1 Mental addition

Target

- **To add a pair of two-digit numbers, such as 78 + 56**

Current understanding

Pupils should already be able to:

- recall addition facts to 20;
- add a multiple of 10 to a whole number, such as 67 + 30.

Common errors

Pupils may calculate:

- 58 + 26 = 74 instead of 84;
- 34 + 29 = 34 + 30 + 1 instead of 34 + 30 – 1.

In 46, pupils may refer to the digit 4 rather than its value, 40.

What to do

Vocabulary

digit

What you need

Number line marked 0 to 100

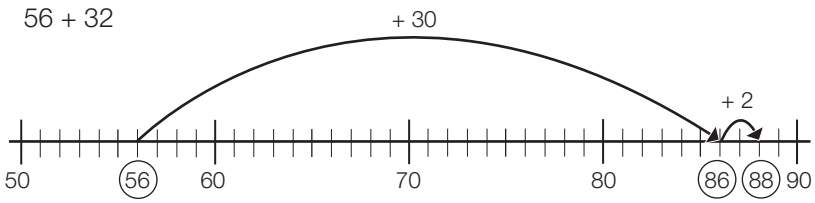
Make sure that the pupil understands the target.

Check that you are clear about the following stages of increasing difficulty in adding a pair of two-digit numbers:

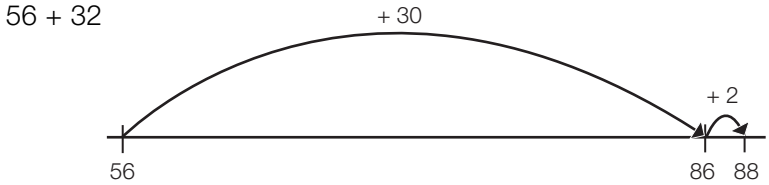
Stage 1	46 + 50	adding tens
Stage 2	43 + 52	units within 10
Stage 3	43 + 58	units greater than 10
Stage 4	63 + 52	tens greater than 100
Stage 5	63 + 58	units greater than 10 and tens greater than 100

For each stage:

- Try a question, then demonstrate as necessary using a 0 to 100 number line. For example, for stage 2:



- Give the pupil similar examples to try.
- When the pupil is confident in using a marked number line, try some examples with first you and then the pupil drawing an empty number line.



- When the pupil is confident, ask them to explain their working without a number line. For example:

  56 + 32 = 86 + 2 = 88

  or 56 + 30 = 86
  86 + 2 = 88

- Practise further examples so that the pupil can explain the calculation without any written support.
- Move on to the next stage in the progression and repeat the process.

At the end of stage 5, use the key questions to check that the pupil has reached the target and is confident.

Key questions

23 + 44

46 + 38

74 + 87

63 + 58

56 + 43 + 8

I have 76p and you have 47p. How much do we have altogether?
How did you work out the answer?

Discuss the methods the pupil uses. For example:

63 + 58 = (63 + 50) + 8 = 113 + 8

or 63 + 58 = (60 + 50) + (3 + 8) = 110 + 11

or 63 + 58 = (63 + 8) + 50 = 71 + 50

2 Mental subtraction

Target

- **To subtract a pair of two-digit numbers, such as 73 – 48**

Current understanding

Pupils should already be able to:

- recall subtraction facts within 20;
- subtract a multiple of 10 from a whole number, such as 68 – 50.

Common errors

Pupils may calculate:

- 73 – 48 = 35 instead of 25;
- 74 – 29 = 74 – 30 – 1 instead of 74 – 30 + 1.

What to do

Vocabulary

digit
difference

What you need

Number line
marked 0 to 100

Make sure that the pupil understands the target.

Check that you are clear about the following stages of increasing difficulty in subtracting a pair of whole numbers:

Stage 1 86 – 50 subtracting tens

Stage 2 86 – 5 units within 10

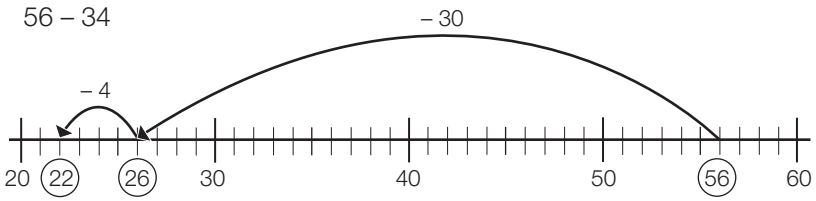
Stage 3 86 – 8 units cross 10 boundary

Stage 4 78 – 52 tens and units within 10

Stage 5 93 – 58 units cross 10 boundary

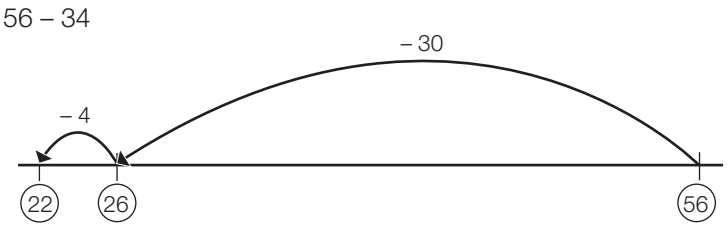
For each stage:

- Demonstrate an example using a 0 to 100 number line. For example, for stage 4:



- Give the pupil similar examples to try.

- When the pupil is confident in using a marked number line, try some examples with first you and then the pupil drawing an empty number line.



- When the pupil is confident, ask them to explain their working without a number line. For example:

  56 – 34 = 26 – 4 = 22

  or 56 – 30 = 26
  26 – 4 = 22

- Practise further examples so that the pupil can explain the calculation without any written support.
- Move on to the next stage in the progression and repeat the process.

At the end of stage 5, use the key questions to check that the pupil has reached the target and is confident.

Key questions

68 – 25

93 – 58

67 – 18

165 – 137

In a school hall there are 83 pupils. 38 leave.
How many are left?
How did you work out the answer?

Discuss the methods the pupil uses. For example:

93 – 58 = (93 – 50) – 8 = 43 – 8

or 93 – 58 = (93 – 60) + 2 = 33 + 2

or 93 – 58 + 2 (to 60) + 33 (to 93)
answer: 35 (by counting on)

3 Multiplication tables

Target

- **To recall multiplication facts up to 10 × 10, such as 9 × 7**

Current understanding

Pupils should already be able to:

- recall multiplication facts in the 2, 5 and 10 times tables.

Common errors

Pupils may calculate:

- 8 × 7 = 54 instead of 56 (to avoid this, think 7 × 7 = 49, and 7 more is 56);
- 0 × 9 = 9 instead of 0.

What to do

Vocabulary

multiply
multiplication
product (the product of 6 and 7 is 6 × 7 = 42)

What you need

Flash cards for key multiplication facts

Written lists of tables (sheet 3.1)

Tricky facts (sheet 3.2)

Make sure that the pupil understands the target.

To find a starting point, check what the pupil already knows. The suggested order for learning the tables is 2, 10, 5, 3, 4, 6, 8, 9, 7.

Concentrate initially on **one table** at a time. Give the pupil a copy of the table, cut from **sheet 3.1**.

- Chant the table, e.g. 1 × 8 = 8, 2 × 8 = 16, 3 × 8 = 24, …
- Chant the sequence, e.g. 7, 14, 21, 28, …
- Highlight square numbers, e.g. 9 × 9 = 81
- Build up understanding that 3 × 4 = 4 × 3 etc.

To check each table

Prepare flash cards or write down key facts, for example:

8 × 7 = ?

□ × 7 = 42

Ask table facts in a jumbled order.

To check several tables

Prepare random lists of multiplication facts for the pupil either to answer orally or to complete in a timed session.

Strategies

Help the pupil to reduce the work by using **links between tables**.

- The 2, 5 and 10 times tables should already be known.
- The 3 times table needs to be learned.

- The 4 times table is double the 2 times table.
- The 6 times table is double the 3 times table.
- The 8 times table is double the 4 times table.
- The 9 times table follows from the 3 times table.
- The 7 times table needs to be learned, but because $3 \times 7 = 7 \times 3$, much of it is already done when learning other tables.

Encourage the pupil to **work out the ones they don't know from the ones they do**. For example: *'I don't know 8 × 6, but 6 × 6 = 36 and 2 × 6 = 12, so 8 × 6 = 48.'*

Use **doubling**. For example:

7×8 Double 7 to get 14 (7×2)

Double 14 to get 28 (7×4)

Double 28 to get 56 (7×8)

Sheet 3.2 may help the pupil **learn the tricky tables**; give them a copy when they think they know most of the tables.

At the end, use the key questions to check that the pupil has reached the target and is confident.

Key questions

8×7

7×6

9×8

How many 8s are in 48?

I have 60p. How many 7p pens can I buy?
Did you know the answer?
How did you work it out?

4 Fractions

Target

- **To recognise the decimal equivalents of simple fractions (tenths, half, fifths, quarters)**

Current understanding

Pupils should already be able to recognise a fraction ($\frac{1}{2}$) and a decimal (0.5).

Common errors

Pupils may think that, because $\frac{1}{10}$ is equivalent to 0.1, $\frac{1}{4}$ is equivalent to 0.4.

What to do

Make sure the pupil understands the target.

Equivalent fractions

Use the fraction wall (**sheet 4.1**) to show the pupil that $\frac{1}{10}$ is equivalent to 0.1.

Discuss other equivalent fractions and decimals (tenths). For example, show that $\frac{3}{10}$ is equivalent to 0.3, 0.7 is equivalent to $\frac{7}{10}$ and $\frac{9}{10}$ is equivalent to 0.9.

Show that $\frac{1}{5}$ is equivalent to $\frac{2}{10}$ and to 0.2.

Ask the pupil what is equivalent to $\frac{2}{5}$ ($\frac{4}{10}$ and 0.4).

Ask similar questions to cover other fifths and one half.

- $\frac{1}{10}$, $\frac{3}{10}$, $\frac{7}{10}$ and $\frac{9}{10}$ are equivalent to 0.1, 0.3, 0.7, and 0.9.
- $\frac{2}{10}$ is equivalent to $\frac{1}{5}$ and 0.2.
- $\frac{4}{10}$ is equivalent to $\frac{2}{5}$ and 0.4.
- $\frac{5}{10}$ is equivalent to $\frac{1}{2}$ and 0.5.
- $\frac{6}{10}$ is equivalent to $\frac{3}{5}$ and 0.6.
- $\frac{8}{10}$ is equivalent to $\frac{4}{5}$ and 0.8.
- $\frac{10}{10}$ is equivalent to 1.

Ask the pupil:

- What is a half of a half?
- What is a half of 0.5?

Then use the fraction wall (sheet 4.1) to show that $\frac{1}{4}$ is equivalent to 0.25 and that 0.75 is equivalent to $\frac{3}{4}$.

It is useful for pupils to know the fraction and decimal equivalents listed above.

Vocabulary

numerator
denominator
equivalent
simplify
order

In the fraction $\frac{2}{5}$, 2 is the numerator and 5 is the denominator.

$\frac{1}{2}$, $\frac{5}{10}$ and 0.5 are equivalent.

What you need

Fraction wall (sheet 4.1)

Ordering fractions

Use the fraction wall (sheet 4.1) to show that $\frac{1}{2}$ is smaller than $\frac{3}{5}$.

Help the pupil recognise which of a pair of fractions is the smaller or the larger. For example, $\frac{1}{2}$ and 0.4, $\frac{1}{4}$ and 0.4, 0.3 and $\frac{2}{5}$.

Finally, use the key questions to check that the pupil has reached the target and is confident.

Key questions

What is $\frac{1}{2}$ as a decimal?

Which decimal is equivalent to $\frac{7}{10}$?

What fraction is equivalent to 0.4? ($\frac{4}{10}$)

Can you simplify your answer? ($\frac{2}{5}$)

Which is larger, 0.8 or $\frac{3}{4}$? Explain your answer.

5 Using a calculator

Target

- **To use a calculator and interpret the display, checking that the answer is reasonable**

Current understanding

Pupils should already be able to:

- round numbers to the nearest 10, 100 or 1000;
- enter numbers into a calculator.

Common errors

- Pupils may misread the calculator display.
- Pupils may input the wrong repeated digits, e.g. 23445 instead of 23345.
- Pupils may not recognise 2.9999999 as 3 or 1.6666667 as $1.\dot{6}$ (1.6 recurring).
- Pupils insecure with place value may enter seven hundred and fifty-six as 70056.

What to do

Vocabulary

approximate,
approximately equal to (≈)
estimate
recurring decimal

What you need

Basic (non-scientific) calculator

Calculator problems (sheet 5.1)

Make sure that the pupil understands the target.

For each question, first estimate the answer.

Check the sequence the pupil uses to enter information into the calculator.

Finally, compare the calculator result with the estimate and sort out any errors.

Start with a question like 32.7 × 86.2. Help the pupil first to approximate to estimate the answer:

This is approximately 30 × 90, which is 2700.
Using a calculator gives 32.7 × 86.2 = 2818.74.
Does this look correct?
Why can't 28 187.4 be correct?

Give the pupil a variety of calculations such as:

Calculation	Estimate	Answer
234 × 126	200 × 130	29484
11 088 ÷ 56	11000 ÷ 50	198
98.6 × 23.8	100 × 20	2346.68
2740.92 ÷ 36.4	2800 ÷ 40	75.3
(274 ÷ 18) × 6	(300 ÷ 20) × 6	91.333333
6789 – 8753 + 7605	7000 – 9000 + 8000	5641

Calculator problems (sheet 5.1)

For each question on **sheet 5.1** the final calculator display is shown. Ask pupils to give the correct answer to the problem. These are:

1 (a) £25.30
(b) Approximating, 20 × £1.10 = £22, so the answer looks about right.

2 (a) 76
(b) Approximating, 200 ÷ 20 = 10 and 10 × 6 = 60, so the answer looks about right.

3 (a) 13 boxes (and some spare eggs)
(b) Approximating, 160 ÷ 10 = 16, so the answer looks about right.

4 (a) £2069.15
(b) £2400 – £1000 = £1400, so the answer looks wrong.
(c) Sandip typed in £2993.50 instead of £2399.50.

At the end, use the key questions to check that the pupil has reached the target and is confident.

Key questions

How would a calculator show an answer of £37.70?

When on a calculator the answer to a calculation is shown as 56.999996, what do you think the answer should be?

How would you write 12.33333333 as an answer?

Is 24 an approximate answer to 9621 ÷ 40.1?

Use a calculator to work out 94.2 × 23.1.
How did you work out the answer?
Does the answer look correct?

6 Word problems

Target

To solve word problems by:

- extracting key information;
- choosing the correct mathematical operation (+, –, ×, ÷);
- using an appropriate method of calculation.

Current understanding

Pupils should already be able to complete numerical calculations.

Common errors

- In a word problem, pupils may not recognise which numbers to use or which operation is needed.
- Pupils may mix units (for example, pence and pounds).
- Pupils may not link their answer back to the original problem.

What to do

Vocabulary

sum (23.4 + 67)
product (45 × 12)
total
more than
operation (+, –, ×, ÷)

What you need

Calculator

Word problems (sheet 6.1)

Springboard 7 (pp. 350–2) has further questions.

Make sure that the pupil understands the target.

For each question on **sheet 6.1**, ask the pupil to:

- read the question out loud;
- tell you what they have to find out;
- decide which bits of information in the question will help them (ask them to circle this information or write it down);
- tell you how they are going to use the numbers to work out the answer;
- write down the calculation they need to do to work out the answer;
- tell you how they are going to do the calculation (in their head, using a written method or using a calculator);
- work out the answer;
- write down the answer;
- check whether the answer is sensible (ask them: 'Is this what you expected? Can you do an approximate calculation to check it?).

Hints for calculations on sheet 6.1

Make sure that the pupil writes down the answer to any intermediate steps.

1 Addition 342 + 129 = 471
The pupil may be able to work this out mentally but may use a calculator.
Note: 27 is superfluous information.

2 Multiplication, using a calculator, followed by a subtraction
$42 \times 63 = 2646$
$2646 - 423 = 2223$ Answer: £2223

3 Take care with the units
$2 \times 1.23 = 2.46$ (mentally) £2.46
$6 \times 21 = 126$ 126p or £1.26
The total is $2.46 + 1.26 = 3.72$ Answer: £3.72

4 Division $960 \div 8 = 120$
The pupil may recognise that $12 \times 8 = 96$ but would probably want to use a calculator.

5 Many pupils try to add the two numbers; however, they should do a subtraction.
$2319 - 1250 = 1069$

Try some further examples from *Springboard 7* pages 350 to 352.

At the end, use the key questions to check that the pupil has reached the target and is confident.

Key questions

Find the sum of two numbers, for example, the sum of 234 and 621.
What operation should you use? (addition)

480 pupils are split into four equal groups.
How many pupils are in each group?
How did you work out the answer? (division)

Anne travels 14 512 miles in a year. Raj travels 27 863 miles in the same year. How many more miles does Raj travel than Anne?
How did you work out the answer? (subtraction)

Find the product of 23 and 54.
What does 'product' mean? (multiplication)

7 Graphs and charts

Target

- **To read and interpret a range of charts and graphs**

Current understanding

Pupils should already be able to interpret bar charts and will have met information presented in tables.

Common errors

Pupils may misinterpret scales, for example, thinking that each division always represents one unit.

What to do

Vocabulary

graph
bar chart
pie chart
table

What you need

Examples of graphs and charts and linked prompts (sheets 7.1 to 7.8)

Charts from recent newspapers or magazines

Make sure that the pupil understands the target.

Choose some of the questions on **sheets 7.1 to 7.8** and use the prompts to work with the pupil. Make sure that you use a wide range of graphs and charts. Alternatively, charts from recent newspapers or magazines are a good resource for questions.

For each example, help the pupil to answer the questions using the chart or table. Check that the pupil understands what the graph or chart is about. Follow these stages:

- Explain what the graph or chart is showing.
- Work things out from the graph or chart.

Useful prompts

- What does the graph show?
- How many are …? *(Read this from the graph or table.)*
- How many more are …? *(Calculate this from values on the graph or in the table.)*
- Why might there be more of …? *(Explain.)*

At the end, check that the pupil has reached the target and is confident – do they have the key skills listed below?

Key skills

Can the pupil explain what the graph or chart is showing?

Can they extract information from the graphs and charts?

Can they figure out answers to questions, based on simple calculations using the data?

Are they able to begin to suggest reasons to explain the figures?

8 Angles

Target

- **To know that there are 90° in a right angle and 180° on a straight line**
- **To solve simple angle problems**

Current understanding

Pupils should already be able to recognise right angles.

Common errors

Pupils may measure angles incorrectly because they do not understand that the angle is the amount that the pointer has turned through.

They may confuse the degree (°) and percentage (%) symbols.

What to do

Vocabulary

right angle
degree (°)
rotate

What you need

Angle maker (sheet 8.1 cut out and assembled)

Ruler

Make sure that the pupil understands the target.

Right angles and the angle on a straight line

Demonstrate how to use the angle maker (**sheet 8.1**). Using a pencil to fix the centre, rotate the pointer around the point to make an angle.

Ask the pupil to rotate the pointer to show you a right angle. Draw a line on the paper to fix this (a feint line is already there).

Ask the pupil to continue to rotate the pointer, from this new line, for another right angle. Draw another line on the paper where the pointer stops (a feint line is already there).

Explain that there are 90° in a right angle. Demonstrate that two right angles will be 2 times 90°, which is 180°, and that this is the angle on the straight line.

Pairs of angles that make 90°

Work practically through problems on pairs of angles that add to make 90°.

Ask the pupil to rotate the pointer from the thick line through roughly 50°. Discuss how many more degrees you need to turn the pointer to make a right angle of 90°.

Repeat this, starting with other angles such as 40°, 30°, 70°, 85° and moving on to angles of 37°, 64°.

Pairs of angles that make 180°

Now do this with angles on a straight line.

Ask the pupil to rotate the pointer from the thick line through roughly 50°. Discuss how many more degrees the pointer needs to turn to make a straight line.

Repeat the questions. Starting with 40°, 80°, 110°, 145°, ask for the angle still needed to turn through to make 180° altogether.

At the end, use the key questions to check that the pupil has reached the target and is confident.

Key questions

Show me a right angle on this sheet of paper.

Make a right angle with the pointer. Tell me how many degrees are in a right angle.

How many right angles are on a straight line?

If I turn 50°, how much more do I have to turn to make a right angle?

Tell me the sizes of two angles that will add together to make a straight line (180°).

9 Sequences

Target

- **To recognise and extend number sequences, such as 1, 9, 17, 25, …**

Current understanding

Pupils should already be able to count on and back in 2s, 3s and 4s from any small positive whole number.

Common errors

Pupils may make arithmetical errors when counting back, particularly across zero.

Pupils may call zero by the letter 'O'.

What to do

Vocabulary

sequence

What you need

Counting stick marked with ten divisions

45 small counters

Make sure that the pupil understands the target.

Counting stick sequences

Use the counting stick and, together with the pupil, count in 2s starting from zero or nought (at the left-hand end of the stick from the pupil's point of view): 0, 2, 4, 6, … Allow the pupil to continue to 20.

Repeat, this time counting in 3s from zero, and then in 9s from zero. Continue beyond the end of the stick.

Point to the centre of the counting stick and name the point as 12. Count up in 2s and then back in 2s from 12. Repeat with other start points and steps.

Label the right-hand end of the stick as 15 and count back in 5s. Carry on into negative numbers. Repeat, but counting back in 3s. Next, label the right-hand end as 12 and count back in 4s, then in 2s. Repeat with other start points and steps.

Label the left-hand end of the stick as 5 and count up in 3s. Ask the pupil to begin to write down the sequence of numbers: 5, 8, 11, 14. Ask them to explain what comes next and why (look for 'It goes up in 3s' or 'It's 3 more each time'). Repeat, starting with 3 and steps of 4.

Ask the pupil to make up a sequence of their own, telling you what the start number is and the step length. Repeat until you are sure that the pupil has understood about using the same step length.

Sequences without the counting stick

Write down the sequence 1, 6, 11, 16, … and ask the pupil to explain how the sequence is built up and what will come next. Repeat with 2, 6, 10, 14, … and other sequences.

Sequences from patterns

Make a pattern of counters.

Ask the pupil to demonstrate what the next pattern will look like and write down how many counters there are in each pattern.

Write down the number pattern 1, 5, 9, 13, 17 and discuss how it is growing.

Make a different pattern of counters and repeat the questions.

Ask the pupil to demonstrate what the next pattern will look like. Write down how many counters are in each pattern.

Discuss how the sequence is growing.

At the end, use the key questions to check that the pupil has reached the target and is confident.

Key questions

What comes next? 3, 7, 11, 15, …

Explain how the sequence 1, 9, 17, 25 is built up. What numbers come next?

In the sequence 20, 17, 14, 11, what comes next and why?

In the sequence 301, 305, 309, 313, 317, what comes next and why?

In the sequence 8, 5, 2, what comes next and why?

10 Length and perimeter

Target

- **To measure length accurately to the nearest mm**
- **To work out the perimeters of shapes**

Current understanding

Pupils should already:

- be able to use a ruler to measure lengths to the nearest cm;
- know that the perimeter of a shape is the distance round it.

Common errors

Pupils may measure from the end of a ruler and not from zero.

Pupils may confuse perimeter and area.

What to do

Vocabulary

perimeter

What you need

Shapes (sheet 10.1)

Ruler and pencil

Plain paper

Make sure that the pupil understands the target.

Demonstrate how to draw a line of length 11 cm.

Ask the pupil to draw lines of length 6 cm 3 mm, 12 cm 5 mm and 17 cm 4 mm. Check the accuracy.

Draw three lines and ask the pupil to measure each of them to the nearest mm. Check for accuracy and the correct use of the ruler. If there are any problems, repeat the activity and teach the pupil how to use the ruler correctly.

Ask the pupil to explain what the perimeter of a shape is (the distance around the shape). Demonstrate perimeter: start at one corner and trace round all the sides with your finger.

Use shape A, a quadrilateral, on **sheet 10.1**. Ask the pupil to measure each of the four sides and to write the lengths on the diagram. Ask them to write the lengths in a list and add them up. (Encourage the separate addition of cm and mm, then recombining, converting the total number of mm to cm and mm.)

Repeat this for each of the other quadrilaterals (shapes B, C and D) on sheet 10.1.

At the end, use the key questions to check that the pupil has reached the target and is confident.

Key questions

Measure this line accurately (e.g. 14 cm 8 mm).

Draw a line 11 cm 4 mm long.

Explain how you would work out the perimeter of a straight-sided shape.

What is the perimeter of the triangle on sheet 10.1?