are listed for convenience at the end of the main part of the lesson, they can be offered at any point, especially between the 'episodes' that form the main activity.

The lesson starters are of two kinds: practice starters and teaching starters. The former are opportunities to rehearse skills that will be needed later in the lesson. Teaching starters introduce an idea that is then developed in the main activity.

You will need to tell pupils what they will learn in the lesson, either in the starter or at the beginning of the main activity. Use the plenary to check pupils' learning against the lesson's objectives and to draw attention to the key points that pupils should remember.

Interactive teaching programs (ITPs)

Interactive teaching programs are interactive animated visual aids that can be used with a laptop and data projector or with an interactive whiteboard. As extra support for this unit, you may find it useful to download and use these ITPs from the website www.standards.dfes.gov.uk/numeracy:

for lesson A1.1: *Number facts*

for lesson A1.2: *Number grid*
20 cards

A1.1 Number patterns

objectives

- Know addition and subtraction facts to 20.
- Count on and back in steps of constant size.
- Generate terms of a simple sequence.
- Express simple functions in words.
- Solve problems and investigate in number.

starter

Vocabulary
add
subtract
addition
subtraction
difference

Resources
interlocking cubes
mini-whiteboards
ITP *Number facts* (optional)

Hold up a stick of 10 cubes. Break it into 1 and 9, and ask how many are in each piece. Repeat with 2 and 8. Take the stick of 10 cubes again and this time hold it behind your back. Break it into two pieces and hold up one piece. Ask:

Q How many cubes can you see? (e.g. 7) **How many are hidden?** (e.g. 3)

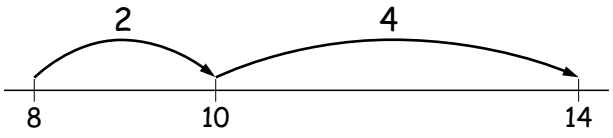
Repeat a couple more times. Remove one cube from the stick to make a stick of 9 cubes and show it to the class. Repeat the activity of holding it behind your back and breaking it into two pieces. Repeat for a stick of 8 cubes.

Ask a few questions to check pupils' understanding of the vocabulary, with an expectation that pupils answer quickly. For example:

Q What must I add to 7 to make 9?
Q Subtract 3 from 8.
Q What is the difference between 4 and 10?

Write 6 + 8 on the board. Say that you are going to show them some quick ways to add pairs of numbers between 5 and 10 in their heads.

Say that addition can be done in any order. It is usually easier to put the largest number first when adding. Write 8 + 6 on the board. Model on an empty number line how to start with 8, and then jump on 2 to reach 10. Say that you have used up 2 out of the 6, so there is still 4 to add on. Jump on 4 to reach 14.



Model what you have done with the cubes. Take a stick of 8 and a stick of 6 and hold them up. Take 2 from the 6 and attach them to the stick of 8 to make a stick of 10. Show the class that there are now sticks of 10 and 4, or 14.

Write on the board: 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14.

Start again with sticks of 8 and 6 cubes. Show a different way to add 8 and 6. Take 5 cubes from the 8 cubes to leave 3 cubes, and 5 cubes from the 6 cubes to leave 1 cube. Combine the two sticks of 5 to make 10, then the 3 and the 1 to make 4.

10 + 4 = 14

Write on the board: 8 + 6 = 5 + 3 + 5 + 1 = 10 + 4 = 14.

Repeat the two methods, including the modelling with cubes, for 7 + 9.

Ask a few more questions to emphasise the vocabulary, allowing a few seconds for pupils to make jottings on their whiteboards. Stress that they do not need to draw the diagrams but they may wish to jot down and rearrange the numbers.

Q Add 7 to 6. Find the sum of 8 and 9. What is the total of 5 and 8?

Now show how to add a pair of numbers when one is more than 10. Write on the board: 12 + 6. Show that this is the same as 10 + (2 + 6). Since the pupils know the answer to 2 + 6 is 8, the answer to 10 + 2 + 6 is 18.

You may wish to support the work above by using the ITP *Number facts*. Use it as an extra model to illustrate adding and subtracting small numbers.

Q What is the answer to 52 + 6? Explain how you worked it out.

Demonstrate how to use patterns to add a single digit to any two-digit number:

7 + 5 = 12
17 + 5 = 22
27 + 5 = 32
37 + 5 = 42, and so on.

Stress that all the basic number facts need to be remembered but that it is useful to know ways of working them out quickly if you don't know the answer straight away.

main activity

Vocabulary
pattern
rule

Resources
OHTs A1.1a (two copies), A1.1b, A1.1c

Ask the class to imagine a machine that will help them to find patterns. Show **OHT A1.1a**. Explain how the first machine will add 3 to any number, and work through the three inputs and outputs with the class.

Choose a rule for the second machine, such as '+ 5', and ask the pupils to suggest three numbers for the input boxes. Complete each output with the class.

Enter three inputs and corresponding outputs for the third machine (e.g. 4, 8, 10 and 10, 14, 16). Ask pupils to identify the rule. Prompt with questions like:

Q What do you need to do to 4 to make 10?

Use the second copy of **OHT A1.1a** for more examples. For example, identify the rule for these sets of numbers: 2, 4, 6 and 6, 12, 18 (the rule is ' ×3'), and for 35, 55, 95 and 26, 46, 86 (the rule is '– 9').

Show **OHT A1.1b**, with two linked machines. Fill in the first input boxes with 3, 5, 7, and rules of '+ 1' and '÷ 2'. Work with the class to identify the outputs (2, 3, 4). For the second example, you could use inputs of 1, 5, 7, a first rule of '– 1', and outputs of 0, 12, 18. Ask pupils to identify the second rule (' ×3').

Take the opportunity to stress that multiplication of 0 results in 0.

Work through the problem on **OHT A1.1c** with the class.

other tasks

Springboard 7
Unit 1

Unit 1 section 2: Simple sequences

1 Counting on and back in 6s page 50
2 Counting on and back in 9s page 50

You may wish to provide some further examples of function machines.

plenary

Vocabulary
sequence

Resources
OHT A1.1d

Display **OHT A1.1d**. Write 'add 2' in the first box in the rule column and 21 in the starting number box. Say that 21 is the start of a pattern or *sequence.*

Q **What will be the next number in the sequence?** (23)
Q **What will be the next number after 23?** (25)

Establish that there is a difference of 2 between each number, so the numbers go up in twos. Complete the other three terms with the class.

Q **How would you describe the numbers in this sequence?** (they are a sequence of odd numbers)

On OHT A1.1d write the rule '+ 11' and the starting number of 0.

Q **What will be the next five numbers?**

Establish that they will be 11, 22, 33, 44 and 55.

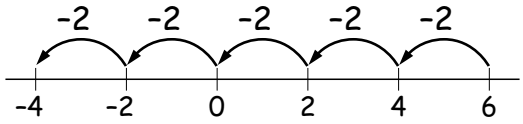
Q **What do you notice about the terms of this sequence?** (the first digit and the last digit are the same) **Will 99 be in this sequence?** (yes)

On OHT A1.1d write the rule '– 1' and the starting number of 13. Complete the next five numbers with the class.

Q **How would you describe the numbers in this sequence?** (they are counting numbers)

Q **What is the difference between this sequence and the previous two?** (the first two were going up; this one is going down)

On OHT A1.1d write the rule '– 2' and the starting number of 6. Complete 4, 2, 0 with the class. Establish that the next two terms term would be –2 and –4 (negative two and negative four). Confirm by using an empty number line.



Complete the last sequence, with a rule of ' $\times 2$' and a starting number of 3.

Q **What do you notice about the terms of this sequence?** (they are all even)

Remember

- Try to remember addition and subtraction facts to 20 so that you know them 'by heart'. If you forget one, there are strategies for working them out quickly.
- Addition can be done in any order. It often helps to put the larger number first.
- Decide whether a sequence is going up or going down.
- Look for patterns in the numbers of a sequence.
- Check whether the difference between each term in a sequence is always the same. If so, the rule is to add or subtract the number that is the difference.

A1.2 Generating sequences

objectives

- Count on and back in steps of constant size.
- Generate and describe terms of a simple sequence.
- Make general statements about odd and even numbers.
- Recognise and use multiples.

starter

Vocabulary
odd
even
multiple
two-digit number
three-digit number

Resources
OHT A1.2a
mini-whiteboards

Show **OHT A1.2a**, a grid of numbers 1 to 60. Say together the even numbers: two, four, six, eight, …, twenty. Point to each number on the grid as pupils say it. Then say the odd numbers from one to nineteen. Ask:

Q Is 9 odd or even? (odd) **Is 24 odd or even?** (even)
Q How can you tell if a number is odd or even?

Establish that the units digit of an odd number is 1, 3, 5, 7 or 9, and that of an even number is 0, 2, 4, 6 or 8. Write a two-digit odd number on the board, e.g. 87. Ask:

Q Is this number odd or even? What is the next odd number after 87? What is the odd number before 87?

Repeat with 236, 5731.

Ask pupils to write on their whiteboards a two-digit odd number greater than 60, then a three-digit even number.

Explain that the even numbers are *multiples* of 2 and are numbers that divide exactly by 2. They include the numbers in the two times table. We also count 0 as even.

Count together in fives to 60, pointing to the numbers on the grid. Explain that pupils have been counting in multiples of 5. Multiples of 5 divide exactly by 5.

Q What do you notice about the units digits of multiples of 5? (0 or 5)
Q Is 87 a multiple of 5? (no) **Is 95 a multiple of 5?** (yes) **How do you know?**
Q What is the next multiple of 5 after 95? How did you work it out? (add 5)
Q How can you tell if a number is a multiple of 10? (its units digit is 0)
Q What is the next multiple of 10 after 150? (160) **Before 150?** (140)

main activity

Vocabulary
pattern
sequence
term
rule

Resources
OHTs A1.2a, A1.2b
OHP pens (red, blue)
ITP *Number grid* (optional)

You may wish to support the main activity of this lesson by using the ITP *Number grid*, downloaded from www.standards.dfes.gov.uk/numeracy. Use it instead of or in addition to the OHTs. Select options and ask questions to consolidate pupils' understanding.

Leave **OHT A1.2a** on the projector. Say together the three times table: one three is three, two threes are six, … Ring the multiples 3, 6, 9, … up to 30 on the grid with a red pen as you say them.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60

Remind pupils that multiples of 3 are numbers that divide exactly by 3 and include the numbers in the three times table.

Q Are the multiples of 3 odd or even? (they alternate)

Q Can you describe the pattern the ringed numbers make? (sloping lines) **How will it continue?** (the sloping lines will extend across the grid)

Q Will 36 be in the extended three times table? How do you know?

Encourage pupils to look down the sloping lines of ringed numbers to judge whether 36 would be included. Repeat for 57 and 41.

Repeat this process for the four times table, marking them with a blue square. Draw out that all the multiples of 4 are even.

Q Which numbers have both a red circle and blue square round them?

Write the sequence of numbers on the board: 12, 24, 36, 48, 60.

Q What do you notice about these numbers? Describe the sequence.

Establish that the numbers: go up in twelves; are the 12 times table numbers; are in both the three times and four times tables.

Q What would be the next number in the sequence? (72) **And the next?** (84) **How did you work it out?**

Explain that 'add 12' is the *rule* for the sequence.

Show **OHT A1.2b**. Write 2 and 5 in the first two boxes of the first row. Say that these numbers are the start of a sequence. They are called the first two *terms*.

Q What might be the next number in the sequence? How would you describe the rule for the sequence?

Take responses, asking pupils to explain their reasoning. Say that there are many possibilities but that you were thinking of a sequence in which the third term is 8.

Q What might be the next term after 8? (11)
Q How would you describe the rule for this sequence?

Establish that there is a difference of 3 between each term, so the numbers go up in threes. The rule is 'add 3'. Write this on the OHT.

On **OHT A1.2b** write the sequence 17, 13, 9, 5, 1.

Q How is this sequence different from the last sequence?

Establish that the numbers are going down and the rule is 'subtract 4'.

Q What will be the next term in the sequence?

Establish that the next term would be –3. Confirm if necessary by using an empty number line.

Complete and extend some more sequences:

3, 8, ☐, 18, 23, ☐, 33

Q What do you notice about the terms of this sequence? Will 95 be in this sequence? (no, its last digit is not 3 or 8)

105, ☐, 101, ☐, 97

Q Will 49 be in this sequence? (yes, it is an odd number)

1, 2, 4, ☐, 16, 32

Q Will 999 be in this sequence? (no, it is not an even number)

other tasks

Springboard 7
Unit 1

Unit 1 section 2: Simple sequences

plenary

Resources
OHT A1.2c
ITP *20 cards* (optional)

Work through the examples on **OHT A1.2c** with the class. Ask questions such as:

Q Will … be in the sequence?

Ask pupils to justify their answers by explaining their reasoning.

You could, if you wish, extend the plenary by using the ITP *20 cards*, asking pupils to predict the next number in the sequence from the stack of cards.

Remember

- Even numbers always end in 0, 2, 4, 6, 8. They are multiples of 2 so they divide exactly by 2. They include the numbers in the two times table.
- Odd numbers always end in 1, 3, 5, 7, 9. They leave a remainder of 1 when they are divided by 2.
- Multiples of 10 end in 0. They include numbers in the ten times table.
- Multiples of 5 end in 0 or 5. They include numbers in the five times table. Every other multiple of 5 is a multiple of 10.
- Check whether the difference between each term in a sequence is always the same. If so, the rule is to add or subtract the number that is the difference.

A1.3 Square numbers

objectives

- Recall multiplication facts to 10 × 10 and derive associated division facts.
- Recognise squares to at least 10 × 10.
- Generate sequences from practical contexts.
- Solve problems and investigate in number.

starter

Vocabulary
multiple
sequence
rule
term

Resources
OHT A1.3a

Display **OHT A1.3a**. Ask:

Q Who knows the two times table?

Invite a pupil to write the multiples of 2 along the second row of the grid, so that 2 is below 1, 4 is below 2, 6 is below 3, and so on.

Q Who knows the three times table?

Invite a pupil to write the multiples of 3 along the third row: 3, 6, 9, 12, …

Quickly complete the first column, then point to the first three numbers in the second column: 2, 4, 6.

Q How will the numbers continue down this column?

Invite a pupil to complete the second column.

Q How would you describe the rule for this sequence? What do you notice about the terms?

Establish that the rule is 'add 2', and the terms are the even numbers or multiples of 2. It is the two times table again. Repeat for the third column. Point out that each set of multiples appears twice on the grid: across a row and down a column.

Q Who knows the five times table?

Repeat the process for the five times table, writing the multiples in the fifth row and the fifth column. Do the same for the ten times and four times tables.

Point out that there are only 16 facts left to fill in, from 6 × 6 to 9 × 9. Of these, six occur twice because 7 × 8 = 8 × 7. Call out the ten remaining facts that need to be learned, ask pupils for the answers, and complete the grid.

6 × 6	7 × 6	8 × 6	9 × 6
	7 × 7	8 × 7	9 × 7
		8 × 8	9 × 8
			9 × 9

Practise chanting the multiples of 7 to 70.

Q What is the rule for this sequence? (add 7) **What would be the next term in the sequence?** (77) **And the next?** (84)

Stress strategies for helping to work out awkward facts:

- reversing the multiplication, e.g. $6 \times 7 = 7 \times 6$;
- doubling, e.g. $7 \times 3 = 21$ so $7 \times 6 = 42$;
- using one multiple to find the next, using the rule for the sequence, e.g. $8 \times 5 = 40$, so $8 \times 6 = 40 + 8 = 48$.

main activity

Vocabulary
square number
arrangement

Resources
OHT A1.3b
coloured OHP pen
mini-whiteboards

Show **OHT A1.3b**.

Point to the 3 by 3 pattern of dots. Ask:

Q How many dots?

Write 9 in the empty box below the dots.

Q Can you describe to me the arrangement of the pattern? (a 3 by 3 square)

Write 3×3 in the box below 9. Continue in this way up to 6×6.

Point to the row of numbers 1, 4, 9, 16, 25, 36. Tell the class that these are called *square numbers* – each of them is the result of multiplying a number by itself and can be represented by counters arranged in a square shape.

Say that there is a special way of reading and writing square numbers. Point to 1^2 and 2^2 in the boxes below 1×1 and 2×2, saying 'one squared, two squared'. Ask pupils to write on their whiteboards three squared, four squared, five squared and six squared, and enter these in the table.

Replace **OHT A1.3a**, the multiplication square, with all multiples entered. Ring the numbers 1, 4, 9, 16, 25, 36 with a coloured pen. Point out how 16 lies in the fourth column and the fourth row, and is worked out as 4×4.

Say that you have ringed the first six numbers in the sequence of square numbers.

Q What is the next number in the sequence? (49) **And the next?** (64)

Q How would we write seven squared? (7×7 or 7^2)

Ring 49 and 64.

Q What number do you square to get 81? (9)
Q What is the square of ten? (100)
Q What is the next square number after 100? (11^2 or 121)

Ring 81 and 100.

Chant the sequence of square numbers, pointing to each one on the OHT as you say it: one, four, nine, sixteen, … Remove the OHT and chant again.

Say to the class that they are going to investigate sums of two square numbers.

Q What is six squared? (36) **What is two squared?** (4) **What is the sum of six squared and two squared?** (40)

Say to the class that 40 is an interesting number. It is the sum of two square numbers. Say that 13 is another interesting number. It is also the sum of two squares. Ask pupils to discuss in pairs what the two squares might be, then ask:

Q What are they? (2 and 3)
Q Which other numbers up to 30 are the sum of two different squares?

Ask pupils to work on the problem in pairs for a few minutes, then draw the class together. Say that when they investigate numbers it helps to work systematically. They could start by adding 1^2 to each of 2^2, 3^2, 4^2 and 5^2. When 1^2 is added to 6^2 the answer is too big, so they could now try adding 2^2 to each of 3^2, 4^2 and 5^2.

Q Why don't we need to try adding 2^2 to 1^2? (it is the same as 1^2 added to 2^2)

Leave the pairs to work for a few more minutes, then gather the complete set of results: 5, 10, 13, 17, 20, 25, 26, 29.

Q How do we know that we have found all the possibilities?

Establish that pupils have worked systematically through all the possible pairs and so have checked them all.

other tasks

Springboard 7
Unit 1

Unit 1 section 1: Square numbers

plenary

Resources
calculators
OHP calculator

Say that you are looking for a mystery number which, when squared, gives the answer 225. Ask the class:

Q Is the mystery number less than 10?

Establish that squares of numbers less than 10 are all less than 100. The mystery number must be bigger than 10.

Q Is the mystery number more than 20?

Confirm on the OHP calculator that $20 \times 20 = 400$, so the mystery number must be less than 20.

Q Could the mystery number be even?

Establish that an even number multiplied by an even number is always even, so the mystery number cannot be even. Say that this leaves 11, 13, 15, 17 and 19. Write these five numbers on the board.

Q Which of these five numbers could it be? Explain why you think so.

Invite pupils to try their suggestions on the OHP calculator (or their own calculator). Draw out that the only possibility is 15, since it is the only number which, when multiplied by itself, will result in a number with a units digit of 5.

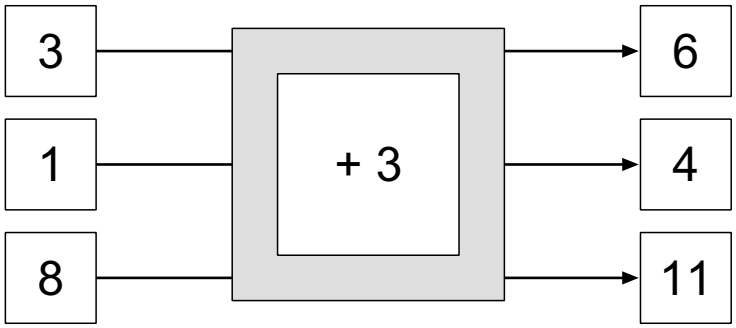
Write $15^2 = 15 \times 15 = 225$ on the board, and confirm on the OHP calculator.

Remember

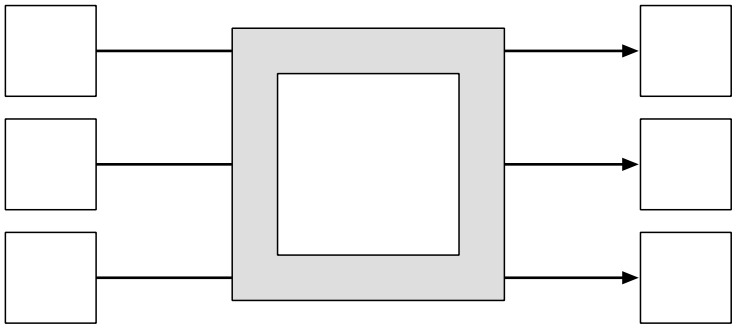
- A square number is the result of multiplying a number by itself.
- 49 is the square of 7. It can be written as 7^2.
- A square number can be represented by dots arranged in a square.
- When solving a problem, it helps to work systematically.

OHT A1.1a

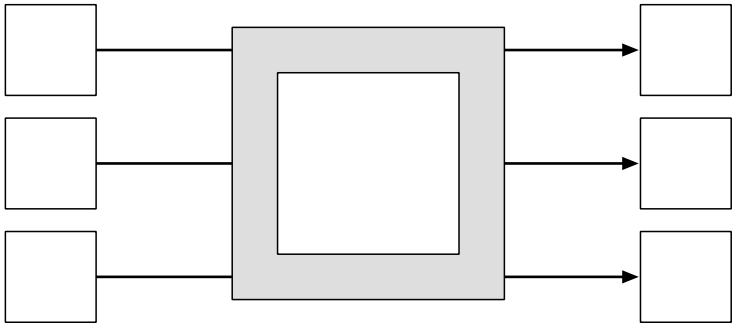
This machine adds 3 to any number.



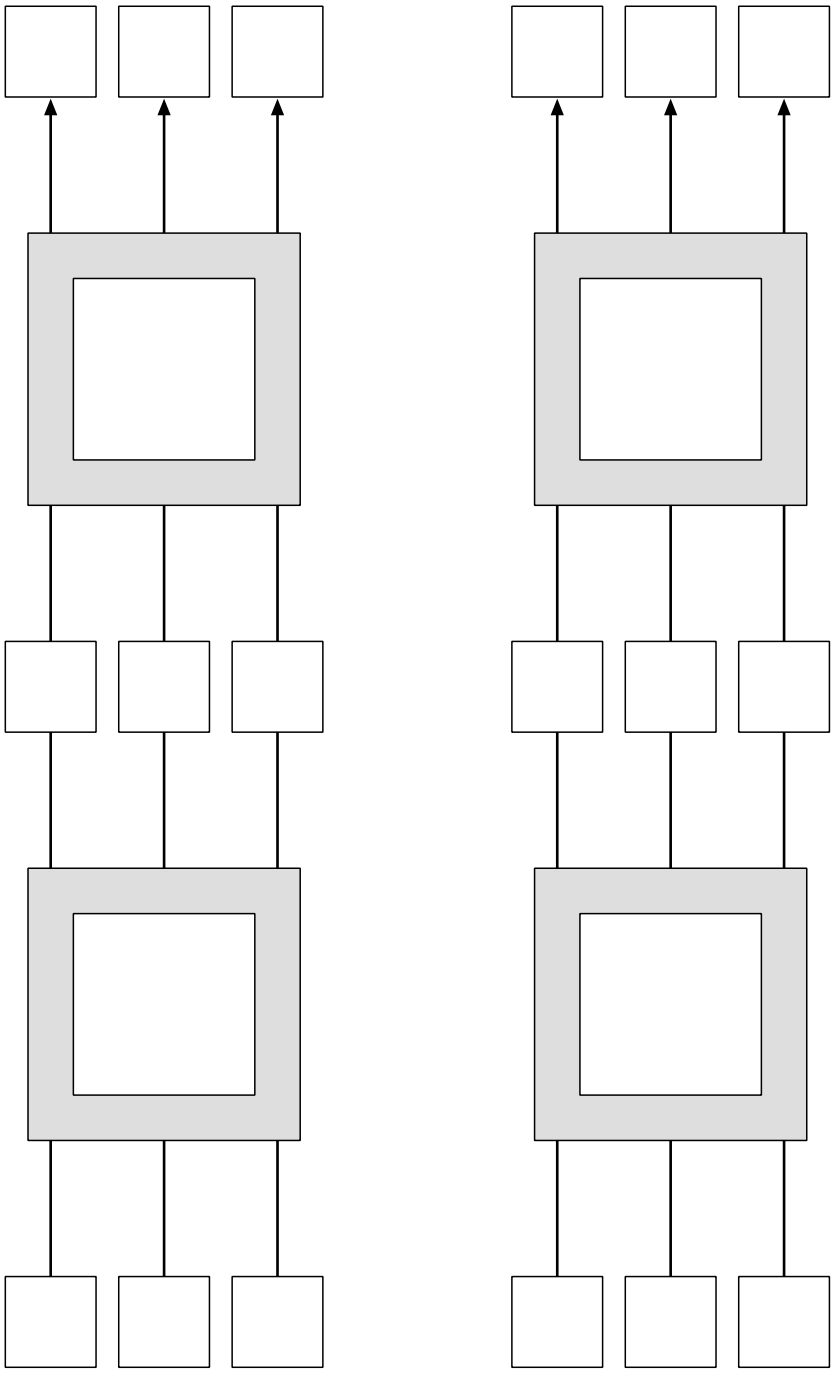
What answers will this machine give?



What is the rule for this machine?

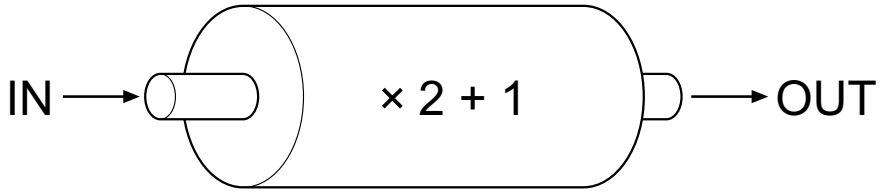


OHT A1.1b



OHT A1.1c

This number machine multiplies all numbers by 2, and then adds 1.



Write the missing numbers in the table.

IN	OUT
5	11
13	
	117

OHT A1.1d

Rule	Starting number	Next five numbers				

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60

OHT A1.2b

Rule					
Sequence of numbers					

OHT A1.2c

The rule for these number sequences is
'double then subtract 1'.

Write in the missing numbers.

2 → 3 → 5 → 9 → .....

..... → 13 → 25 → 49

Some number chains start like this: 1 → 5 →

Show three different ways to continue this chain.
For each chain write down the next three numbers.
Then write down the rule you are using.

First chain

1 → 5 → ..... → ..... → ..... Rule .....

Second chain

1 → 5 → ..... → ..... → ..... Rule .....

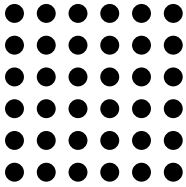
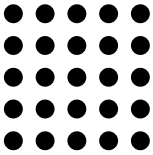
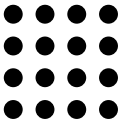
Third chain

1 → 5 → ..... → ..... → ..... Rule .....

OHT A1.3a

1	2	3	4	5	6	7	8	9	10

OHT A1.3b

			
			
			
			
	4	2×2	2^2
	1	1×1	1^2