Shape, space and measures 1

contents

There are three lessons in this unit, **Shape, space and measures 1**.

objectives

The objectives covered in this unit are:

- Convert between m and cm, and cm and mm.
- Use a ruler to measure and draw lines to the nearest centimetre or millimetre.
- Understand that area is measured in square centimetres (cm^2).
- Know and use the formula for the area of a rectangle; calculate the perimeter and area of shapes made from rectangles.
- Solve problems and investigate in shape, space and measures.
- Explain and justify methods and conclusions.

Using the lesson plans in this unit

These lesson plans supplement the *Springboard 7* materials for Key Stage 3 pupils working toward level 4 in mathematics. All the lessons are examples only. There is no requirement to use them. If you decide to use the lessons, you will need to prepare overhead projector transparencies (OHTs) and occasional resource sheets for pupils to use.

The lessons consolidate work at level 3 and extend into level 4. They are suitable for a group of pupils or a whole class. Whatever the size of the group, the pupils are referred to as 'the class'.

Each lesson will support about 30 to 40 minutes of direct teaching. To help match the time to your timetable, each plan refers to 'other tasks' for pupils, based on *Springboard 7* resources. Select from these, textbook exercises or your own materials to provide practice and consolidation in the main part of a lesson and to set homework. Aim to choose tasks that vary in their level of demand, to suit pupils' knowledge, confidence and rate of progress.

Although the 'other tasks' are listed for convenience at the end of the main part of the lesson, they can be offered at any point, especially between the 'episodes' that form the main activity.

The lesson starters are of two kinds: practice starters and teaching starters. The former are opportunities to rehearse skills that will be needed later in the lesson. Teaching starters introduce an idea that is then developed in the main activity.

You will need to tell pupils what they will learn in the lesson, either in the starter or at the beginning of the main activity. Use the plenary to check pupils' learning against the lesson's objectives and to draw attention to the key points that pupils should remember.

Interactive teaching programs (ITPs)

Interactive teaching programs are interactive animated visual aids that can be used with a laptop and data projector or with an interactive whiteboard. As extra support for this unit, you may find it useful to download and use these ITPs from the website www.standards.dfes.gov.uk/numeracy:

for lesson S1.1: *Polygon*
Ruler

for lesson S1.2: *Area*

for lesson S1.3: *Area*

S1.1 Lines, length and perimeter

objectives

- Convert between m and cm, and cm and mm.
- Use a ruler to measure and draw lines to the nearest centimetre or millimetre.
- Calculate the perimeter of shapes made from rectangles.
- Solve problems and investigate in shape, space and measures.

starter

Vocabulary
line
stroke
straight
curved

Ask pupils to think about the letters of the alphabet – just the capital letters. Say that some of these can be drawn with a single stroke without taking your pen from the paper or going over a line twice. Explain that by 'stroke' you mean a continuous line with straight or curved parts. Write L on the board as an example. Ask pupils to close their eyes and imagine. Tell them that you are going to name some capital letters. They have to say to themselves whether the letters can or can't be drawn with a single stroke.

Read out slowly a list of letters, pausing between each one: M, A, T, H, S. Ask pupils to discuss briefly with a partner what they thought, then take feedback. Add M and S to the L on the board as two more examples. Ask pupils:

Q Which other capital letters can be drawn with a single stroke?

Allow a couple of minutes for pupils to discuss the question with a partner, then gather the remaining letters: C, I, J, N, O, U, V, W, Z (D and P are also possibilities).

main activity

Vocabulary
metre (m)
centimetre (cm)
millimetre (cm)
length
height
width
distance
perimeter

Resources
ITPs *Polygon* and *Ruler* (optional)
metre stick, marked in cm
mini-whiteboards
transparent ruler, marked in cm and mm
(continued on next page)

You could, if you wish, support the main activity by using the ITPs *Ruler* and *Polygon*, downloaded from www.standards.dfes.gov.uk/numeracy.

Tell the class that this lesson is about drawing, measuring and estimating the length of straight lines. Ask pupils to stand up and hold one hand one metre from the floor. Choose a pupil to take the metre stick and to compare it with a few of the estimates.

Say that you want pupils to estimate some lengths in metres, and to write their estimates on their whiteboards. Remind them to include the abbreviation *m* for metres when they write their estimates.

Q What is your estimate in metres of the height of this room? Of the classroom door? Of the top of the display board? Of your table?

Q What is your estimate in metres of the length of this room? The width?

Show the metre stick again and point to the divisions for centimetres.

Q How long is the space between each small division? (1 cm)

Q How many centimetres in one metre? (100 cm)

Write on the board: 1 m = 100 cm.

Place on the overhead projector a transparent ruler, making sure that the zero mark and the 15 cm mark are visible. Point out the centimetre markings, and the half-centimetre markings. Place a pen or pencil against the ruler to measure the length,

Resources (cont.)
30 cm ruler and pencil for each pupil
OHTs S1.1a, S1.1c
blank OHT
Resource S1.1b

demonstrating how to line up one end with the zero mark on the ruler. Show how to measure to the nearest centimetre by rounding up or down as appropriate.

Refer pupils to their own rulers. Ask them to measure the length of their pencil to the nearest centimetre. Stress that the zero mark on the ruler must line up with the end of the pencil. Discuss the lengths and record one or two of them on the board, including the abbreviation *cm* for centimetres. Repeat the process, asking pupils to measure one or two more lengths to the nearest centimetre, such as the height and width of their exercise books.

Refer again to the transparent ruler on the projector.

Q **What are the divisions between 7cm and 8cm?** (millimetres) **How many of them are there?** (10)

Write on the board: 1cm = 10mm. Explain that *mm* is a short way of writing millimetre, and that each millimetre is one tenth of a centimetre. Say that using millimetres can help to measure lengths more accurately.

Show **OHT S1.1a**. Demonstrate how to measure the length of line A to the nearest millimetre, positioning the transparent ruler's zero mark against one end.

Q **How many whole centimetres long is this line?** (e.g. 8cm) **How many extra millimetres is it?** (e.g. 4mm) **What is its length?** (e.g. 8cm 4mm)

Q **How many millimetres long is the line?** (e.g. 84mm)

Write on the board: 8cm 4mm = 84mm. Invite pupils to the projector to measure lines B, C and D.

Now use a blank transparency to show the class how to draw a line XY of length 7cm. Position the transparent ruler and draw along it from the zero mark on the left to the 7cm mark on the right. Label the line X at one end and Y at the other, and write 7cm about halfway along it. Repeat by drawing a line 97mm long.

Give out **Resource S1.1b** and ask pupils to measure and draw the lines indicated.

Display **OHT S1.1c**. Explain that the grid is in centimetres.

Q **What is the name of shape A?** (a rectangle) **And of shape D?** (a square)
Q **How do we find the perimeter of shape A?**

Establish that perimeter is the distance around the shape. Ask pupils to work out the perimeters of rectangles A, B, C and D and to write the answers on their whiteboards in centimetres. Discuss the answers, writing on the board:

perimeter of A is 5 + 3 + 5 + 3 = 16cm
perimeter of B is 2 + 10 + 2 + 10 = 24cm
perimeter of C is 4 + 6 + 4 + 6 = 20cm
perimeter of D is 5 + 5 + 5 + 5 = 20cm

Establish that another way to find the perimeter of a rectangle is to add together the length and width, and multiply by 2. Record on the board next to the above:

$(5 + 3) \times 2 = 16$cm
$(10 + 2) \times 2 = 24$cm
$(6 + 4) \times 2 = 20$cm
$(5 + 5) \times 2 = 20$cm

other tasks

Springboard 7
Unit 3

Unit 3 section 1: Perimeter

Unit 3 section 2: Centimetres and millimetres

plenary

Resources
mini-whiteboards

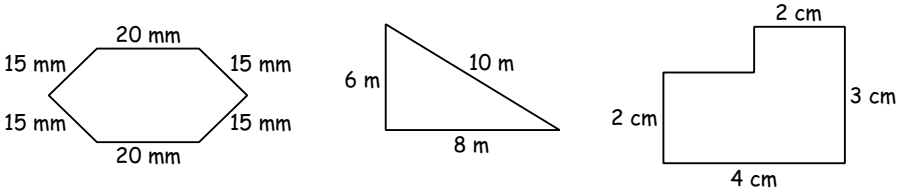
Tell the class you are going to sketch a rectangle with a perimeter of 10 cm.

Q What could be the length and the width of my rectangle?

Establish that half the perimeter is 5 cm, so that the length plus the width must be 5 cm. The possibilities (for whole numbers of centimetres) are 4 cm and 1 cm, and 3 cm and 2 cm. Sketch both the rectangles and label the length and width in each case. Explain that your drawings do not show true measurements.

Ask pupils to sketch on their whiteboards a rectangle with a perimeter of 12 centimetres, and to label the length and the width of the rectangle.

Now sketch on the board a series of shapes, one by one. Label each side with a suitable length, in metres, centimetres or millimetres. For example:



Ask pupils to calculate the perimeter of each shape and to write the answer on their whiteboards. In the case of the third diagram, invite a pupil to explain how to calculate the two missing lengths of edges.

Remember

- When you are using a ruler to measure or draw lines, make sure that you position the zero mark on the ruler at the end of the line.
- Perimeter is the distance all the way round a shape. It is measured in units of length such as metres, centimetres or millimetres.
- To find the perimeter of a rectangle, add length and width, then multiply by 2.
- 1 metre is the same as 100 centimetres, and 1 centimetre is the same as 10 millimetres.

S1.2 Area of a rectangle

objectives

- Understand that area is measured in square centimetres (cm^2).
- Know and use the formula for the area of a rectangle; calculate the area of shapes made from rectangles.
- Explain and justify methods and conclusions.

starter

Vocabulary
area
square centimetres (cm^2)

Resources
OHT S1.2a

Tell the class that this lesson is about finding areas. Explain that area is the space inside a shape and is measured in square units: for example, centimetre squares, square centimetres or cm^2.

Say that one way to estimate an area is to count the squares that it covers. Show **OHT S1.2a** and invite a pupil to count the squares covered by the wood. Count any whole squares first, then look for parts of squares that would combine to make a whole square. Show pupils how to mark off the parts of squares that they use to avoid counting them twice.

Work with the class to establish an estimate for the area of the island in unit squares.

main activity

Vocabulary
area
length
breadth
width

Resources
OHTs S1.2b, S1.2d
Resource S1.2c (one OHT and copies for individual pupils)
mini-whiteboards
ITP *Area* (optional)

You could, if you wish, support the main activity by using the ITP *Area*, downloaded from www.standards.dfes.gov.uk/numeracy. Use it as an alternative to or in addition to the OHTs. Select options and ask questions to consolidate pupils' understanding.

Show **OHT S1.2b**. Tell the class that it is a grid of 1 cm by 1 cm squares. Ask pupils to find the area of rectangle A by counting the rows of squares and to write the answer on their whiteboards. Repeat with rectangles B and C. Write on the board:

area of A is $6 \times 4 = 24\,\text{cm}^2$
area of B is $12 \times 2 = 24\,\text{cm}^2$
area of C is $8 \times 3 = 24\,\text{cm}^2$

Point out that the areas of rectangles A, B and C are the same but that their perimeters are all different. Stress that to find the area of a rectangle we need to know its length and width, or breadth. Write on the board:

area of rectangle = length $\times$ breadth

Q What is the name of shape D? What are its area and perimeter?

Take responses. Point out that the square has the same perimeter as rectangle A: $(5 + 5) \times 2 = 20\,\text{cm}$, but it has a larger area. Write:

area of D is $5 \times 5 = 25\,\text{cm}^2$

Q What is the name of shape E? How do we find its area?

Establish that the area of this triangle is half the area of a 5 cm by 5 cm square. Write:

area of E is $\frac{1}{2}$ of $25\,\text{cm}^2 = 12.5\,\text{cm}^2$

Confirm by counting the squares of the triangle.

Give out copies of **Resource S1.2c**. Explain that the grid is in centimetres. Ask pupils to find the area of the square in the top left corner. Encourage them to calculate the area by multiplying the length by the width, $4 \times 4 = 16\,cm^2$. Emphasise that the units are square centimetres. Ask them to write the area in the centre of the square, reminding them to include the units.

Q How can we work out the area of the triangles in the top row?

Get the pupils to draw rectangles to surround the triangles and to use the fact that the area of the triangle is half the area of the rectangle.

Ask pupils to find the areas of the other shapes. Encourage them to draw in surrounding rectangles, and to find and write on the sheet the areas of any rectangles or triangles that form part of the original shape.

Display an OHT made from **Resource S1.2c**. Collect answers and invite individual pupils to explain their methods and conclusions to the rest of the class.

Show **OHT S1.2d**, with two rectangles made up from squares. Establish the area and the perimeter for each rectangle.

Indicate the compound shape that combines the two rectangles.

Q What is the area of this new shape? And the perimeter?

Q The area of the new shape is the same as the areas of the original two shapes added together. Why?

Q The perimeter of the new shape is not the same as the perimeters of the original two shapes added together. Why? What has happened to part of the perimeter of the original two shapes? (parts are now inside the shape and are not part of the new perimeter)

Now refer to the outlines of shapes on the bottom row, pointing out the dimensions. Ask pupils to sketch the compound shape on their whiteboards and to mark in all the dimensions for each side.

Q What is the perimeter of this shape? (20 cm)

Q How could we find the area of this shape? (add the areas of the two rectangles)

Q Is there another way to find the area of the compound shape?

Ask pupils to discuss this question in pairs. Remind them that in their last lesson they found the areas of shapes by drawing the surrounding rectangle. Ask them to draw in the rectangle that surrounds the compound shape, to work out its dimensions and also the dimensions of the two extra small rectangles. Show them how they can subtract the areas of the two small rectangles from the area of the surrounding rectangle to find the area of the compound shape.

Establish that each method produces the same result. Stress that pupils can choose the method that they think will work best, depending on the problem.

other tasks

Springboard 7
Unit 3

Unit 3 section 4: Area

plenary

Resources
Resource S2.1e
mini-whiteboards

Tell the class you are going to sketch a rectangle with an area of 20 square centimetres. Each side is a whole number of centimetres.

Q What could be the length and the width of my rectangle?

Establish that the length multiplied by the width must be 20 square centimetres. The possibilities are 1cm by 20cm, 2cm by 10cm, and 4cm by 5cm. Sketch all three rectangles on the board and label the length and width in each case. Explain that your drawings do not show true measurements.

Ask pupils to sketch on their whiteboards a rectangle with an area of 12 square centimetres, and to label the length and the width of their rectangle. Point out the three possibilities: 1cm by 12cm, 2cm by 6cm, and 3cm by 4cm.

Q What is the same about all three rectangles? (the area)

Q What is the perimeter of each rectangle? (26cm, 16cm, and 14cm)

Give out copies of **Resource S1.2e**. Ask pupils to complete the two questions. Invite individual pupils to explain their methods to the rest of the class.

Remember

- Area is the space inside a flat shape. It is measured in square units such as square centimetres, written as cm^2.
- Areas of shapes can be estimated or worked out by counting squares or part squares.
- Areas of some shapes can be found by surrounding them by a rectangle, working out the area of the extra parts, and subtracting this from the area of the surrounding rectangle.
- Shapes that have the same area do not necessarily have the same perimeter.

S1.3 Solving problems

objectives

- Calculate the perimeter and area of shapes made from rectangles.
- Solve problems and investigate in shape, space and measures.
- Explain and justify methods and conclusions.

starter

Vocabulary
square
perimeter
area

Ask the class to imagine a single, square paving stone, of the kind used to make pavements. The paving stone is 1 metre by 1 metre. Ask them to imagine 12 of the paving stones placed in a line to make a long thin path.

Next, ask them to think of a path 2 stones wide and 10 long.

Q **How many paving stones would you need to make the path?**

Q **What would be the perimeter of the path?** (24 m) **And its area?** (20 m^2)

Q **If you had 15 paving stones, what square or rectangular pavements could you make? What if you had 25 paving stones?**

main activity

Vocabulary
identical
tetronimo
pentomino
symmetry
area
perimeter

Resources
cm squared paper
mini-whiteboards
OHTs S1.3a (two copies), S1.3b
ITP *Area* (optional)

You could, if you wish, support the main activity by using the ITP *Area*, downloaded from www.standards.dfes.gov.uk/numeracy.

Say to the class:

Q **Imagine two identical 1 centimetre by 1 centimetre squares, side by side, joined by one edge. What is the area of the whole shape?** (2 square centimetres) **What is the perimeter of the whole shape?** (6 cm)

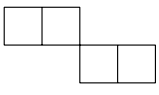
Ask the class to imagine adding a third identical square to the shape. Invite a pupil to the board to sketch the new shape.

Q **Does anyone have a different shape?**

Establish that there are two possibilities.

Q **Which shape has the longest perimeter?** (they are the same)

Ask pupils to work in groups of four. Give each pupil a piece of squared paper and ask them to draw secretly a shape made from four squares touching edge to edge. They should make sure that nobody else can see their shape. When they have drawn it they should turn over their piece of paper. Stress that the shapes must connect with each other along a whole edge, so that a shape such as this would not be acceptable.

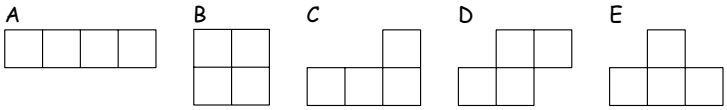


In turn within the group, each pupil then looks at their shape without letting others see it, and describes it for the rest of the group to draw. When the three other pupils have finished their drawing, they should compare with the original shape.

Tell the class that shapes made from four squares are called tetrominoes. Ask each

group to cooperate and to draw as many different tetromino shapes as they can. Tell them that just turning a shape around or flipping it over does not count as different.

Put the first copy of **OHT S1.3a** on the projector. Invite different pupils to come to the projector to draw a tetromino. Continue adding tetrominoes until all five different shapes have been collected. Label the five shapes A to E. Discuss how to check the different possibilities by considering systematically adding one more square to the two shapes they made from three squares.



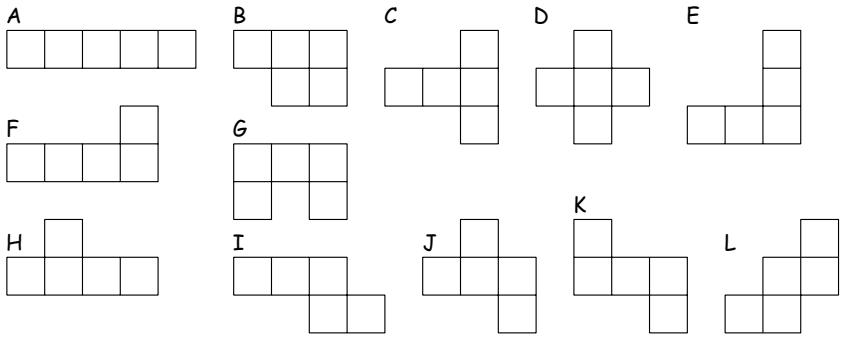
Ask the class to answer the following questions using their whiteboards.

Q **What is the area of each tetromino?** (4 square centimetres)

Q **Which tetromino has the shortest perimeter?** (B)

Q **Which of the tetrominoes have a line of symmetry?** (A, B, E) **Do any have more than one line of symmetry?** (A, B)

Repeat by asking the groups to make shapes from five identical squares (pentominoes). Invite different pupils to add different shapes to the second copy of **OHT S1.3a**. When they have run out of suggestions, display the top part of **OHT S1.3b** to show the complete set of 12 different shapes.

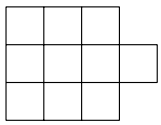


Q **What is the area of each pentomino?** (5 square centimetres)

Q **Which pentomino has the shortest perimeter?** (B)

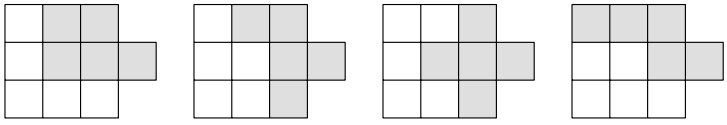
Q **Which of the pentominoes have a line of symmetry?** (A, C, D, E, G, L) **Do any have more than one line of symmetry?** (A, D)

If time allows, reveal this shape on **OHT S1.3b**.



Tell pupils to work again in their groups of four and ask them:

Q **Can you find four different pairs of pentominoes to fit into this shape?**



If any groups finish quickly, ask them to choose any pentomino and to fit six of the same shape together to form a pattern. There should be no holes or spaces between the six shapes. They should try to do this so that the perimeter of the shape formed from the six pentominoes is as small as possible.

Q **What is the area of your shape made from six pentominoes?** (30 square centimetres) **What is the perimeter of your shape?**

other tasks

Springboard 7
Unit 3

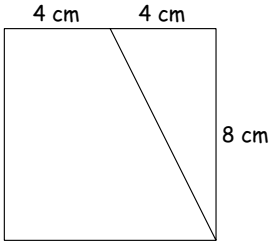
Unit 3 section 1: Perimeters
Star challenge 3: Perimeter puzzle page 112

Unit 3 section 4: Area
Star challenge 9: Wire rectangles page 129

plenary

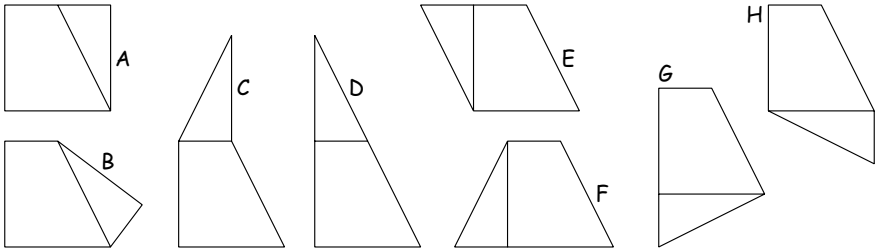
Resources
squared paper
rulers
scissors

Give each pupil a piece of squared paper. Tell them that they will need their rulers. Ask them to use a ruler to draw an 8cm by 8cm square on the squared paper, and to join the midpoint of one side to the opposite corner. They should cut out the two pieces formed.



Working with a partner, they should put the pieces together, joining edges that are the same length.

Q **How many different shapes can you make?** (8 including the original)



Q **How do you know that you have found them all?** (each possible pair of matching edges has been joined in the two possible ways)

Q **What is the same about all the shapes that you have made?** (they all have the same area) **How do you know?** (they were all made from the same original 8 cm by 8 cm square)

Q **Can you explain why the perimeter of shape B is the same as the perimeter of the original square?**

Q **Which shape has the smallest perimeter?** (A and B) **And the longest?** (C and D)

Ask pupils to use their rulers and to find out by measuring and calculating the length of the longest perimeter (about 42cm).

Remember

- Shapes with the same area can have different perimeters.

OHT S1.1a

line A

line B

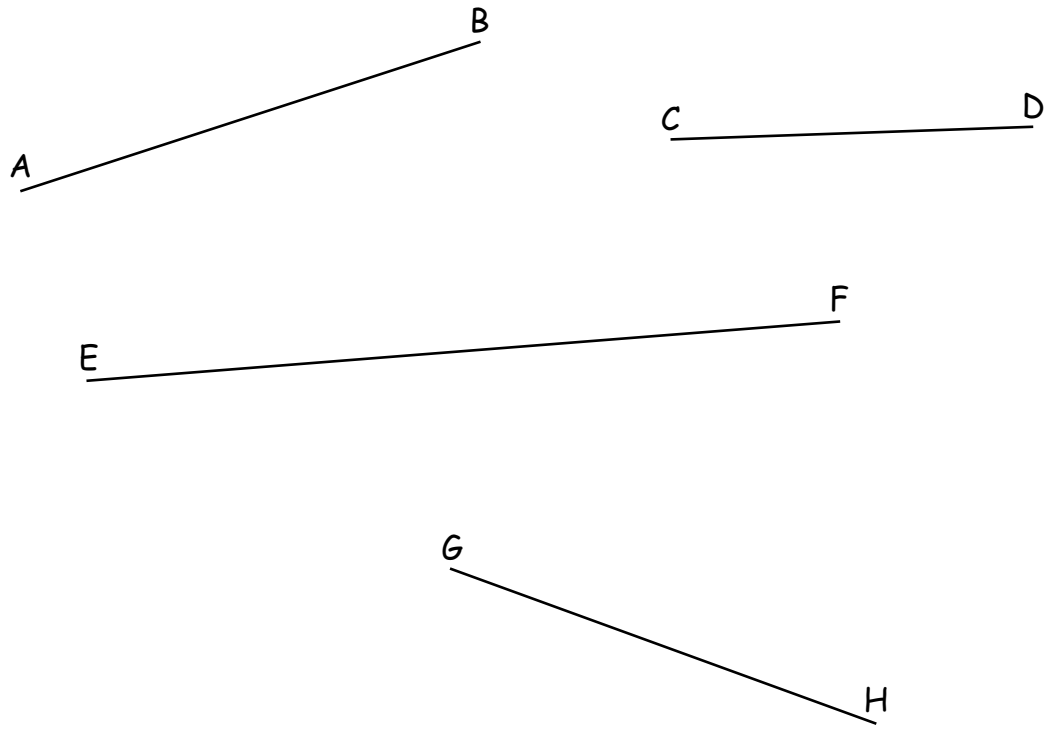
line C

line D

Resource S1.1b

Estimate the length of each line in centimetres. Write your estimate above each line.

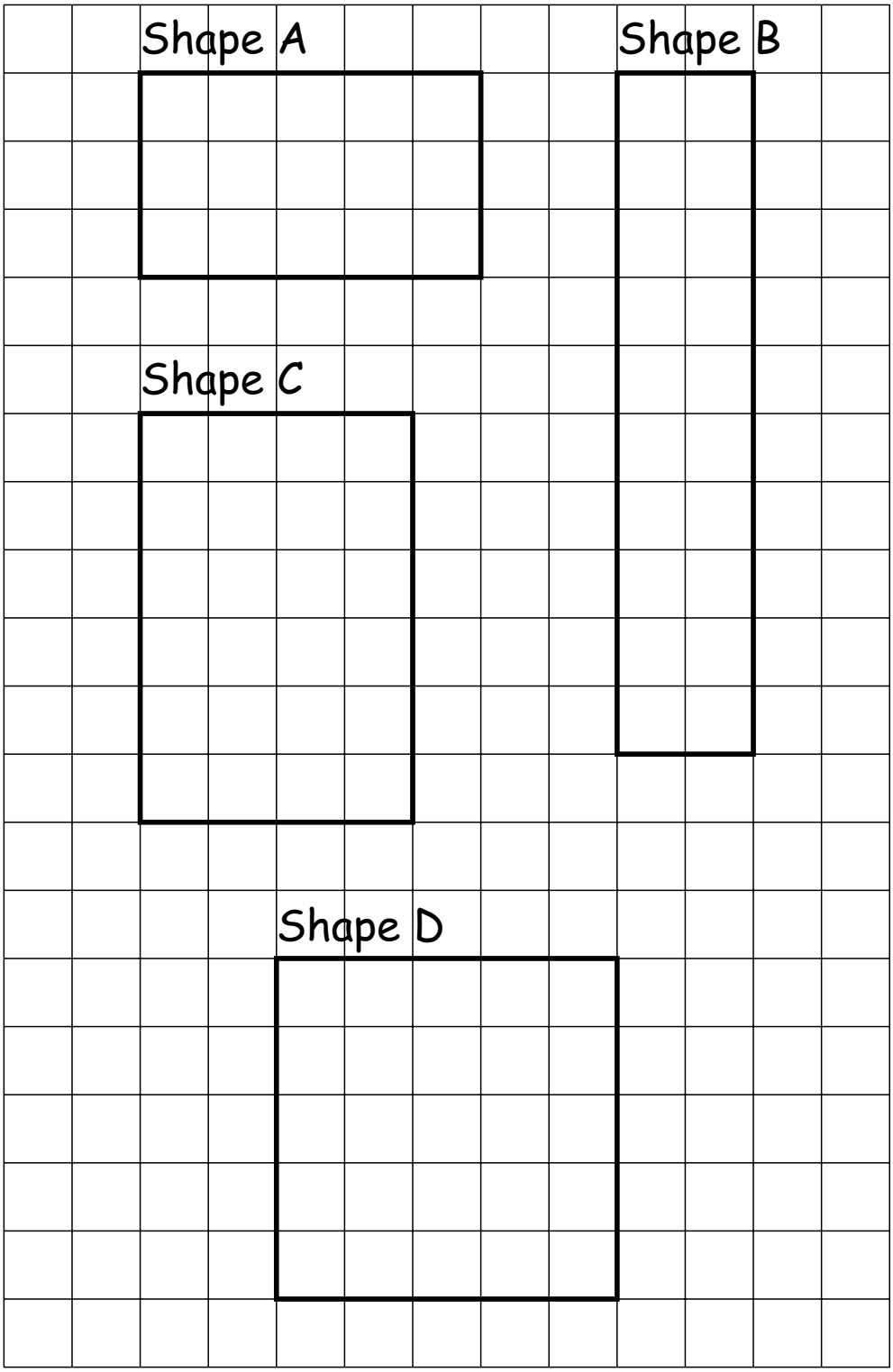
Measure the length of each line to the nearest millimetre. Write the measured length below each line.



Using the space below, draw a line 64mm long.
Label the line PQ. Write the length on the line.

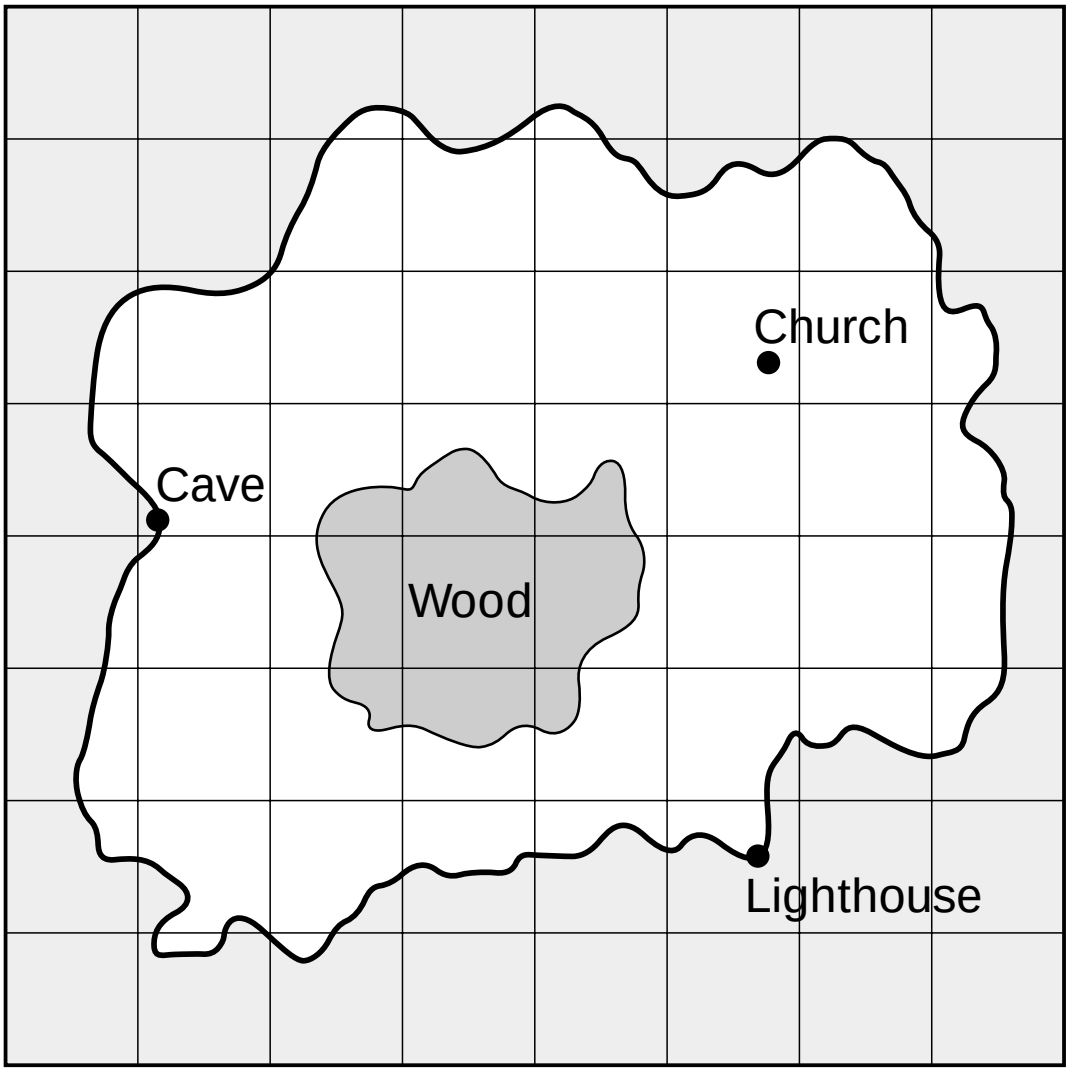
Now draw a line 8.5cm long. Label the line RS.
Write the length on the line.

OHT S1.1c



OHT S1.2a

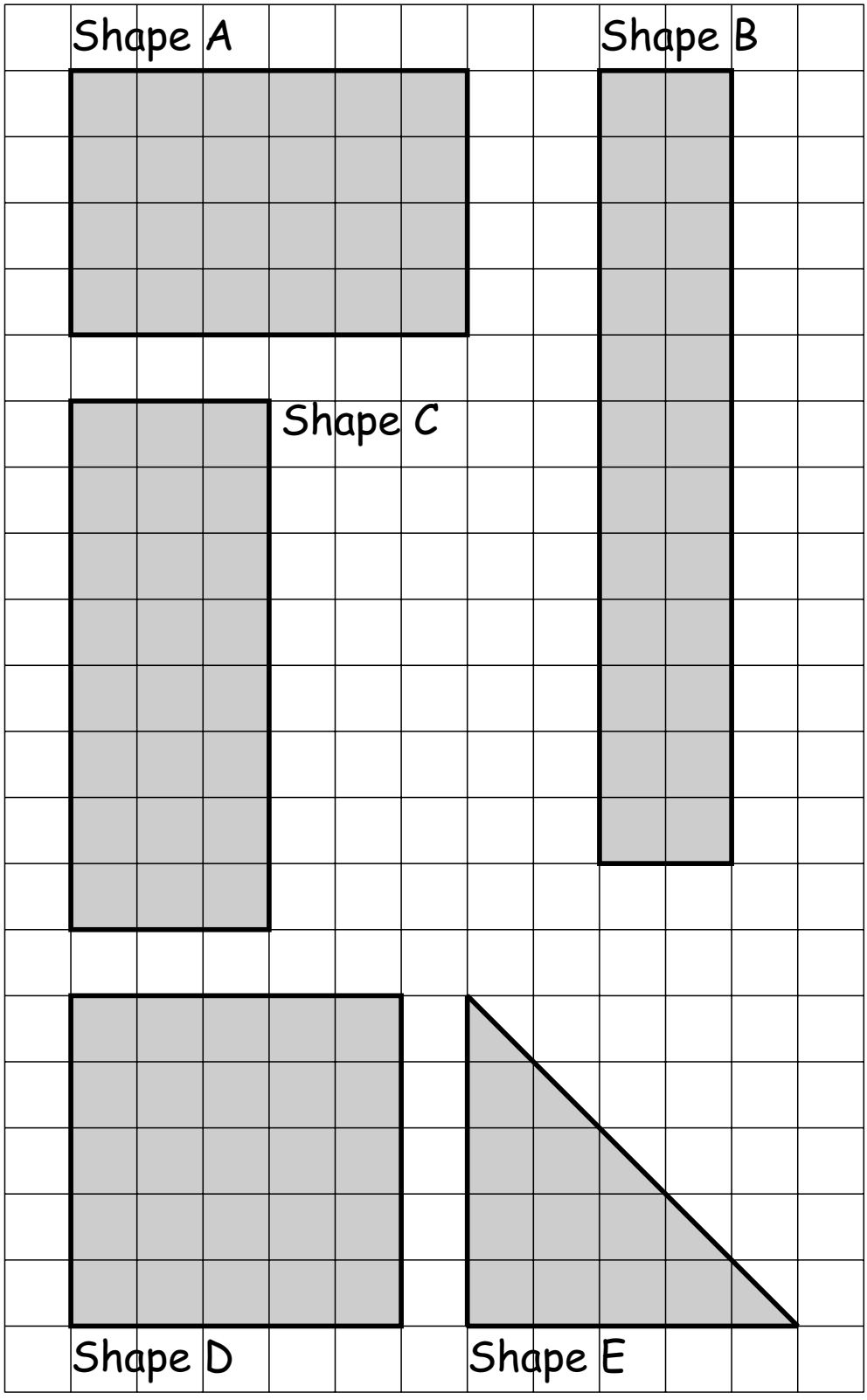
Here is a map of an island.



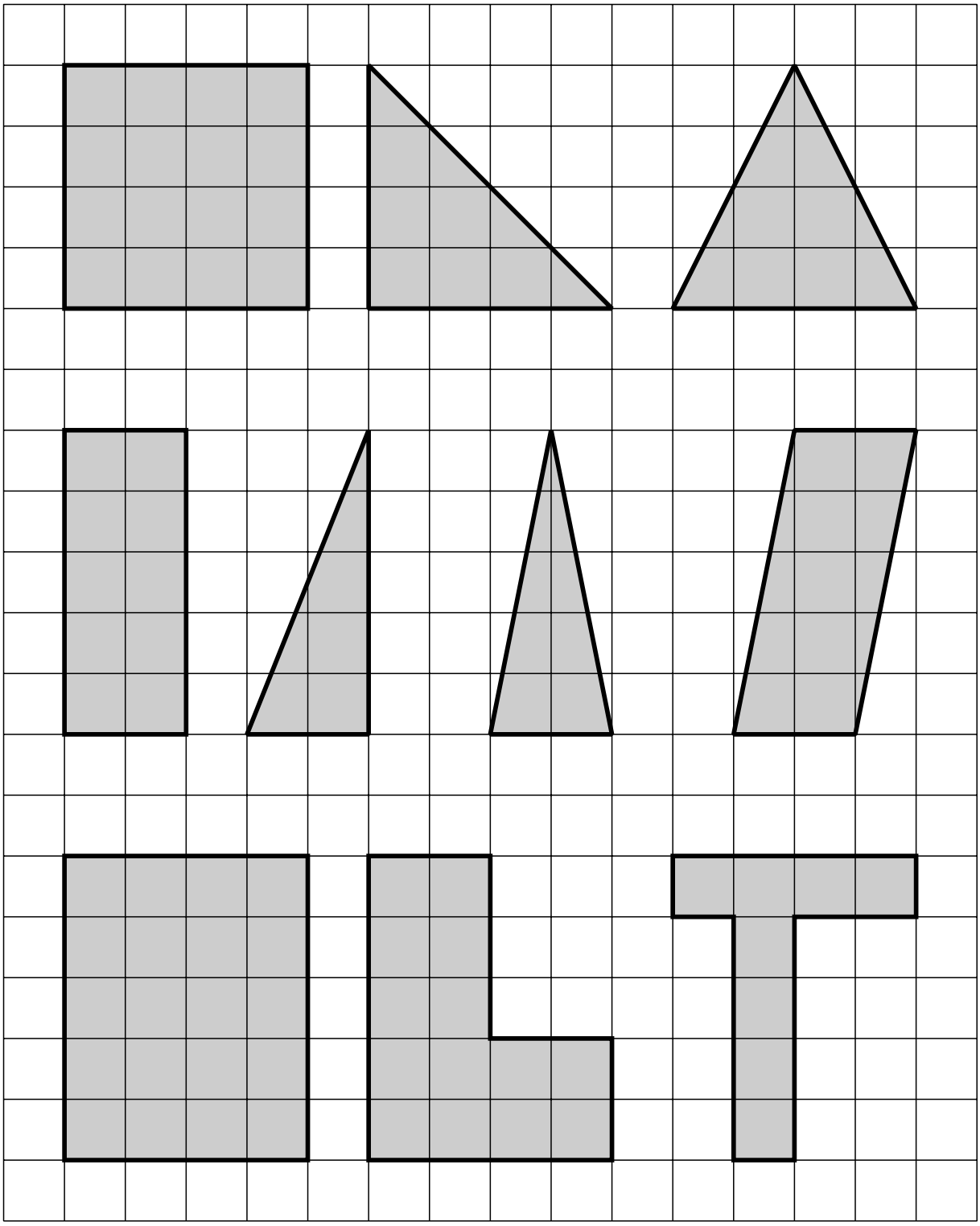
Estimate the area of the wood. ..... squares

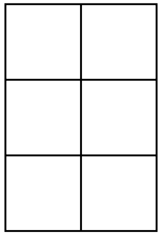
Estimate the area of the island. ..... squares

OHT S1.2b

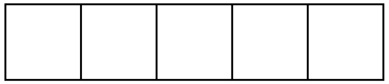


Resource S1.2c

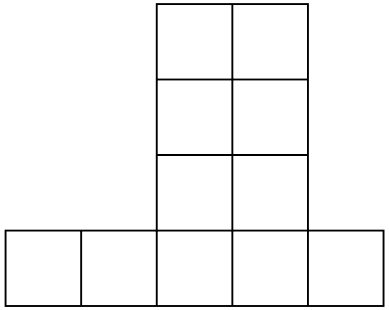




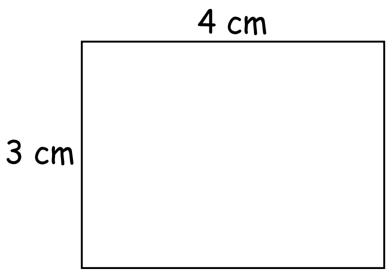
perimeter = …… cm
area = …… cm^2



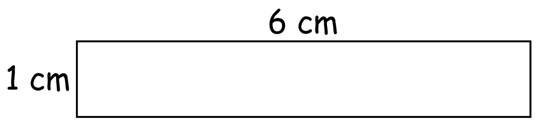
perimeter = …… cm
area = …… cm^2



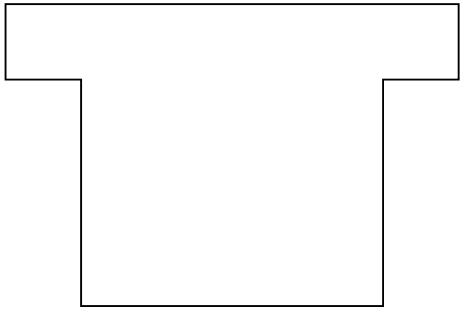
perimeter = …… cm
area = …… cm^2



perimeter = …… cm
area = …… cm^2



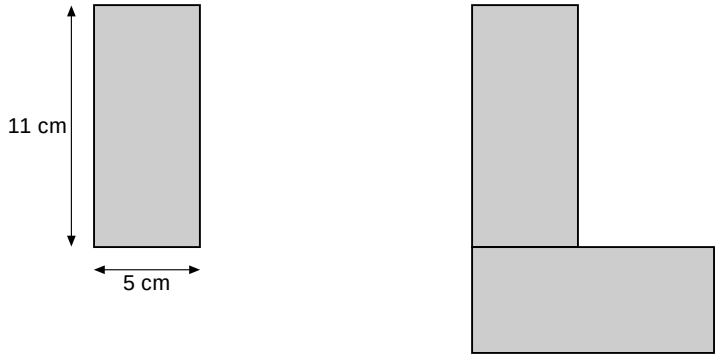
perimeter = …… cm
area = …… cm^2



perimeter = …… cm
area = …… cm^2

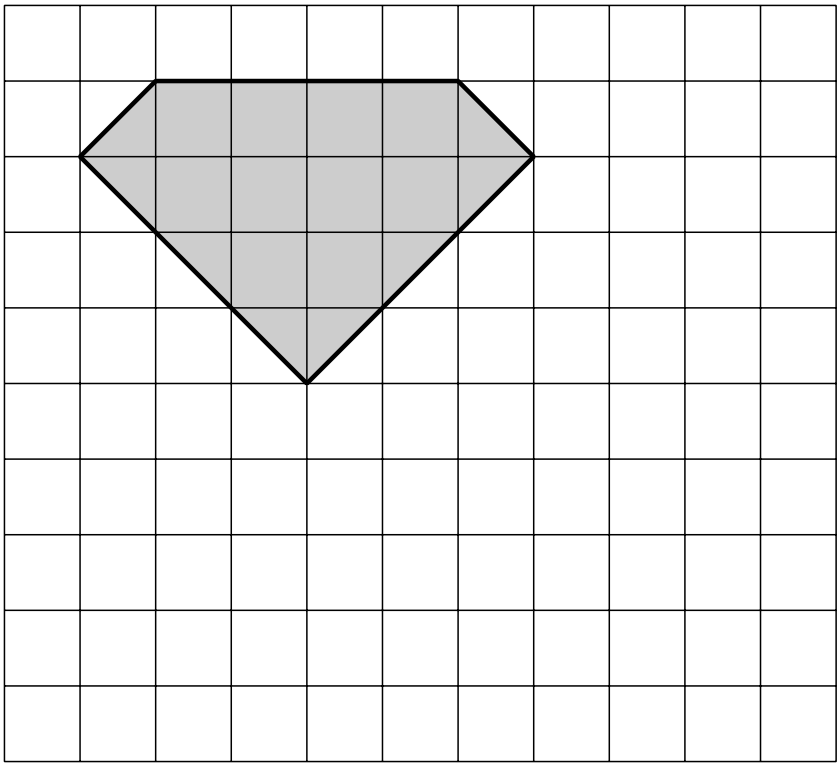
Resource S1.2e

The L-shape is made from two rectangular tiles like this.

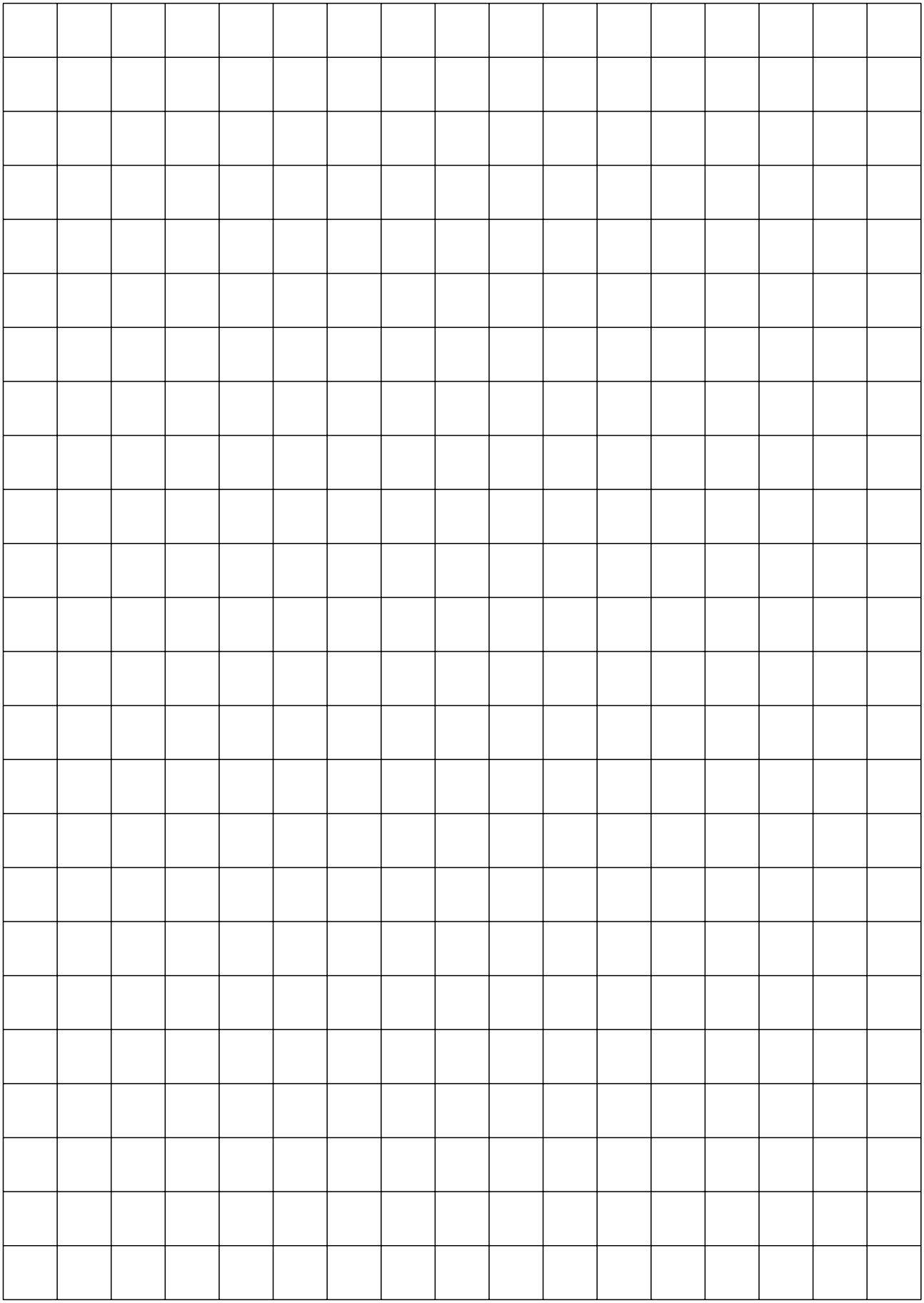


What is the perimeter of the L-shape? ..... cm

On the grid, draw a rectangle which has the same area as this shaded pentagon. Use a ruler.

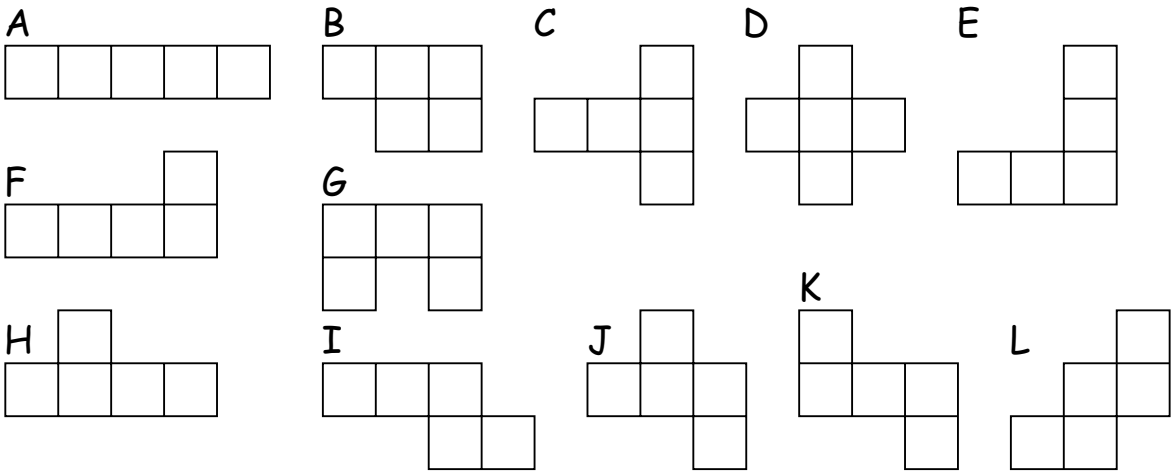


OHT S1.3a

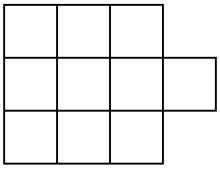


OHT S1.3b

There are 12 different pentominoes.



Which two pentominoes will fit into this shape?



Find four different pairs.