# Number 1

#### contents There are eight lessons in this unit, **Number 1**. N1.1 Positive and negative numbers 3 N1.2 Adding and subtracting whole numbers 6 9 N1.3 Subtracting whole numbers N1.4 Inverse operations: addition and subtraction 12 N1.5 Multiplying and dividing by 10 and 100 15 N1.6 Doubling and halving 18 N1.7 Multiplication and division facts and TU × U 21 24 N1.8 Reasoning about numbers

## objectives

The objectives covered in this unit are:

Resource sheets for the lessons

- Read and write whole numbers in figures and words; know what each digit represents.
- Extend beyond zero when counting back in steps of constant size.
- Understand negative numbers as positions on a number line; order negative integers.
- Calculate temperature differences across 0°C.
- Know addition and subtraction facts to 20.
- Recall multiplication facts to 10 × 10 and derive associated division facts.
- Recall two-digit pairs that total 100.
- Add and subtract mentally pairs of two-digit numbers.
- Calculate mentally a difference such as 8006 2993.
- Find doubles and halves of numbers.
- Use jottings to support or explain mental calculations.
- Add and subtract whole numbers using standard column methods.
- Understand and use the inverse relationship between addition and subtraction.
- Multiply and divide whole numbers by 10 and 100 (whole-number answers), and explain the effect.
- Multiply  $TU \times U$ .
- Use a calculator effectively; enter numbers and interpret the display in different contexts.
- Carry out calculations with more than one step.
- Solve word problems and investigate in number.

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## Using the lesson plans in this unit

These lesson plans supplement the Springboard 7 materials for Key Stage 3 pupils working toward level 4 in mathematics. All the lessons are examples only. There is no requirement to use them. If you decide to use the lessons, you will need to prepare overhead projector transparencies (OHTs) and occasional resource sheets for pupils to use.

The lessons consolidate work at level 3 and extend into level 4. They are suitable for a group of pupils or a whole class. Whatever the size of the group, the pupils are referred to as 'the class'.

Each lesson will support about 30 to 40 minutes of direct teaching. To help match the time to your timetable, each plan refers to 'other tasks' for pupils, based on Springboard 7 resources. Select from these, textbook exercises or your own materials to provide practice and consolidation in the main part of a lesson and to set homework. Aim to choose tasks that vary in their level of demand, to suit pupils' knowledge, confidence and rate of progress.

Although the 'other tasks' are listed for convenience at the end of the main part of the lesson, they can be offered at any point, especially between the 'episodes' that form the main activity.

The lesson starters are of two kinds: practice starters and teaching starters. The former are opportunities to rehearse skills that will be needed later in the lesson. Teaching starters introduce an idea that is then developed in the main activity.

You will need to tell pupils what they will learn in the lesson, either in the starter or at the beginning of the main activity. Use the plenary to check pupils' learning against the lesson's objectives and to draw attention to the key points that pupils should remember.

# Interactive teaching programs (ITPs)

Interactive teaching programs are interactive animated visual aids that can be used with a laptop and data projector or with an interactive whiteboard. As extra support for this unit, you may find it useful to download and use these ITPs from the website www.standards.dfes.gov.uk/numeracy:

for lesson N1.1: Number line

Thermometer

for lesson N1.2: Number line

for lesson N1.7: Multiplication grid

# Positive and negative numbers

#### objectives

- Extend beyond zero when counting back in steps of constant size.
- Understand negative numbers as positions on a number line; order negative integers.
- Calculate temperature differences across 0°C.

#### starter

#### Vocabulary

negative positive less than (<) more than (>) lies between difference

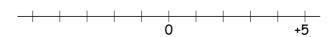
#### Resources

OHP calculator mini-whiteboards ITP Number line (optional)

You could, if you wish, support this lesson by using the ITP Number line, downloaded from www.standards.dfes.gov.uk/numeracy.

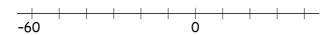
Use an OHP calculator. Start at 5. Count down in ones to below zero, asking pupils to predict the next number before you press the equals sign. Explain that negative numbers are referred to as 'negative one', 'negative two', and so on.

Draw on the board a number line with 10 intervals. Mark 0 in the centre of the line, and +5 at the right-hand end.



Point to different positions on the line, and invite individual pupils to the board to write in the numbers. When the numbers are written in, count backwards together from +5 to -5 and back to +5.

Erase the numbers, mark in -60 at the left-hand end, and 0 at the sixth division.



Point to different positions on the line, asking the class:

Q What number is this?

Label the numbers as they are identified. Ask some questions such as:

Q Tell me a number that is less than negative twenty ... a number that is more than negative thirty ... a number that lies between negative twenty and ten.

Record answers on the board, for example:

$$-40 < -20$$
  $-10 > -30$   $-20 < 0 < 10$ 

Explain that, just like on a number line with only positive numbers, it is possible to add and subtract by counting the steps along the line.

- Q What is five more than negative two?
- Q What is six less than four?
- Q What is the difference between five and negative three?

Stress that a difference between two numbers is measured by the number of steps or distance between them.

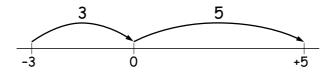
Write on the board:

-3 -2 -1 0 1 2 3

Ask pupils:

Q Which pair of numbers has a difference of 8? A different pair? And another pair?

If necessary, illustrate with an empty number line.



Now write on the board:

-0.50 0.5 1.5

Q Which pair of numbers has a difference of 2? And a different pair?

### main activity

#### Vocabulary

temperature degrees Celsius

#### Resources

OHTs N1.1a, N1.1b ITP Thermometer (optional) mini-whiteboards

You could, if you wish, use the ITP Thermometer to introduce pupils to temperatures above and below zero. Select options and ask questions to consolidate pupils' understanding.

Show pupils OHT N1.1a. Explain that a thermometer contains a substance such as mercury or alcohol that expands or contracts as it gets warmer or colder. Say that the thermometers on the slide measure temperature in degrees Celsius, and point out the °C abbreviation on them. Discuss the scales on the thermometers, explaining that they usually show only some numbers, leaving the others unmarked. Identify the positive and negative numbers on the scale. Ask pupils:

Q What is the temperature in York? In Rome?

Show how to record these temperatures as 2°C and 7°C.

Stress that as you move down the scale and pass zero (equivalent to nought on the number line), the temperature is falling and that the air in the room or outdoors is getting colder. Point out where -5°C lies on each scale and invite a pupil to indicate where -7°C lies. Explain that, with temperatures, this is read as 'minus seven degrees Celsius' not 'negative seven degrees Celsius' and that it means that the temperature is seven degrees Celsius below zero.

Ask a few questions, inviting pupils to write the answers on their whiteboards as appropriate:

- Q The temperature is 2°C and it is getting warmer. Which way will the indicator move? Which way will it move if it is getting colder?
- Q What does minus three degrees Celsius mean? How would you write

Remind pupils that they should always include the units when they write a temperature.

Q The temperature starts at four degrees Celsius and goes down by ten degrees. What is the temperature now? How did you work it out?

Now complete the questions on the slide.

Show and work through the questions on OHT N1.1b. Invite pupils to the projector to write the answers, making sure that they include the units.

#### other tasks

#### Unit 2 section 3: Positive and negative numbers

#### Springboard 7

Unit 2

1 Winter weather page 76 A cold night page 76 Comparing temperatures page 77 page 78 Star challenge 5: Temperature differences

#### plenary

Write randomly on the board a selection of temperatures. Ask the class to order these temperatures from the hottest to the coldest.

92°C	 hottest
37°C	
–12°C	
73°C	
12°C	
−2°C	 coldest

Use the temperatures to ask questions such as:

- Q What is the difference between 12°C and 37°C? Between 12°C and -12°C? Between -12°C and -2°C?
- Q The temperature is -2°C. How many degrees must it rise to reach 12°C?
- Q The temperature falls from 37°C to -12°C. How many degrees has the temperature fallen?

After each question, ask pupils how they worked out the answer.

#### Remember

- A number and its negative are the same distance from nought or zero.
- Numbers get less as you count back along the number line beyond nought or zero, so -10 < -5 (negative 10 is less than negative 5).
- Temperature is measured in degrees Celsius (°C). A temperature of six degrees below zero is minus six degrees Celsius (-6°C).
- -10°C is a lower temperature than -5°C.
- Always include the units when you write a temperature.

# Adding and subtracting whole numbers

#### objectives

- Know addition and subtraction facts to 20.
- Know what each digit of a whole number represents.
- Add and subtract mentally pairs of two-digit numbers.
- Use jottings to support or explain mental calculations.
- Add whole numbers using a standard column method.

#### starter

Ask four or five quick-fire questions about complements in 10, such as:

#### Vocabulary

add

subtract plus

minus

sum

difference

total

altogether

how many more?

how many less?

#### Resources

OHT N1.2a

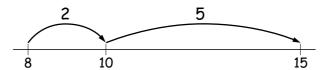
- Q What must be added to 4 to make 10?
- Q What is 2 plus 8? 10 minus 3?
- Q Subtract 5 from 10.
- Q How many more is 10 than 1?

Vary the vocabulary as much as possible.

Write 8 + 7 on the board.

Q Imagine that you have a friend who has forgotten the answer to this sum. How could your friend work out the answer?

Acknowledge pupils' suggestions, which may include that 8 + 7 is double 8 minus 1, or double 7 plus 1, or other strategies. Draw out that, if all else fails, the answer can be worked out in two steps by bridging through 10. First work out how much of the 7 must be added to the 8 to make 10, then add on the remaining amount. Illustrate with an empty number line.



#### Q How could you work out the answer to 6 + 9 if you had forgotten it?

Explain that, when adding, it is usually easier to put the larger number first. Write 9 + 6 on the board. Invite a pupil to the board to explain how to work out the answer by bridging through 10. Repeat with 6 + 7.

Write 16 – 7 on the board. Explain that with subtraction we can count up from the smaller number to the larger number, again by bridging through 10.



Now ask pupils to work out the answer to 8 + 5, this time by imagining the number line. Repeat with 7 + 9, 12 - 5 and 15 - 8.

Stress that there are other ways to work out addition and subtraction facts. For example, 16 - 7 might be done as 16 - 6 - 1, and 9 + 6 as 10 + 6 - 1. It doesn't matter which method is used but is important to be able to work out the answers quickly. Show the addition table on OHT N1.2a. Fill in the 'doubles' across the diagonal, asking pupils to call them out for you. Invite pupils to fill in some of the other facts, making sure that they recognise that they can fill in 8 + 6 at the same time as they fill in 6 + 8.

#### main activity

#### Vocabulary

addition subtraction

#### Resources

OHT N1.2b mini-whiteboards ITP Number line (optional)

Write 80 on the board. Count on in tens around the class to 230. Then begin at 1000, and count back in tens to 880. Ask:

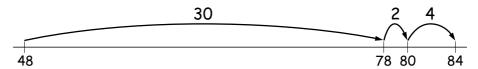
- Q What is thirty plus fifty? Ninety take away forty? How did you know the answer?
- Q Add seven hundred to two hundred. How did you work it out? Subtract five hundred from eight hundred. Explain how you did it.

Extend to 56 + 30, 93 - 50. Refer to the 100-square on **OHT N1.2b** to confirm answers, counting on or back in tens. Model the calculation on an empty number line.

Write 357 on the board.

- Q What do we add to change 357 into 397? To change 357 into 407?
- Q What answer will we get if we take away 50 from 357? If we take away 60? How did you work it out?

Write 36 + 48 on the board. Ask pupils to do the calculation mentally and to explain their method. Show them that, if necessary, they can use an empty number line as a jotting to support their thinking. Say that they can also use empty number lines to explain their working in tests.



Repeat with 82 - 27, counting up from 27 to 77, to 80, to 82 (or other methods that pupils are confident with). Give some more examples.

You could, if you wish, provide extra support for this activity by using the ITP Number line, downloaded from www.standards.dfes.gov.uk/numeracy. Select options and ask questions to consolidate pupils' understanding.

Say that some calculations are too awkward to be done mentally. Write 369 + 138 on the board. Ask:

#### Q What will the answer be, approximately?

Establish that 369 can be rounded up to 400, and 138 can be rounded to 140. An approximate answer to the calculation is 400 + 140 = 540.

Invite pupils to come to the board to do a column addition. They may use an expanded form, adding either the hundreds or the ones or units first. Stress the importance of lining up the columns to help avoid errors.

Pupils will probably have moved beyond this stage, and will be carrying out column addition in a contracted form, with carrying figures below the total. If they make persistent errors with this method, revert to the expanded form above, asking them to explain the steps in their calculations.

### other tasks

#### Unit 1 section 3: Mental addition

### Springboard 7

Units 1 and 2

ı	Adding and subtracting in your nead	page 55
2	Adding pairs of numbers in your head	page 55

#### Unit 1 section 4: Mental subtraction

Adding and authtracting in your hand

1	Subtracting across the tens boundary	page 56
2	Bigger jumps across the tens boundary	page 57
3	Subtracting across the hundreds boundary	page 58

#### **Unit 2 section 4: Addition**

1 Addition page 81

#### plenary

#### Resources

OHT N1.2c mini-whiteboards Show OHT N1.2c, a table showing the numbers of goals scored for and against some top teams in the 2001–02 football season. Pose the problem:

Q Some fans went to all of Liverpool's matches. What was the total number of goals they saw?

Establish that the key word is 'total' and the calculation is addition.

- Q What other questions could you ask that involve addition?
- Q What questions can you ask that involve subtraction?

Give each pair of pupils two of the words or phrases on the list at the bottom of the slide. Ask them to make up questions using the words and the information in the table. They should record the questions on their whiteboards.

Take feedback on at least one question for each key phrase. Ask pupils to calculate the answers, mentally if possible. Check and correct any errors.

#### Remember

- When 10 is added to a number, the digit in the tens place changes.
- It can help to bridge through 10 when adding pairs of single-digit numbers.
- When adding numbers mentally, it is usually easier to start with the larger
- An empty number line can be used to support mental calculations or to explain working, including in tests.

# **Subtracting whole numbers**

#### objectives

- Read and write whole numbers in figures and words; know what each digit represents.
- Subtract whole numbers using a standard column method.
- Calculate mentally a difference such as 8006 2993.
- Use jottings to support or explain mental calculations.

#### starter

#### Vocabulary

digit value

write in figures

add

subtract

plus

minus

difference

what must be added

to?

#### Resources

mini-whiteboards

Write 7041 on the board. Ask the class to read the number aloud in words. Discuss the value of different digits.

- Q What is the value of the digit 7? Of the 4?
- Q Why is there a zero or nought in the hundreds column?

Explain that the words 'zero' and 'nought' are used interchangeably, and that the zero is used as a place holder.

Q What number is 100 more than 7041? 100 less than 7041?

Talk through writing 4357 in an expanded form.

$$4357 = 4000 + 300 + 50 + 7$$

Ask pupils to write these numbers in figures on their whiteboards: four thousand two hundred; five thousand and ninety; two thousand and seven.

Check and correct any errors by writing numbers in an expanded form.

Write 97 on the board. Ask pupils to add 10 and write the answer on their whiteboards. Count on in tens round the class from 107 to 227.

Write 1012 on the board. Ask pupils to subtract 10 and write the answer (1002). Ask them to subtract 10 again and write the answer (992).

Begin at 6520, and count back in hundreds to 6120. Ask a pupil to write six thousand one hundred and twenty in figures on the board. Ask the class to subtract 100 and to write the answer on their whiteboards. Check the answers, write 6020 on the board, and ask the class again to subtract 100. Check the answers again. If necessary, use a number line to confirm.

### main activity

#### Vocabulary

multiple

#### Resources

mini-whiteboards OHT N1.2b two card strips to cover multiples on OHT

Show OHT N1.2b, from lesson N1.2, using the card strips to cover up the multiples of 5 and the multiples of 10. Ask:

Q What are the missing numbers? Explain how you know.

Revise addition and subtraction facts to 20 by asking questions such as:

- Q Seven add nine.
- Q Subtract 6 from 14.

Remind pupils that it is usually easier to put the larger number first when adding. Remind them also that they can imagine bridging through 10 if they don't know the fact immediately. Encourage pupils to think quickly.

Ask pupils to use their whiteboards to answer questions such as:

$$76 + 8 = 84$$
  $65 - 7 = 58$ 

Prompt by referring to OHT N1.2b (with the card strips removed) and asking:

- Q What is the next multiple of 10 after 76? (80)
- Q What must be added to 76 to make 80?

Each time, ask pupils to explain how they worked out the calculation.

Ask pupils to think about the mental methods of subtraction that they have used previously. Ask them to work out in their heads examples such as:

They can make jottings, or sketch a number line. Prompt by saying:

- Q Imagine a number line. What multiples of 10 can you see on it? What other numbers can you see?
- Q What is the answer? How did you work it out?

Explain that some calculations are too awkward to be done mentally. Write on the board:

784

- 159

Ask:

#### Q What will the answer be, approximately?

Establish that 784 can be rounded up to 800, and 159 can be rounded to 150. An approximate answer to the calculation is 800 - 150 = 650.

Invite pupils to come to the board to tackle the subtraction. They may already use a method accurately and reasonably quickly. If there is no clearly established approach, you could introduce complementary addition, which relates closely to the countingup method on the number line.

Pupils may be able to shorten this to:

If pupils have made sound progress with mental methods of subtraction, and are confident with addition and subtraction facts to 20, introduce them to a standard written method such as decomposition, first in an expanded form.

$$784 = 700 + 80 + 4 = 700 + 70 + 14$$
 $-159 = 100 + 50 + 9$ 
 $-100 + 50 + 9$ 
 $-100 + 50 + 5 = 625$ 

With either method, draw attention to the hundreds, tens and units columns and the need to keep digits in their correct columns. With decomposition, stress that the calculation begins with the ones or units column on the right. To begin with, restrict 'exchanging' or 'borrowing' to the tens and units.

Demonstrate two more examples, then ask pupils to try for themselves.

#### other tasks

#### Unit 1 section 4: Mental subtraction

## Springboard 7

Units 1 and 2

4 Subtracting across the thousands boundary	page 58
Star challenge 5: Mental subtraction	page 59
Star challenge 6: Bigger jumps across the hundreds boundary	page 59

#### Unit 2 section 1: Putting numbers into words

1	Numbers and words	page 69
St	ar challenge 1: Which of these numbers is?	page 70

#### **Unit 2 section 5: Subtraction**

1	Subtraction 1	page 83
2	Subtraction 2	page 84

#### Unit 2 section 7: Addition and subtraction problems

#### plenary

Write a set of subtractions on the board, or prepare an OHT. For example:

Discuss which of these questions can be done entirely mentally, which can be done mentally with some jottings, and which might need a written method in columns. Work through each calculation, inviting a pupil to explain their method to the class. Stress that the last example, in particular, is one that can be done mentally. Demonstrate on the number line one or two further examples such as 8006 – 2993.

#### Remember

• Look at the numbers carefully before deciding which way to do a subtraction.

# **Inverse operations:** addition and subtraction

#### objectives

- Recall two-digit pairs that total 100.
- Understand and use the inverse relationship between addition and subtraction.
- Carry out calculations with more than one step.

#### starter

#### Vocabulary

what must be added to?

#### Resources

one set of digit cards

Hold up the digit 6 and ask the class to tell you what must be added to the number to make 10. Repeat with other examples.

Using an 'empty box' format, practise examples of multiples of 10 that total 100, such as  $20 + \square = 100$ , then pairs of whole numbers that total 100, such as  $41 + \square = 100$  and  $\square + 36 = 100$ . Each time ask:

- Q How did you work out your answer?
- Q Is there another strategy that you could use to decide what goes in the empty box?
- Q How could you check your answer?

#### main activity

#### Vocabulary

equation number sentence more than less than difference plus

minus total

sum

inverse

#### Resources

OHT N1.4a mini-whiteboards calculators

Write  $14 + 18 = \square$  on the board, with a box large enough to write in.

Establish that the answer is 32 and write it in the box. Explain that this number sentence or equation is part of a family of four.

Q What are the other associated number sentences or equations?

Collect the other three: 18 + 14 = 32

32 - 14 = 18

32 - 18 = 14

Emphasise that for each equation of this type there are usually three others. Work through 17 + 17 = 34 to show that in this case there is only one associated equation: 34 - 17 = 17.

Write  $23 + 64 = \square$  on the board. Establish that the answer is 87. Ask pupils to use their whiteboards to write an associated number sentence or equation. Check for all possibilities from the responses.

Write 58 – 16 = 42 on the board and obtain the other three number sentences or equations from the pupils.

Q How can we describe the relationship between 58, 16 and 42?

Encourage pupils to use terms such as 'more than', 'less than', 'difference between', 'sum of' and 'total of'.

Write  $\square$  – 23 = 45 on the board. Ask pupils for the other three number equations.

 $\Box$  - 45 = 23

23 + 45 =  $\square$ 

45 + 23 =  $\square$ 

Q Which of these equations can you complete?

Reinforce that knowing that 45 plus 23 equals 68 means that we know that 68 minus 23 equals 45, and that 68 minus 45 equals 23. Knowing that 45 + 23 = 68 is enough. This fact can be used to fill in the other boxes.

Say that addition and subtraction are inverse operations. Addition is the inverse of subtraction and subtraction is the inverse of addition.

Ask pupils to work in pairs. Each pupil writes down one addition and one subtraction calculation involving three-digit numbers and uses a calculator to complete them. For example:

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317 + 468 = 785
682 - 294 = 388
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They then write out their calculations again on a separate slip of paper, this time replacing one of the first two numbers with an empty box.

$$317 + \square = 785$$
  
\[ \sum - 294 = 388

They then swap these with their partner, who works out the answers without a calculator. Remind them to make use of the other three number sentences. The pair then checks each other's calculations.

Tell the class that the same strategies can be used to solve problems involving money. Give the class a problem.

The watch that Mary wants to buy costs £74.38. Mary has saved £42.54 towards the cost. How much more must Mary save?

Show pupils how to write an equation to model the problem.

$$42.54 + \square = 74.38$$

Ask pupils for the other three number equations.

Ask pupils to use their calculators to work out the answer to the problem by completing one of the equations.

#### Q How can we check the answer?

Establish that the answer can be checked by putting it back into the original equation:  $42.54 + \square = 74.38$ .

Write a different problem on the board.

Raj thinks of a number. He says: 'If I subtract 14 from it, I get 29.' What is Raj's number?

Tell pupils to work in pairs and to discuss the answer. Ask different pairs to describe to the rest of the class how they worked out their answer.

Show OHT N1.4a, a set of 'I am thinking of a number' problems. Work through the problems, asking pupils to explain how they worked out the answers. Each time, write an equation to represent the problem and solve it by using an inverse operation.

### other tasks

#### **Unit 2 section 5: Subtraction**

Star challenge 7: Add or subtract? Star challenge 8: Find the missing digits page 85 page 85

## Springboard 7

OHT N1.4b

Unit 2

### plenary

Show **OHT N1.4b**. Point to one of the questions in the right-hand column. Ask: Resources

Q Which of the statements in the left-hand column can be used to solve this problem? Explain why.

#### Remember

- Addition and subtraction are the inverse of each other (one 'undoes' or 'reverses' the other).
- When finding a missing number, write down the other three number sentences and then decide which one to use to find the missing number.
- An inverse operation can be used to check the answer to a calculation.

# Multiplying and dividing by 10 and 100

#### objectives

- Know addition and subtraction facts to 20.
- Recall multiplication facts to 10 x 10 and derive associated division facts.
- Multiply and divide whole numbers by 10 and 100 (whole-number answers), and explain the effect.
- Use a calculator effectively; enter numbers and interpret the display in different contexts.

#### starter

#### Vocabulary

subtract minus take away

#### Resources

mini-whiteboards 100-square or OHT N1.2b

Start by asking pupils for pairs of numbers with a total of 10. Then ask a series of questions for pupils to answer on their whiteboards. Keep the units digit the same, and vary the wording.

- Q What is ten take away six? Twenty take away six? Ninety take away six? Fifty take away six?
- Q What is thirty minus sixteen? What is seventy minus sixteen? How did you work that out?
- Q Subtract twenty-six from fifty. Subtract twenty-six from eighty. How did you work that out?
- Q What do you notice about the answers to all these questions? (the units digit is always four) Why? (because ten take away six is four)

If pupils have difficulty with the questions, refer to a 100-square, for example, on **OHT N1.2b**. Remind them that subtract and minus mean the same as 'take away'.

Repeat with a different units digit.

Q What is ten minus three? Forty minus three? Eighty minus three?

#### main activity

#### Vocabulary

multiply divide ten times larger ten times smaller inverse convert

#### Resources

OHT N1.5a OHP calculator As a class, chant the ten times table, forwards and backwards (one ten is ten, two tens are twenty, three tens are thirty, and so on). Write on the board:

$$6 \times 10 = 60$$

Explain that each individual unit in the six ones has been multiplied by 10, or made 10 times larger, so each one becomes ten. Use the place value grid on OHT N1.5a to show how the six ones or units have become six tens, and the digit 6 has moved one place to the left.

Now write on the board:

$$40 \times 10 = 400$$

Explain that this time each individual ten has been multiplied by 10, or made ten times larger. Demonstrate, using the place value grid, how the four tens have become four hundreds, and the digit 4 has moved one place to the left.

Write on the board:  $46 \times 10$ . Ask:

Q What do you think the answer will be? How did you work it out?

Establish that each of the digits has moved one place to the left and that 0 has been put in the units place as a place holder. Explain that 'add a 0' is not acceptable (it does not work with decimals). Repeat for one or two more two-digit numbers.

Use the OHP calculator, and multiply whole numbers under 100 by 10. Each time ask the class to predict the answer before you display it. Ask:

#### Q What will happen if I now divide the answer by 10?

Establish the generalisation that dividing by 10 makes the number ten times smaller and that each of its digits moves one place to the right.

Draw out, through a few different examples, that multiplying by 10 and dividing by 10 are inverse operations (one undoes the effect of the other).

As a class, chant the 100 times table. Write on the board:

$$61 \times 100 = 6100$$

Read the equation aloud together. Explain that each individual unit has been multiplied by 100, or made 100 times larger. Demonstrate on the board using thousands, hundreds, tens and units boxes, how each has moved two places to the left, or use **OHT N1.5a**.

Write on the board:  $205 \times 100$ . Ask:

#### Q What do you think the answer will be? How did you work it out?

Establish again that each digit has moved two places to the left. Repeat for one or two more two- or three-digit numbers.

Use the OHP calculator, and multiply numbers under 100 by 100. Each time ask the class to predict the answer. Ask:

#### Q What will happen if I now divide the answer by 100?

Establish the generalisation that dividing by 100 makes the number one hundred times smaller and that each digit has moved two places to the right.

Draw out, through more examples, that dividing by 100 and multiplying by 100 are inverse operations.

Remind the class that there are 10 millimetres in 1 centimetre. To change or convert centimetres to millimetres they must multiply by 10, and to change millimetres to centimetres they must divide by 10. Ask:

- Q How many millimetres are there in nine centimetres?
- Q Change one hundred and ninety millimetres into centimetres.

To change metres to centimetres, they must multiply by 100, and to change centimetres to metres they must divide by 100. Ask:

- Q A table is two hundred centimetres long. How many metres is that?
- Q Change nine hundred centimetres into metres.

#### other tasks

#### Unit 2 section 2: Multiplying and dividing by 10 and 100

#### Springboard 7

Units 2 and 6

1	Multiplying whole numbers by 10 and 100	page 72
2	Dividing whole numbers by 10 and 100	page 72
3	Multiplying and dividing by 10 and 100	page 72
Star challenge 4: Multiplying in your head page 74		

#### Unit 6 section 5: Multiplication

2 Multiples of 10 and 100

page 231

#### plenary

Write on the board:

 $6 \times 10 = 10 \text{ lots of } 6$ 

 $6 \times 9 = 9$  lots of 6

Q How can we work out nine times a number from ten times the number? (subtract the number from ten times the number)

Demonstrate how to work out  $6 \times 9$  as:

$$(6 \times 10) - 6 = 60 - 6 = 54$$

Then demonstrate how to work out  $16 \times 9$  as:

$$(16 \times 10) - 16 = 160 - 16 = 144$$

Use this principle to complete the nine times table, writing it on the board. Ask:

Q What do you notice about the digits of the multiples of nine in the table? (they always add to nine)

Chant the table forwards and backwards (one nine is nine, two nines are eighteen, three nines are twenty-seven, and so on). Finally ask:

Q How can we work out 99 times a number from 100 times the number? (subtract the number from 100 times the number)

Practise one or two examples.

#### Remember

- Multiplying a number by 10 makes it ten times larger; the digits move one place to the left. Dividing a number by 10 makes it ten times smaller; the digits move one place to the right.
- Multiplying a number by 100 makes it one hundred times larger; the digits move two places to the left. Dividing a number by 100 makes it one hundred times smaller; the digits move two places to the right.
- To multiply a number by 9, multiply the number by 10 and subtract the number.

# **Doubling and halving**

#### objectives

- Recall multiplication facts to 10 × 10 and derive associated division facts.
- Double and halve two- and three-digit numbers.

#### starter

#### Vocabulary

multiplied by divided by product multiple remainder

#### Resources

counting stick

As a class, recite the two times table, forwards and backwards. Ask a few random questions, varying the wording.

- Q What is 8 multiplied by 2?
- Q What is 18 divided by 2?
- Q What is the next multiple of 2 after 14?
- Q What is the remainder when 13 is divided by 2?

Explain that 20 is the *product* of 2 and 10, or  $2 \times 10$ .

Q What is the product of 5 and 2? Tell me two other ways of saying this. (5 multiplied by 2 and 5 times 2)

Use a counting stick.

Tell pupils that one end is nought or zero. Count along the stick and back again in twos. Point randomly at divisions on the stick, saying:

Q What is this number? How do you know?

Encourage pupils to use 'multiplied by' and 'divided by' in their answers. Point out that they can use the midpoint of the stick as a reference point, for example: 'I know that halfway is 5 multiplied by 2, or 10, and the next point is 2 more, or 12.'

Now ask the class to count along the stick and back again in 20s: nought, twenty, forty, ... Once again, point at divisions on the stick, saying:

Q What is this number? How do you know?

#### main activity

#### Vocabulary

double halve

Practise doubling some numbers up to 10: double 3, double 9, double 7. Remind pupils that doubling is the same as multiplying by 2. Say that double 40 is the same as double 4 multiplied by 10, and extend to doubling multiples of 10: double 30, double 80, double 60.

Write on the board a selection of whole numbers under 50:

17 19 21 24 28 32 35 38

Ask pupils if they can double any of the numbers straight away (e.g. 21, 32). Cross out these numbers and record on the board, for example,  $21 \times 2 = 42$ ,  $32 \times 2 = 64$ .

Ask pupils to use their exercise books to double the remaining numbers. Allow a couple of minutes, then go through the numbers one by one, inviting pupils to the board to explain their method to the class. Look for these methods:

using known facts, e.g.  $19 \times 2$  is 2 less than double 20;

- splitting the number into tens and ones or units, e.g.  $28 \times 2$  is double 20 + double 8;
- splitting the number in other ways, e.g.  $38 \times 2$  is double 35 plus double 3.

Use a diagram to show pupils how they can always double a two-digit number by doubling the tens and doubling the ones or units.

Ask pupils to use this method to double 28, then 36, doing as much as possible mentally. Show them how the method can be extended to doubling a sum of money such as £27.38 by splitting the pounds and the pence.

£27 38p

£20 + £7 + 30p + 8p

$$\checkmark$$
  $\checkmark$   $\checkmark$   $\checkmark$   $\checkmark$   $\checkmark$   $\checkmark$   $\checkmark$  £40 + £14 + 60p + 16p

£54 76p = £54.76

Give one or two examples to practise, such as £13.09 and £36.75.

Repeat the above for halving numbers, starting with some simple practice of halving numbers to 20, including odd numbers. Check that pupils can express an answer to half of 15 both as a mixed number (seven and a half or  $7^{1}/_{2}$ ) and as a decimal (seven point five or 7.5).

Write  $14 \div 2 = 7$  on the board, and  $140 \div 2 = 70$ . Explain that:

$$140 \div 2 = 10 \times 14 \div 2 = 10 \times 7 = 70$$

Stress that the answer is the same as 10 times 14 ÷ 2. Practise halving a few more multiples of 10 to 200, and multiples of 100 to 2000.

Give the class some two-digit numbers under 100 to halve, inviting them to explain their strategies. Show them how they can always halve two-digit numbers by partitioning into tens and ones or units, and how to halve sums of money by partitioning into pounds and pence, using diagrams similar to those for doubling.

Give one or two examples of amounts of money to halve, such as £18.54 and £21.38.

#### other tasks

#### Unit 6 section 1: Doubles and halves

### Springboard 7

Units 6 and 13

1	Doubles and halves	page 215
2	Doubles and halves of multiples of 10 and 100	page 215
3	Doubles and halves of 2-digit numbers	page 216
Sta	ar challenge 1: Double yummy	page 217
Sta	ar challenge 2: Doubles and halves in disguise	page 217

#### Unit 6 section 2: Mental calculation

1 Repeated doubling and halving page 219

Level 3 to level 4 lessons Number 1

1 Halves and quarters

#### plenary

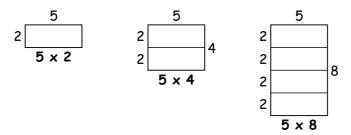
#### Resources

OHP calculator OHT N1.6a

Show pupils how they can use doubling to work out the 4 times table from the 2 times table, and the 8 times table from the 4 times table.

Use an OHP calculator to demonstrate that any number multiplied by 4 is the same as the number multiplied by 2 or doubled, and then again multiplied by 2 or doubled.

Demonstrate also by using a diagram to show equivalent areas.



Complete OHT N1.6a using doubling to generate the 4 times and 8 times tables, working across the rows. Finish by asking questions such as:

Q What is 6 multiplied by 2? What is 6 multiplied by 4? What is 6 multiplied by 8?

Do this initially with the OHT in place, then remove it. Encourage pupils to use doubling strategies if they cannot remember a fact in the 4 or 8 times tables.

#### Remember

- Double, twice and multiply by 2 mean the same thing.
- Halving is the same as dividing by 2.
- You can multiply a number by 2 by doubling it. You can multiply it by 4 by doubling it again, and by 8 by doubling again.
- To find the product of some numbers, you multiply them together.

# **Multiplication and division** facts and TU×U

#### objectives

- Recall multiplication facts to  $10 \times 10$  and derive associated division facts.
- Use a written method to calculate TU × U.

#### starter

#### Vocabulary

multiplied by divided by product multiple remainder

As a class, recite the eight times table, forwards and backwards. Ask a few random questions, varying the wording.

- Q What is 32 divided by 8?
- Q What is the product of 5 and 8?
- Q What is the next multiple of 8 after 40?
- Q How many eights make 56?
- Q What is the remainder when 50 is divided by 8?

Discuss ways to remember awkward facts. For example, to remember ten times a number is always easy. To find five times a number is also easy, as it is half of ten times the number. For example, 10 times 8 is 80, so 5 times 8 is half of 80, or 40.

Remind the class that they can always work out eight times a number by starting with twice the number, doubling it to get four times the number, and doubling again to get eight times the number.

#### main activity

#### Vocabulary

commutative

#### Resources

OHT N1.7a counting stick ITP Multiplication grid (optional)

Use a counting stick and count along it and back again in multiples of 3. Then chant the three times table forwards and backwards: 'one three is three, two threes are six, three threes are nine. ...'.

Show pupils how they can use doubling to work out the six times table from the three times table by completing OHT N1.7a, working across the rows. Ask questions such as:

- Q What is 3 multiplied by 5? What is 6 multiplied by 5?
- Q How many threes make 24? How many sixes make 24?

Do this first with the OHT in place. Use the counting stick to practise counting along it and back again in multiples of 6. Remove the OHT and continue the questioning. Remind pupils to use their knowledge of the three times table and doubling strategies if they cannot remember a fact in the six times table.

Remind pupils of the commutative law of multiplication: seven twos are the same as two sevens. Remind them that if they know a fact one way round, they know it the other way round as well.

Build up the seven times table on the board, using facts that pupils already know. Chant the table, forwards and backwards.

Remind the class that they know how to multiply a number by 10 or 100, and that the digits will move one or two places to the left accordingly.

#### Q How could we multiply a number by 20?

Demonstrate that  $7 \times 20 = 7 \times 2 \times 10 = 14 \times 10 = 140$ .

#### Q How could we multiply a number by 60?

Demonstrate that  $8 \times 60 = 8 \times 6 \times 10 = 48 \times 10 = 480$ .

#### Q How could we multiply a number by 600?

Demonstrate that  $9 \times 600 = 9 \times 6 \times 100 = 54 \times 100 = 540$ .

You could, if you wish, use the ITP *Multiplication grid* downloaded from www.standards.dfes.gov.uk/numeracy to introduce pupils to multiplication in this part of the lesson. Select options and ask questions similar to those below.

Write  $7 \times 53 = 53 \times 7$  on the board.

#### Q What is an estimate of the answer?

Establish that the answer will lie between  $7 \times 50 = 7 \times 5 \times 10 = 350$  and  $7 \times 60 = 7 \times 6 \times 10 = 420$ . It will be closer to 350 than to 420, since 53 is closer to 50 than to 60.

Point out that 53 can be written as 50 + 3. Draw a grid on the board.

×	50	3
7		

×	50	3
7	350	21

Work through the left-hand grid with the class to get the right-hand grid. Ask:

#### Q How can we get the answer to $53 \times 7$ from the grid?

Add 350 and 21 mentally to get the answer 371. Work through one or two more examples, e.g.  $37 \times 4$ ,  $72 \times 6$ . Explain that the grid can be used as a jotting to support or explain a mental calculation.

Say that they may be able to do simple examples mentally without writing anything. Try  $13 \times 4$  and  $16 \times 6$  as mental calculations.

#### other tasks

#### Springboard 7

Units 6 and 15

#### Unit 6 section 2: Mental calculations

3 Multiplying in your head using partitioning page 220

#### Unit 6 section 5: Multiplication

Multiples of 6, 7, 8 and 9
 Multiplication for TU x U
 Star challenge 9: Little problems
 page 231
 page 232

#### Unit 6 section 6: Division

1 Division page 2352 Ways of asking the same thing page 235

#### Unit 15 section 3: Multiplication - written methods

1 TU x U page 482

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#### plenary

#### Resources

100-square or OHT N1.2b mini-whiteboards Show OHT N1.2b, a 100-square. Ask pupils to use their whiteboards and to answer questions such as these.

- Q I am thinking of a multiple 8 lying between 50 and 60. What is it?
- Q I am thinking of a multiple of 9 lying between 30 and 40. What is it?
- Q I am thinking of a multiple of 7 that is greater than 40 and less than 50. What could it be?

Invite individual pupils to justify their answers by asking:

Q How do you know?

Expect them to justify by stating the relevant multiplication or division fact: 'I know that 56 is a multiple of 8 because  $8 \times 7 = 56$  and  $56 \div 8 = 7$ .

#### Remember

• Use multiplication facts that you know to work out other multiplication facts.

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# Reasoning about numbers

#### objectives

- Know addition and subtraction facts to 20.
- Recall multiplication facts to 10 x 10 and derive associated division facts.
- Solve problems and investigate in number.

#### starter

Write on the board:

#### Vocabulary

fact

equation

#### Resources

OHT N1.8a

$$6 + 7 = 13$$

Q We know that six add seven equals thirteen. What other facts can you work out from this?

Write pupils' suggestions on the board as they make them.

$$13 - 7 = 6$$

$$13 - 6 = 7$$

$$6 + 8 = 14$$

$$16 + 7 = 23$$

$$23 - 7 = 16$$

$$106 + 7 = 113$$

$$6 + 17 = 23$$

$$23 - 17 = 6$$

$$56 + 7 = 63$$

$$60 + 70 = 130$$

$$600 + 700 = 1300$$

Show **OHT N1.8a**. Ask pupils to use the fact  $6 \times 7 = 42$  to complete the other equations. Stress that using facts that you know to deduce new information is an important mathematical skill.

#### main activity

Show the first problem on **OHT N1.8b**. Explain that each box represents a digit. Say:

#### Vocabulary

problem solution possibilities digit

#### Resources

OHT N1.8b OHT N1.8c (optional) Q Explain in your own words what the question is asking you to do.

Confirm that we are looking for a two-digit number and a single-digit number with a difference of 25. The digits must be chosen from 1, 3 and 6 and each digit must be used.

- Q How many times can we use each digit? (once)
- Q How could we begin to solve this problem?

Discuss pupils' responses.

- Q What could the first digit of the two-digit number be? Explain why the digit 1 would be impossible.
- Q Could 61 be the first number? If not, why not? What about 63?
- Q Could 36 be the first number? Why not?
- Q What is the solution to the problem? Explain why.
- Q Are there any other solutions? (no)

Say that this time pupils can use any digits but each digit must be different.

Q How many different solutions can you find?

Ask pupils to work in pairs for two or three minutes on the problem, then ask the pairs to pause. Ask:

#### Q What could the second number be?

Establish that it could be any of the digits from 0 to 9 (we know the solution for 6 from earlier work in the lesson).

- Q If the second number is 9, what is the first number? Explain why.
- Q If the second number is 8, what is the first number? (33) Is this possible, given the rules of the problem? (no)

Ask them now to complete the problem by working systematically through the digits 0 to 7. When they have completed the task, establish that there are a total of 8 solutions (the second number cannot be 2 or 8).

Show the second problem on OHT N1.8b. Remind them that the < sign means 'is less than'. Explain that we are looking for a three-digit number which, when subtracted from 550, leaves a positive number less than 200. The digits must be chosen from 1, 3, 5, 7 and 9, without repeats.

- Q Could the first digit of the three-digit number be 7 or 9? Why not? (the answer would be negative)
- Q Could it be 1? Why not? (the answer would be too big)
- Q Imagine that the first digit is 5. What could the second digit be?

Draw out that the second digit could be 1 or 3, but that 5 would be a repeat and 7 or 9 would result in a negative answer. Ask pupils in pairs to work out all the solutions if the first digit is 5 (513, 517, 519, 531, 537, 539).

Q What if the first digit is 3? What could the second digit be?

Establish that the second digit could be 5 or 7 or 9, but that 1 or 3 would make the answer too big. Ask the pairs to work out all the solutions if the first digit is 3 and the second digit is 5 or 7 or 9. Remind them to work systematically. (Solutions are 351, 357, 359, 371, 375, 379, 391, 395, 397.)

- Q How many solutions are there altogether? (15)
- Q How can we be certain we have found them all? (we have worked systematically through all the possibilities)

If there is enough time, work through the problem on **OHT N1.8c**.

- Q Explain in your own words what the question is asking you to do.
- Q How many times can we use each card? (once)
- Q How could we begin to solve this problem?
- Q Can we put any digits in place immediately? Explain why. (the three-digit number must have a hundreds digit of 4)
- Q What other information can we use? (the two units digits must have a total of 10)

Ask pupils to find at least one solution to the problem.

Q Are there any other solutions? How many solutions are there altogether?

The four solutions are 442 + 58, 448 + 52, 452 + 48, 458 + 42.

#### other tasks

#### Springboard 7

Unit 2

Assess and review 2

#### Unit 2 section 1: Putting numbers into words

Star challenge 2: Making numbers to order page 70

#### Unit 2 section 4: Addition

Star challenge 6: Addition arithmogons page 82

#### Unit 2 section 7: Addition and subtraction problems

Star challenge 11: More difficult word problems page 93

#### Assess and review 2

Number 8: Fill in the missing digits page 293

#### plenary

Invite pairs of pupils to the board to demonstrate their solutions to the problems they did independently as 'other tasks'.

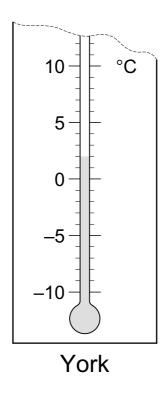
- Q Did anyone have a different way of tackling this problem?
- Q Would your method be different if you had a calculator?
- Q Are there any other solutions?
- Q How can you be sure that you have found all the solutions?

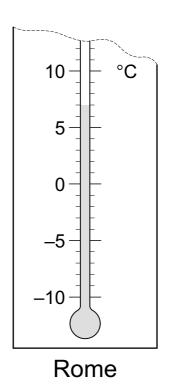
#### Remember

- When finding missing numbers, use the inverse operation to rewrite the equation.
- Look at the last digits to see if you can use your knowledge of number facts to eliminate possible values.
- Try out values that you can work out quickly in your head.
- Work systematically.

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These are the temperatures in York and Rome on a day in winter.





How many degrees colder is it in York than in Rome?

°C

On another day, the temperature in York is 4°C. Rome is 7 degrees colder than York.

What is the temperature in Rome?

°C

#### **OHT N1.1b**

Here is a table of temperatures at dawn on the same day.

Temperatures			
London –4°C			
Moscow	-6°C		
New York	–9°C		
Paris	+6°C		
Sydney +14°C			

What is the difference in temperature between:

- a. London and Paris?
- b. New York and Sydney? .....
- c. London and Moscow?

At noon the temperature in New York has risen by 5 degrees. What is the temperature in New York at noon?

.....

+	5	6	7	8	9
5					
6					
7					
8					
9					

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

#### **OHT N1.2c**

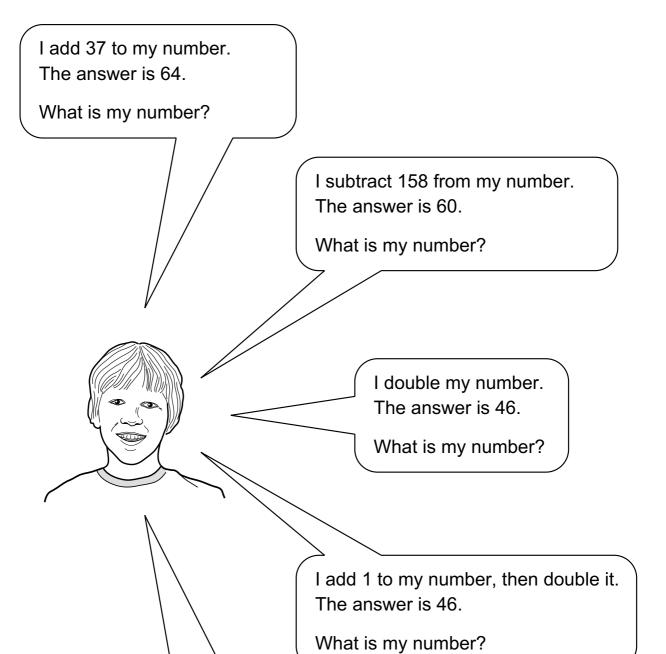
This table shows the numbers of goals scored for and against some teams in the 2001-02 football season.

Team	Goals for	Goals against
Arsenal	79	36
Charlton Athletic	38	49
Chelsea	66	38
Leeds United	53	37
Leicester City	30	64
Liverpool	67	30
Manchester United	87	45
Newcastle United	74	52
Sunderland	29	51
Tottenham Hotspur	49	53
West Ham United	48	57

Make up questions about the teams. Use these words.

total altogether add sum minus how many less? take away plus subtract difference how many more?

## **OHT N1.4a**



I halve my number, then subtract 5. The answer is 20.

What is my number?

### **OHT N1.4b**

Use the facts in the left-hand column to complete the calculations in the right-hand column.

$$1100 - 300 = 800$$

$$97 + 46 = 143$$

$$718 - 525 = 193$$

$$273 - 143 = 130$$

$$143 + 157 = 300$$

$$525 - 368 = 157$$

thousands	hundreds	tens	units

# OHT N1.6a

Ⅱ ∞	II ©	 ∞	II ∞	II ∞	II ∞	 ∞	 ∞	II ©	   &
× —		ო ×	<b>4</b> ×	δ ×	× •	× <b>/</b>	× ∞	× б	10 ×
П	Ш	П	П	П	Ш	Ш	П	II	П
4	4	4	4	4	4	4	4	4	4
×	, Z	ო ×	4 ×	δ ×	× <b>9</b>	× <b>′</b>	× &	× 6	10 ×
П	II	П	П	П	П	П	П	П	П
0	0	0	0	8	0	0	0	0	N
×	$\stackrel{C}{\times}$	$\overset{\circ}{\kappa}$	<b>4</b> ×	Ω ×	× ×	× _	$\infty$	× တ	10 ×

# **OHT N1.7a**

## **OHT N1.8a**

You know that  $6 \times 7 = 42$ .

Use this fact to complete these equations.

# **Problem 1**

Use all of the digits 6, 1 and 3 to complete this subtraction.

# **Problem 2**

Use three of the five digits 1, 3, 5, 7 and 9 to make this statement correct.

# **OHT N1.8c**

Here are five number cards.

5 8

Use all five cards to make an addition that has the answer 500.

