Guidance for Summer Numeracy Schools

Set 1

To support sessions 1 and 5

Recalling number facts and practising mental calculation strategies

These 20 units support two half hour sessions each day on mental calculation strategies and recall of number facts.

Sessions 1 and 5 are intended as whole class teaching sessions, mainly oral, in which you can revise mental calculation strategies with pupils and they can practise using them. They can also rehearse recalling number facts such as addition and subtraction facts to 20 and multiplication tables.

Each of the 20 units is designed as a 'mini-lesson' of 30 minutes, usually with a 5 minute starter, 20 minutes of whole class or group activity, and a 5 minute plenary. Since the time is short, try to make sure that each session starts promptly and is conducted briskly. The ten units for the first week concentrate on addition and subtraction strategies, while the ten units for the second week focus on multiplication and division strategies.

Pupils should find the first few units, in particular, relatively easy. The purpose of this is to help you to build pupils' confidence so that they can succeed and establish an expectation that the pace in these sessions will be brisk.

When you are planning your mini-lessons for sessions 1 and 5, you may find it helpful to look at the QCA publication *Teaching Mental Calculation: Guidance for Teachers at Key Stages 1 and 2.* This suggests a range of suitable classroom activities.

You should also refer to Section 6 of the *Framework for Teaching Mathematics from Reception to Year 6*. Pages 38–47 (for addition and subtraction) and pages 58–65 (for multiplication and division) set out the number facts that pupils are expected to recall rapidly and give a year by year listing of a range of calculations that pupils are expected to be able to do mentally.

The units cover these objectives from the Framework for teaching mathematics

Unit	Objectives	Resources provided with the unit	Resources to be provided by the summer school
1.1	**Simple subtraction** • recall addition and subtraction facts to 20 • put the larger number first • understand subtraction and use the associated vocabulary	none	• large display cards with subtraction phrases written on them • large 0–30 number cards
1.2	**Instant recall of subtraction bonds** • recall addition and subtraction facts to 20 • understand subtraction and use the associated vocabulary	none	• sets of 0–20 number cards or number fans • two 1–20 dice • individual number lines (optional)
1.3	**Adding several small numbers** • recall addition and subtraction facts to 20 • add several small numbers	none	• 0–30 number line • sheets of plain paper • four 0–9 dice • sets of 0–9 number cards or number fans
1.4	**Adding and subtracting 9, 11...** • count on and back in tens • add and subtract mentally a ‘near multiple of 10’	• Sheet 1.A	• individual 100-grids (optional)
1.5	**Using an empty number line 1** • add and subtract mentally any pair of two-digit numbers • put the larger number first	• Sheet 1.B	• 0–50 wall number line (optional) • individual number lines
1.6	**Addition and subtraction facts** • recall addition and subtraction facts to 20	none	• sheets of A4 paper with a number written in large digits on each
1.7	**Using an empty number line 2** • add and subtract mentally any pair of two-digit numbers • know that an addition fact can be interpreted as a subtraction fact	none	• set of 0–9 number cards per pair • 0–100 number line
1.8	**Adding and subtracting 1, 10 and 100** • count on and back in 1s, 10s, 100s • use known facts and place value to consolidate mental addition and subtraction	• Sheet 1.C	none
1.9	**Adding and subtracting money** • count on and back in 1s, 10s, 100s • add and subtract mentally any pair of two-digit numbers • use addition and subtraction to solve word problems involving money	none	none

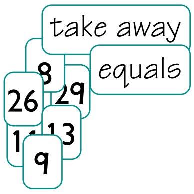
Unit	Objectives	Resources provided with the unit	Resources to be provided by the summer school
1.10	**Solving word problems** • use addition and subtraction to solve word problems involving money • explain methods and reasoning	• Sheet 1.D • Sheet 1.E1 • Sheet 1.E2	• individual number lines and 100-grids (optional)
1.11	**Multiplication tables** • know by heart multiplication facts to 10×10	none	• large multiplication square with some numbers blanked out
1.12	**Patterns in tables** • know by heart all multiplication facts up to 10×10 and derive quickly corresponding division facts	• Sheet 1.F	• large multiplication square with some numbers blanked out
1.13	**Multiplying by 9** • use known facts (eg to multiply by 9, multiply by 10 and adjust) • use multiplication to solve word problems involving money	none	• 0–100 number line (optional) • 100-grid (optional)
1.14	**Multiplying bigger numbers** • use known facts and partitioning to multiply larger numbers mentally	none	• multiplication squares
1.15	**Multiplying and dividing by 10** • use decimal notation for tenths and hundredths • multiply and divide whole numbers and decimals by 10 and explain the effect	• Sheet 1.G	none
1.16	**Division with remainders** • find remainders after division • round up or down after division, depending on the context	none	• 2–9 number cards and 1–6 dice for each group • counters
1.17	**Doubling two-digit numbers** • derive quickly doubles of two-digit numbers	none	none
1.18	**Finding pairs of factors** • know by heart all multiplication facts up to 10×10 • find all the pairs of factors for any number up to 100 • understand and use division as the inverse of multiplication	• Sheet 1.H	• OHP • squared paper • two 1–6 dice • counters in two colours
1.19	**Approaches to division** • know by heart all multiplication facts up to 10×10 • understand and use division as the inverse of multiplication • explain methods and reasoning	none	• 1–10 spinner or 1–10 number cards
1.20	**More money problems** • use multiplication and division to solve word problems involving numbers in ‘real life’ and money • explain methods and reasoning	• Sheet 1.I • Sheet 1.J	none

Simple subtraction

Resources

large display cards with subtraction phrases written on them

large 0–30 number cards, one of each number



Language

leaves, equals, equal to, makes difference, subtract, minus, take away take from, less than, more than, count on count back, number sentence write in figures

5 mins

Starter: whole class

Explain to the pupils that they are going to do some work on subtraction. Give them some subtraction calculations to do in their heads, phrased in different ways, such as:

Take 3 from 7. What is 4 less than 8?...And 9 subtract 3?
What is the difference between 10 and 18?...What is 11 take away 5?
What do you get if you count back 5 from 14?
What is 21 minus 6?...What does 4 taken from 7 leave?
6 taken from a number leaves 5. What is the number?

From time to time ask:

How did you work it out? How did you know what to do?
Who did something different from that?
Did anyone just know it?

20 mins

Main activity: whole class

Now write up some subtraction calculations on the board, and ask pupils to find different ways of saying these, using words such as 'less', 'difference', 'take away' and 'count back'.

21 – 5

When we are doing subtraction we usually say the bigger number first: '11 take away 5' or '11 minus 5'. But sometimes we say the smaller number first: 'take 5 from 11' or 'the difference between 5 and 11' or 'subtract 3 from 6'.

Call four pupils out to the front of the class.

Give two of them large number cards and another two large phrase cards, such as 'taken from' and 'leaves'.

Ask the four pupils to arrange themselves facing the class so that when they hold up their cards they make an unfinished number sentence.

Get the class to chant the incomplete sentence aloud:

14 | taken from | 22 | leaves

Fourteen taken from twenty-two leaves...

Objectives
- recall addition and subtraction facts to 20
- put the larger number first
- understand subtraction and use the associated vocabulary

Invite a fifth pupil to come to the front, pick the appropriate number card, and hold it up so as to complete the sentence.

14 | taken from | 22 | leaves | 8

Get the class to chant the completed sentence aloud:

Fourteen taken from twenty-two leaves eight.

Ask the pupils holding number cards to stay where they are, but replace the other two with pupils holding a different set of phrase cards.

14 | less than | 22 | is | 8

Again, ask the pupils to face the class holding up their cards to make a number sentence – with the same numbers as before but with different words.

Get the class to chant the completed sentence.

Do this several times involving as many pupils as possible. (When using 'the difference between' you will need a total of six pupils: three to hold the number cards, one to hold 'the difference between' and two to hold 'and' and 'is').

the difference between | 14 | and | 22 | is | 8

You could also introduce a 'mystery card' into the number sentence:

14 | taken from | ? | leaves | 6

? | minus | 7 | is | 8

Finish by writing a set of similar number sentences up on the board, with squares where the numbers belong.

Give the pupils a choice as to which trio of numbers they work with: 5, 4 and 9 or 8, 13 and 21. They should work quickly to copy and complete the sentences.

☐ less than ☐ is ☐

☐ take away ☐ leaves ☐

☐ taken from ☐ leaves ☐

☐ minus ☐ equals ☐

☐ count back ☐ comes to ☐

the difference between ☐ and ☐ is ☐

☐ - ☐ = ☐

5 mins

Plenary: whole class

Ask the pupils to mark their own work as you say the answers out loud.

Get the whole class to chant the answers as you give them.

Instant recall of subtraction bonds

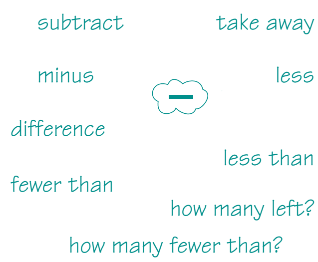
Resources
sets of 0–20 number cards or number fans
two 1–20 dice
individual number lines (optional)

Language
count on, count back
take away, subtract, less, minus
difference, less than, fewer than, more than
equals, number sentence, calculation
lower, smaller
left, right

5 mins

Starter: whole class

Start with some vocabulary work. On the board write the symbol for subtraction, then ask the class to tell you any words associated with this symbol and invite them to spell the words they suggest. Write these words around the subtraction symbol.



Ask the pupils to suggest number sentences using these words.

> Can anyone say a number sentence using the word 'difference'?

Ask pupils, in pairs, to write down a number sentence using any of the words or phrases on the board. After a minute or two invite volunteers to read out their sentence to the class.

20 mins

Main activity: whole class

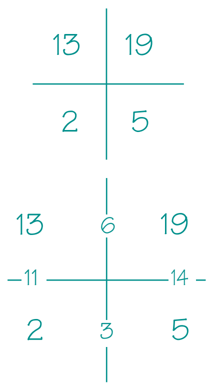
Explain the purpose of the lesson.

> It's important to know all addition and subtraction facts up to 20 in your heads. Now we are going to practise subtraction – and I want you to try and answer the questions really quickly.

Roll two 1–20 dice and call out the numbers. Ask the pupils to show the difference between these two numbers with their number cards or fans. Keep the pace fast as you continue the activity for a few minutes.

Now move on to an investigation involving difference. Draw a 2 × 2 grid on the board and ask the class to suggest any numbers below 20 to write on the grid.

Rub out the bits of grid between next door numbers and ask pupils to tell you the difference between the two numbers. Put a ring around the odd differences – and remind pupils how to distinguish between odd and even numbers.



Pupils work in pairs. Set the challenge: they must arrange any four numbers on a similar grid so that the differences are all odd.

Any arrangement of numbers will work as long as every odd number is next to only even numbers, and every even number is next to only odd numbers.

Challenges

Try to find another solution that works.

Try the same investigation on a 3 × 3 grid.

Try to find a solution using four numbers between 20 and 30.

11	9	3
2	15	7
9	1	17

5 mins

Plenary: whole class

Ask a volunteer to come to the board and show their arrangement. Go over the differences with the class, checking each one.

What is the difference between this number and this one?…
Yes, 5. Is that odd?

Ask the class if anyone found a different solution, and explain that there are several.

Discuss the general rule for making arrangements that 'work' in this activity and help the class check out the rule.

Did anyone realise that every odd number needs to be next to an even one?
Let's find a few more solutions and see if they fit this rule.

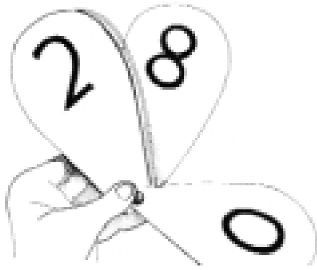
Encourage pupils to talk about the mental strategies they used in finding the differences.

Which pairs of numbers did you just know?
Which pairs of numbers were not easy to remember? Why?
If you're subtracting 9 from a number how might you do it?… Is there any other method you could use?

Adding several small numbers

Resources

0–30 number line
sheets of plain paper
four 0–9 dice
sets of 0–9 number cards or number fans



Language

digit, number
count on
larger, more than, greater than
add, plus, total, sum
altogether, is the same as, equals
calculation, number sentence

5 mins

Starter: whole class

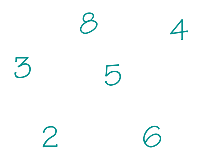
Start by practising instant recall of number bonds for a number below 20, such as 15. Say a number and ask the class to work out what to add to make it up to 15. They should show the answer with their cards as quickly as possible.

Twelve. How many steps is it to 15? Yes… three. 12 and 3 make 15.

20 mins

Main activity: whole class

Explain that today the class is going to practise adding several small numbers together.



Ask four or five pupils each to suggest a single-digit number; write these numbers on the board, and add one or two of your own – the aim is to have one or two pairs or trios of numbers that add to 10.

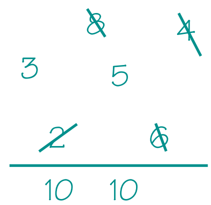
Spend a few minutes adding those numbers in your head. But I'm not going to ask you for the answer – I want you to pay attention to *how* you do it.

After a couple of minutes ask the pupils how they added the numbers. Remind them of the strategy of looking for pairs or trios that make 10. If any of the pupils used this strategy, stop and talk about its benefits. If not, outline the strategy yourself.

One really good way of adding several small numbers is to look for pairs or groups of three that make 10. I want you to help me do that; Kaine, can you see any numbers that add up to 10?… Yes, 3, 5 and 2.

Show pupils how to cross out the numbers they have dealt with and record their total so far, then look for more ways to make 10. Finally show them how to add the tens and any spare numbers.

We've got two tens, which makes… Yes, 20. And 5 and 3. That makes… Yes, 28.



Objectives

- recall addition and subtraction facts to 20
- add several small numbers

Now play a lotto game with the whole class. Each pupil needs a sheet of plain paper. Show them how to fold it to make eight 'boxes' when opened out. Then give instructions for filling each box with a number under 36.

> In the top left box write the number you like best below 36… In the next box along write any odd number below 36…

These eight numbers are the pupils' 'lotto' numbers. In order to generate numbers to 'call', throw four 0–9 dice, read out the numbers, and record them on the board. Invite the class to suggest ways of adding the four numbers, reminding them about the strategy of looking for numbers that add up to 10.

When a total matches one of their lotto numbers, pupils may cross that number out. The first one to cross out all eight numbers is the winner.

At appropriate points in the game point out, or remind the class about, other useful strategies such as looking for doubles or starting with the largest number and adding on the smaller ones.

5 mins

Plenary: whole class

Conclude the lesson with a 'chain' calculation to give pupils practice in holding numbers in their heads. Shuffle a set of 0–9 number cards and turn them over one by one.

> The first number is 5. Put 5 in your head.
> The next number is… 3. Add 3 to the 5 you already have.
> Now add… 8… And 2.

Stop after about five numbers and ask pupils to tell their neighbour the final answer. Then get them to show this number to you with their number cards.

Look at the number cards with the class and talk about how they found the activity.

> Mark, how did you do the adding?… You managed to hold the numbers in your head, did you?
> Cerys says she counted on with her fingers. Did anyone else do that?…
> Ruby, you counted on in your head, did you?

Finish with a reminder about the main purpose of the lesson: learning efficient ways of adding several small numbers by:

— looking for pairs or trios that make 10

— starting with the largest number

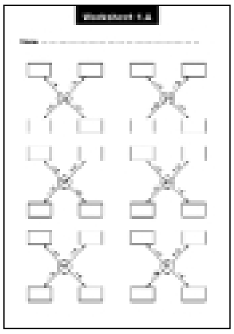
— looking for doubles.

Adding and subtracting 9, 11…

Resources

Worksheet 1.A

individual 100-grids (optional)



Language

count on, count back

digit

add, plus, total, sum

take away, subtract, minus

altogether, equals

number sentence, calculation

5 mins

Starter: whole class

Start with a number problem for all the pupils to tackle at their own level.

Ask four pupils each to write a single-digit number on the board.

4 3 1 9

Now pupils work in pairs. They can use the digits as they are or multiply them by ten. They then add or subtract the numbers made, trying to get as close to 100 as possible.

$90 + 10 + 4 - 3 = 101$

When everybody has had a try, choose one or two pupils to write their solutions on the board and explain them to the class.

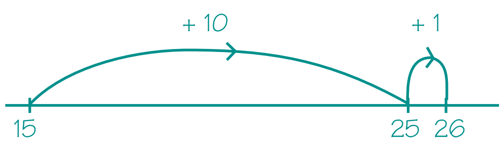
20 mins

Main activity: whole class

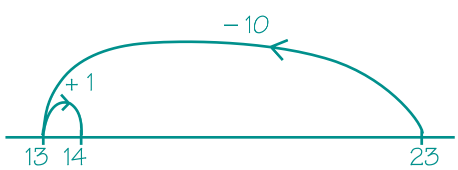
Tell the class that today they are going to work on adding and subtracting 9 and 11.

Start with some practice using numbers below 30 – adding 9 or 11 to, and subtracting 9 and 11 from, these two-digit numbers. Demonstrate these operations by drawing ‘empty number lines’ on the board.

Fifteen add 11. Start with 15 and add on 10… That’s 25. And one more makes… 26.



Twenty-three minus 9. Start with 23 and take off 10… That’s 13. Now what do we do?… Yes, add on 1. That makes… 14.

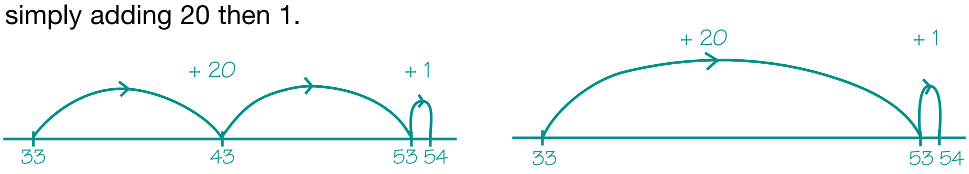


Objectives
- count on or back in tens starting from a two-digit number
- add and subtract mentally a 'near multiple of ten' to or from a two-digit number

Now move on to adding and subtracting numbers near to a multiple of ten, such as 29, 31, 39 or 41. Tell pupils that the same method can be used, adding a multiple of 10 and then adjusting up or down.

> OK, you all know how to add and subtract 9 and 11. What about 19 and 21? Do you think you could use the same method with numbers like these?
> Let's try adding 21 and 33. Start with 33. We can add 21 by jumping on 20 to get… Yes, 53, and going on one more step to… Yes, 54.

Demonstrate this on an empty number line. You may want to support less confident pupils by jumping in tens; if you do, also demonstrate how to do the calculation by simply adding 20 then 1.



Talk about, and demonstrate similar methods for adding 19, and for subtracting 19 or 21.

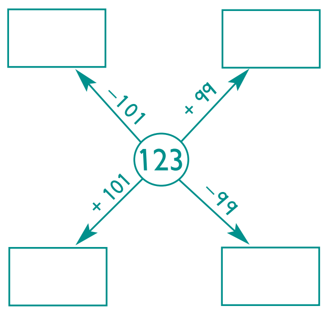
Give each pupil a copy of Worksheet 1.A and go through the first problem with them on the board. Ask them to use the methods you have just been discussing, or any other they prefer, to work out the rest of the answers.

Challenges

Provide problems in adding or subtracting other near multiples of ten such as 28 and 29, 31 and 32, 41 and 42…

Pupils can make up their own problems using two-digit or three-digit numbers. Talk with them about the methods they use.

Draw this diagram on the board. Ask pupils to adapt today's methods to perform the four calculations.



5 mins

Plenary: whole class

Discuss how the pupils tackled some of the problems on the worksheet. Invite individual pupils to write one of their calculations on the board and explain how they did it. Ask if other pupils used the same or another method. Stress that adding numbers that are 'nearly tens numbers' is easy!

Using an empty number line 1

Resources

Worksheet 1.B
0–50 wall number line (optional)
individual number lines

Language

add, total, sum, plus, take away
minus, subtract, difference
how many more/fewer than…?
how many more to make…?
how many left? altogether, equals

5 mins

Starter: whole class

Explain that today's lesson is about adding and subtracting mentally.

Begin with a series of counting activities with the whole class.

> I am going to start counting. Join in as soon as you can: 0, 5, 10, 15… 65, 70.
> What was the rule?… Yes, we were adding on 5 each time, or increasing by 5.
> Now try this one: 3, 13, 23… 93, 103.
> What is the rule this time?… Yes, counting on 10, or increasing by 10.
> Now try this one: 50, 46, 42, 38… 6, 2.
> What is the rule this time?… Yes, counting back 4, or decreasing by 4.

20 mins

Main activity: whole class and individuals

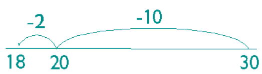
Draw a number line on the board from 0 to 50, drawing only the tens numbers:

Remind pupils about how this can be used to add and subtract tens numbers such as 10, 20, 30, 40 and 50 (by drawing jumps, or making them with a finger or pointer). Practise some calculations with them.

10 + 20 =   30 – 20 =   20 + 30 =   50 – 20 =

Then move on to adding and subtracting where one number is a tens number and the other isn't. Show pupils how they can draw their own line segments and jumps, demonstrating on the board, and inviting individual pupils up to draw some themselves.

30 – 12 =



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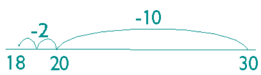
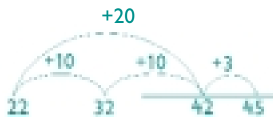


Move on to working with other two-digit numbers, where totals are no more than 100, and where the units, when added or subtracted, do not cross a tens number. (For example, 11 + 34 or 39 – 21 but not 54 + 27.)

When adding, say, 22 and 23, pupils may well draw jumps of ten. This is a perfectly good method, but pupils should also consider the option of drawing larger jumps of, say, 20.



22 + 23 =

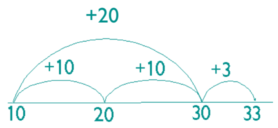
Objectives

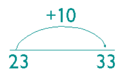
- add and subtract mentally any pair of two-digit numbers
- put the larger number first

Talk about the strategy of putting the bigger number first when adding.

> When you want to add 10 and 23, how can you do it?
> Is there another way to do it?
> Which is quicker and easier?

Ask pupils to demonstrate the two methods on the board:





Then give out Worksheet 1.B.

> I am interested in how you do the questions and later I will be asking you to talk about how you got the answers.
> When you have finished the worksheet, find a partner to swap sheets with, and do the calculations they have invented for you.

Challenges

Pupils who complete the worksheet early could make up any addition problems that they feel are a bit difficult, decide on a strategy for tackling them, and solve them, writing down their method at the same time.

5 mins

Plenary: whole class

Ask the pupils how they worked out the problems. Try to draw out from them what strategies they used.

> How did you do 35 – 13? And 70 + 25?

When the pupils describe the methods they used, point out any special case strategies (for example, adding 19 by adding 20 and subtracting 1). Remind them also of the strategy of putting the bigger number first when adding.

> Was it easier to do 32 + 15 or 15 + 32?

Finish by stressing that when they are adding and subtracting, they can break numbers down into manageable parts.

Addition and subtraction facts

Resources

sheets of A4 paper, with a number written in large digits on each

Language

add, total, sum, plus, increase, minus, decrease take away, subtract, difference, more than less than, how many more/fewer than…? how many more to make…? how many left? altogether, equals

5 mins

Starter: whole class

Start with some class chanting, counting on and back using small numbers.

I am going to start counting. Join in as soon as you can: 5, 10, 15, 20… 45, 50.
What was the rule?… Yes, we were adding on 5 each time, or increasing by 5.
Now try this one: 31, 28, 25… 7, 4, 1.
What is the rule this time?… Yes, counting back 3, or decreasing by 3.

20 mins

Main activity: whole class and individuals

Explain that this lesson is about adding and subtracting mentally – ‘in your heads’.

Now, write some of the words and phrases associated with addition and subtraction on the board: ‘add’, ‘total’, ‘sum’, ‘take away’, ‘subtract’, ‘difference’, ‘plus’, ‘minus’, ‘increase’, ‘decrease’. Read through these with the class.

Can you think of any more words to add to the list?
How do you spell that word?

Two pupils come out to the front of the class and each one chooses a number to hold up in front of them. A third pupil points to one of the words or phrases on the board and the rest of the class try to make up a sentence that uses both these numbers and the phrase:

The total of 5 and 17 is 22.
22 take away 5 equals 17.
Increase 18 by 13.

Let different pupils have a turn at choosing and holding numbers and pointing to the words. If the pupils don’t use a wide range of vocabulary, introduce some of the missing words and phrases yourself:

The difference between 29 and 24 is… Yes, 5.
If we decrease 31 by 3 the answer is 28.

Sometimes, illustrate the numbers, perhaps using the pupils themselves.

13 and 5 more… Thirteen pupils are standing up. Five more stand up… which makes a total of 18.

Objective
- recall addition and subtraction facts to 20

Now spend about five minutes on a writing activity.

Ask pupils to choose two numbers each and write these down. Encourage them to choose single-digit or two-digit numbers, or one of each, depending on their progress. Ask them to write down three sentences, each of which uses both their numbers and one of the phrases or words from the board.

Pupils work in trios of similar attainment.

— one pupil writes down an equation and the other two copy it, then work out the answer

— the first pupil looks at both answers and decides whether or not they are correct (in particular, whether the answers agree)

— if the first pupil decides they are not correct, or if the answers do not agree, all three pupils check the calculations, until they can collectively agree an answer

— they then swap roles, so that everybody has a turn at choosing the calculation

Simplest
The starting number is under 50. Pupils add or subtract a single-digit number.

Next simplest
The starting number is in the 50–100 range. Pupils add or subtract a number in the 10–20 range.

Hardest
The starting number is 150. Pupils add or subtract a number between 10 and 50.

About five minutes before the plenary, give pupils some individual practice in written calculations. Write 10 to 20 mixed number calculations on the board for them to do as speedily as they can.

$17 - \square = 4$

$23 + \square = 39$

$29 - 3 = \square$

$36 - \square = 24$

5 mins

Plenary: whole class

Ask pupils to mark their own work as you say the answers.

Discuss how they got on with the main activity. Encourage them to think about the mental strategies they used.

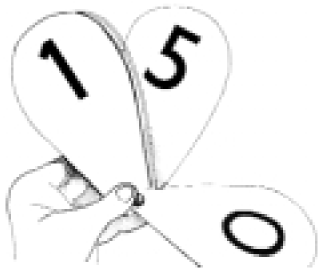
Which pairs of numbers were easy to add?
Which pairs of numbers were not so easy to add? Why?
If the starting number is 50 and you want to add 11, what do you do? Is there any other way you could do it?
If the starting number is 50 and you want to add 19, what do you do? Is there any other way you could do it?
If the starting number is 51 and you want to add 18, what do you do?
If the starting number is 55 and you want to add 18, what do you do?

Using an empty number line 2

Resources

set of 0–9 cards or number fan for each pair

0–100 number line



Language

digit, unit, number
addition, plus, count on, total, sum
altogether, equals
number sentence, calculation

5 mins

Starter: whole class

Each pair of pupils makes a two-digit number with their number fans and holds them up. Meanwhile, make a two-digit number yourself. Now hold up your number and ask the pupils to work out which of their numbers is closest to yours. Refer to the number line if appropriate.

20 mins

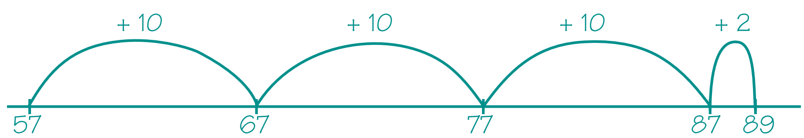
Main activity: whole class

Tell the class that you are going to show them how to use an empty number line to work out subtraction problems involving two-digit numbers. Start with a simple calculation that does not involve crossing the tens. Write the calculation on the board and establish which number to start with.

> I want to do 89 take away 57. I can start with the 57 and count on to 89.
> This will tell me the difference.

Invite suggestions as to how to count on. Choose someone to demonstrate on the board.

> Dawn says start with 57 and add 10 and 10 and 10 and 2.



> So I'll start at 57 and do a jump of 10 to… Yes 67. And another, and another…
> And finally a jump of 2.
>
> The final number is… Yes 89.
>
> Altogether we have jumped on 10 and 10 and 10 and 2. That's 32 altogether.
> So 89 subtract 57 is 32.

Complete the calculation on the board, showing the jumps drawn on the line.

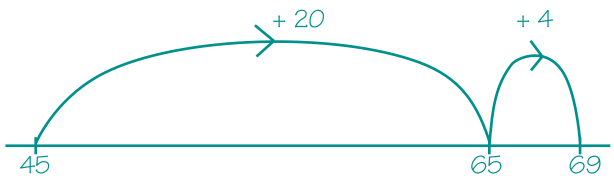
$$\begin{aligned} 57 + 32 &= 57 + 10 + 10 + 10 + 2 \\ &= 87 + 2 \\ &= 89 \end{aligned}$$

Objectives

- add and subtract mentally any pair of two-digit numbers
- know that an addition fact can be interpreted as a subtraction fact

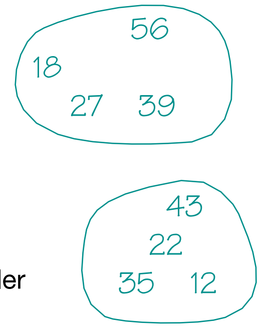
Demonstrate several different subtractions on the board, using the empty number line. Point out that the quickest method of jumping on is to draw as many tens as possible in one jump.

> The next calculation is 69 take away 45. We start with the smaller number, 45.
> So now we need to find out what number to jump on to reach 69.
> A jump of 20 is quicker than two jumps of 10. So we jump 20 to… Yes, 65
> Then 4 more to… 69.



$$45 + 24 = 45 + 20 + 4$$
$$= 65 + 4$$
$$= 69$$

Draw two circles on the board and write four two-digit numbers in each – make sure that in the first circle all the ones digits are 6 or more, and that in the second circle all the ones digits are 5 or less. Pupils choose a number from each circle and subtract them, mentally where possible, or with the help of an empty number line.



Before they start remind them of the strategy 'start with the smaller number and jump on to the bigger'.

Challenges

Provide numbers in the hundreds that do not cross the tens, such as 137 – 82.

5 mins

Plenary: whole class

Invite one or two pupils to demonstrate a subtraction, and their method for working it out, on the board.

Suggest that pupils 'draw' empty number lines in their heads.

> Now I'm going to ask you to do a couple of subtractions in your head. It might help you to close your eyes and imagine drawing jumps on an empty number line as a way of working out the answers.
>
> What is 45 take away 21?… Yes, that's easy, 24. Did anyone imagine an empty number line?
>
> Safia, can you tell me how you worked it out?

Finish by reminding pupils of the strategies: start with the smaller number, and add on the tens all in one go, to get as close as possible to the larger number.

Adding and subtracting 1, 10 and 100

Resources

Worksheet 1.C: 'Stepping stones'

Language

units, ones, tens, hundreds, add
take away, subtract, figure, digit, numeral
count on, count back, count in tens
count in hundreds

5 mins

Starter: whole class

Begin with a series of counting activities with the whole class:

> Start at 28. Count on in ones… Stop at 50 and count back again.
> Start at 5. Count on in tens until you pass 100. Now count back to 5.
> Count on from 100 in tens… Stop when you get to 200 and count back again.
> Count on from 5 in hundreds… Stop when you get past 1000 and count back again.

20 mins

Main activity: whole class

Explain that the lesson today is about adding and subtracting multiples of 10 and 100 – 'tens' numbers and 'hundreds' numbers.

Write several operations on the board (+1, –1, +10, –10, +100 and –100). Then ask the pupils to count around the class in ones. After a few numbers, stop the pupils and point to one of the operations. Now the counting continues according to the operation shown. For example:

> 10, 11, 12, 13, 14, (+10), 24, 34, 44, 54, (–1), 53, 52, 51, 50, 49…

Write some two- and three-digit numbers on the board, and read them out with the class (for example: 26, 34, 78, 708, 630). Ask some quick-fire questions:

> What number is one more than 78?
> What number is ten more than 630?

Repeat for '1 less than', '10 less than', '100 more than', and so on.

Objectives

- count on and back in 1s, 10s, 100s
- use known facts and place value to consolidate mental addition and subtraction

20 120 430 230 559 109

Now draw a set of stepping stones on the board, similar to those on Worksheet 1.C. Say:

> There is a number on the first stepping stone, and then a string of other numbers on the other stones. You have to write in the missing operations to get from one to the next.

Invite the pupils to make suggestions as to the operations, and write these in above the stones.

Try another one, including four-digit numbers.

363 963 993 2003

Now give out Worksheet 1.C: 'Stepping stones'.

5 mins

Plenary: whole class

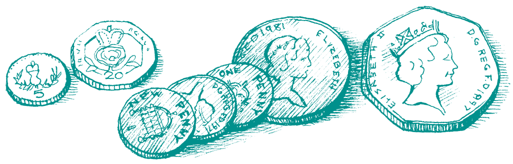
Read out the answers. Ask pupils to mark their own sheets.

Remind pupils that if you add or subtract 10, 100 or 1000, the units digit will stay the same.

Adding and subtracting money

Resources

none



Language

add, total, sum, take away
subtract, figure, numeral
larger, smaller, altogether
equals cost, change
price, value

5 mins

Starter: whole class

Start with a series of counting activities with the whole class.

> I am going to start counting. Join in as soon as you can: 50, 45, 40… 10, 5, 0.
> What was the rule?… Yes, we were taking off 5 each time, or decreasing by 5.
> Now try this one: 7, 27, 47… 107, 127, 147.
> What is the rule this time?… Yes, counting on 20, or increasing by 20.
> Now try this one: £1, 95p, 90p… 10p, 5p, 0p.
> What is the rule this time?… Yes, decreasing by 5p.

20 mins

Main activity: whole class and individuals

Explain that today's lesson is about adding and subtracting with money.

Remind the pupils of different methods of addition and subtraction.

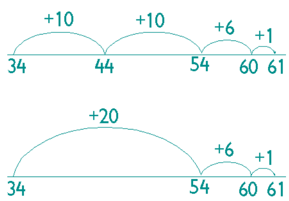
> How would you add 17 + 72?

Write the sum on the board and ask a pupil to come up and draw jumps on a number line to show how they would tackle it. (This is an obvious case for the strategy 'put the larger number first'. If the pupil uses this strategy, point it out to the class; if they don't, remind them about it.)

Then move on to adding and subtracting across the tens, with a calculation such as 27 + 34.

Ask the pupils to suggest ways for tackling this, making sure that they understand there is more than one way of breaking down a number.

> We'll start with the bigger number, 34.
> What jumps shall we make to add on 27?



Invite a pupil up to draw this on the board. If they draw jumps of ten, praise this as useful, and ask if they could try again drawing a jump of 20 – which is even more useful, being quicker and simpler.

Ask the pupils to think of some more additions and subtractions that cross tens numbers. They can come out to the board and show their way of tackling each one.

Objectives

- count on and back in 1s, 10s, 100s
- add and subtract mentally any pair of two-digit numbers
- use addition and subtraction to solve word problems invoving money

Write up on the board a café price list, where items cost under 50p – making sure that at least half the prices have a digit 6, 7, 8 or 9 in the units place. For example:

sandwich	37p	roll and butter	15p	muffin	39p
toasted sandwich	56p	squash	17p	cola	20p
tea	18p				

Give pupils the following scenario:

> A family of four want to have a meal in this café. Each person has £1 to spend. Decide what each person is having, and write it down. Then add up the exact cost of everyone's food.

Invite pupils to do these calculations using the kind of methods you have been demonstrating on the board.

When a pupil has finished their work, say:

> Now you are going to give each member of your family the exact money they need to pay for their meal. Decide what coins you are going to give to each person.

Challenge

What change will each person get from £1?

Let each person choose four items to eat.

Find the total cost of the four people's meals. What will be the change from £5?

5 mins

Plenary: whole class

Ask some of the pupils to share their work with the class.

> Tell me two things that someone in your family chose. How did you work out that addition problem?
> Could that problem be tackled another way?

When the pupils describe the methods they used, remind them that there are different ways to add and subtract. Remind them also of the strategy of putting the bigger amount first when adding and counting up from a smaller number when subtracting money.

Solving word problems

Resources

Worksheets 1.E1 and 1.E2: 'Loop cards'
Worksheet 1.D: 'Word problems'
individual number lines (optional)
100-grids (optional)

Language

add, addition, more, plus
altogether, equals
subtract, take away, minus
how many left? how many fewer than?
number sentence, calculation

5 mins

Starter: whole class

Begin by playing a game of 'I have... Who has?' with the class, using the cards on Worksheets 1.E1 and 1.E2. This is a loop game: the first person reads their card, the words of which indicate the next card to be read. In this way everybody gets to read their card in turn, until the loop is completed; the last card that is read out refers back to the first card.

Shuffle and deal out the cards to the pupils, and yourself. If necessary, give some pupils more than one card. Read out your card:

> I have 21. Who has 8 more?

The pupil who has the card with the appropriate number reads his card out loud, for example, "I have 29. Who has 7 less?" The game continues until the last card is read out.

20 mins

Main activity: whole class

Write a simple addition word problem on the board. Below this write 'Calculation' and 'Answer to problem'.

> Sam and Kim are twins. On their birthday, Sam got £32 and Kim got £27. How much money did they get altogether?
>
> Calculation
>
> Answer to problem

Explain to the class that when they work out the answer to any word problem they are doing two things:

— choosing and doing a calculation

— using the result of a calculation to answer a problem.

Ask pupils to work with a partner and spend a minute or two discussing how to tackle the problem.

Invite one of the pupils to come to the board and cross out any irrelevant information (in this case 'Sam and Kim are twins') then write on the board the calculation needed to solve the problem.

> ~~Sam and Kim are twins.~~ On their birthday, Sam got £32 and Kim got £27. How much money did they get altogether?
>
> Calculation
>
> 32 + 27 = 59
>
> Answer to problem
>
> they get £59

Objectives

- use addition and subtraction to solve word problems involving money
- explain methods and reasoning

Remind the pupils about the two stages:

> We have done the calculation. But we haven't yet got the answer to the problem. What was the question?
> So, what is the answer?… Yes, it's 59, but 59 what?… Yes, 59 pounds.

Check that they think this is a reasonable answer in the context of the problem.

Now repeat the process described above for one or two other examples, both addition and subtraction. Spend about another five minutes on this. Then leave the class to do two or three such calculations on their own (see Worksheet 1.D for suggestions), following the format: calculation and working, answer to problem. Remind pupils to use an empty number line if they think it will help them.

Challenge — Invite pupils to write word problems for each other.

5 mins

Plenary: whole class

Go over a couple of the problems pupils worked on independently. Invite them to offer their solutions. In each case help them establish the calculation and the answer as separate items.

> What information was irrelevant in that problem?
> What calculation did you do to solve the problem?
> What strategy did you use to work out the answer?
> Could you have worked out the answer in a different way?
> What is the answer to this problem – in a number sentence?
> Is the answer reasonable in the context of the problem?

Finally, explain a homework activity for the weekend: pupils are to make up one or two problems, like the ones in the lesson, to try out on their maths partners after the weekend. Tell them to practise their problems on an adult or older brother or sister, and check that they know the answers themselves.

Multiplication tables

Resources

large multiplication square with some numbers blanked out

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20		28	32	36	40
5		15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14		28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18		36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Language

double, halve, times
times table, multiply
multiplied by
divided by multiple
product

5 mins

Starter: whole class

Explain that today's lesson focuses on multiplication. You will be looking at the way three numbers are linked together: a number to start with, another number to multiply it with, and a third number, which is the answer. Remind them that this has a special name: 'product'.

Start by practising counting on and back in twos, threes, fours, fives and tens from 0 to 50 or above, and back again.

Ask the class to respond together to questions about the 2, 3, 4, 5 and 10 times tables. (Make sure you use a wide range of vocabulary: 'double', 'times', 'multiply', 'halve', 'multiplied by', 'multiple', 'product', 'divided by'…)

> Everyone answer together: What are two sevens?
> Now people on this table, all together… What is double three?…
> Is 40 a multiple of five?
> Now all the boys… Three times nine.
> How many fives in 35?
> Everyone, multiply four by eight… What is the product of five and eight?
> What is eight divided by two?

20 mins

Main activity: whole class and individuals

Write some numbers on the board from, say, the 3, 4, 5 and 7 times tables.

7 45 4 9
3 10 6 5 8
2 30 21 36

> There are links between some of these numbers. For instance, three sevens make 21.
> Can you see any other links between any of these numbers?

Some pupils may suggest links other than multiplication bonds, perhaps addition bonds – for example, "6 and 4 makes 10". Don't reject these, but say:

Objective

- know by heart multiplication facts to 10 × 10

> Can you see any multiplication or division links? I particularly want you to look for these kinds of links.

Encourage pupils to use a wide vocabulary, and to be creative in the links they make.

> You linked two, five and ten and said "two fives make ten". Can you think of another way of saying that?… How about "two times five is ten"? or "double five is ten"?
>
> Can you point out all the numbers that are in the same table?
>
> Can you point out all the numbers in the 5 times table? The 2 times table?

Discuss ways of recording some of the links: for example, drawing lines between them, or writing number sentences. Get the pupils to write or draw examples on the board.

7 45 4 9

3 — 10 6 5 8

2 30 21 36

10 x 3 is 30

9 and 36 are in the 9 x table

Double 4 is 8

45 ÷ 9 = 5

Next, put the following numbers on the board and ask pupils to write down as many number sentences as they can using these numbers.

4 12 8 2 5

10 18 3 9

24 6 20 15 16

Finally, put an incomplete multiplication square up on the board, and ask individual pupils to contribute the missing bits (directing the 'easier' bits to those pupils who will find it most difficult).

5 mins

Plenary: whole class

In the final discussion look out for facts that pupils are hesitant about, and write them on the board (or get pupils to do so). Chant them aloud as a whole class.

> Six threes are eighteen. Four sixes are twenty-four…

Finish with a game of Fizz. Everyone sits in a circle and says the number sequence, starting from 1. Every time the sequence comes to a multiple of 5 (or whatever number you choose) the person whose turn it is must say 'Fizz' instead.

> One, two, three, four, Fizz, six, seven, eight, nine, Fizz, eleven…

Patterns in tables

Resources

large multiplication square with some numbers blanked out

Worksheet 1.F

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24		32	36	40
5		15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14		28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18		36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Language

sequence, double
halve, times
times table, multiply
multiplied by
divided by, multiple
factor, product
divisible by, pattern

10 mins

Starter: whole class

Explain that today's lesson focuses on the number patterns that can be found in the multiplication tables.

Ask the pupils to spot the missing numbers in various number sequences from the multiplication tables.

What numbers are missing in this sequence? 3, 6, 9, …, 15, …, 21
And in this sequence? 4, 8, …, 16, …, 24, 28
Can you continue this sequence? 6, 12, 18, 24, …, …, …
How could we describe that sequence?… Yes, increasing by six, or multiples of six.

Make sure you include a variety of sequences, including some that decrease in size, and some involving larger numbers.

What numbers are missing in this sequence? 25, 20, …, 10, …, …
And this one? 90, 80,…, 60, 50, 40, …, 20, …, 0.
Could it continue past zero? How would it go on?

Do an activity on 'holding numbers in your head'.

Ask the whole class to respond together, when you signal with your hand. (This gives slower pupils the opportunity to think, and prevents faster ones from calling out.)

Hold these three numbers in your head: 8, 15, 12.
Which of the numbers are multiples of 2?
Which ones are multiples of 5?
Which ones are in the 4 times table?

Objective

- know by heart all multiplication facts to 10 × 10 and derive quickly the corresponding division facts

Use some division questions such as:

> Hold these three numbers in your head: 8, 11, 20.
> Which of the numbers can you divide by 2?
> Hold these three numbers in your head: 15, 21, 20.
> Which of the numbers are divisible by 3?… What are some of their factors?

15 mins

Main activity: whole class and individuals

Show pupils the large multiplication square with some numbers blanked out.

> Where are all the numbers with a 5 in? What other patterns can you see?

Ask the pupils to suggest the missing numbers by looking for patterns.

> What number goes between 21 and 35?
> How can you work out 7 times 4? Look at the pattern of 4s: 16, 20, 24… What do you think comes next?

It is not necessary to fill in the whole square. Get to the point where pupils are aware of some of the patterns.

Before moving on, ask some division questions such as:

> How can you use the square to work out 30 ÷ 6? And 42 ÷ 6?

Give out Worksheet 1.F for pupils to do individually. On completion, pupils can swap worksheets and check each other's work.

Part 3 of the worksheet is suited to faster and more able pupils.

5 mins

Plenary: whole class

Ask pupils to think about these rows in the multiplication squares.

2	4	6	8	10	12	14	16	18	20
4	8	12	16	20	24	28	32	36	40
8	16	24	32	40	48	56	64	72	80

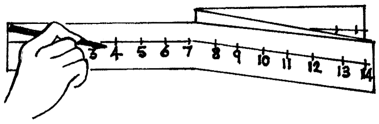
> What relationships can you see between the 2, 4 and 8 times tables?… Yes, each one is double the one before.
> Do you think this works for other tables? Which ones?

Stress that doubling and halving can be useful strategies for working out multiplication and division facts.

Multiplying by 9

Resources

0–100 number line (optional)
100-grid (optional)



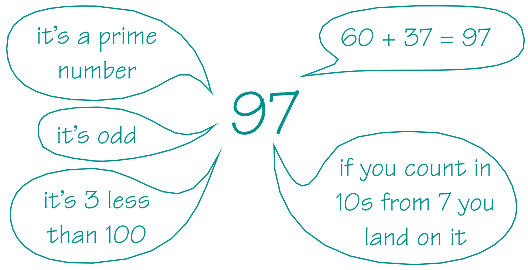
Language

digits, hundreds, tens, units, ones
how much? total, altogether
multiply, calculation

5 mins

Starter: whole class

Choose a number and write it on the board and ask the children to tell you facts about it; encourage each one to think up something different. Choose some of these facts to write around the number.



20 mins

Main activity: whole class and individuals

Explain to the class that today they are going to learn how to multiply by 9, but first…

> We will start with revision of multiplying by 10.
> What is 4 × 10?… And 3 × 10?… What about 19 × 10?

Write the calculations on the board.

$4 \times 10 = 40$
$3 \times 10 = 30$
$19 \times 10 = 190$
$24 \times 10 = 240$

> What happens to a number when it is multiplied by 10?
> Yes, multiplying by 10 moves all the digits one place to the left.

Pupils may say that to multiply by ten you 'add a nought'. This strategy may work for whole numbers, but causes problems later on with decimals, so stress that adding 0 doesn't necessarily change a number.

Now move on to multiplying numbers by 9. Choose the same starting numbers but this time multiply them by 9 instead of 10. Write the equations beside the previous examples.

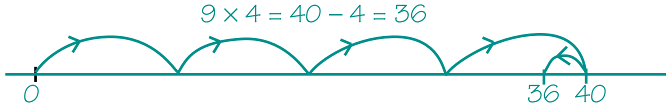
$4 \times 10 = 40$	$4 \times 9 = 36$
$3 \times 10 = 30$	$3 \times 9 = 27$
$19 \times 10 = 190$	$19 \times 9 = 171$
$24 \times 10 = 240$	$24 \times 9 = 216$

Talk about how pupils can use their knowledge of multiplication by 10 to derive multiplication by 9.

> To do 4 × 9 you can do 4 × 10 then just subtract a 4.

Some children may need this demonstrated on an empty number line. Remind them that four 9s are the same as nine 4s; so if ten 4s are 40, then nine 4s are 4 less – that is, 36.

$10 \times 4 = 40$
go back 4
$9 \times 4 = 40 - 4 = 36$



Objectives

- use known facts (eg to multiply by 9, multiply by 10 and adjust)
- use multiplication to solve word problems involving money

Introduce this scenario:

> Sally runs a 'Roll-a-penny' stall at the Summer Fair every year. She has to count up her takings (all in pennies) at the end of the day. She makes this easier by putting the pennies in piles; but she likes to make piles of 9 pennies even though her friend says it would be easier if she made piles of 10.
>
> If she made 18 piles, how many pennies did she take altogether?

Ask the class to discuss in pairs how to solve the problem. Then, as a class, talk about the mental methods used.

Now write up the number of piles Sally made over the last few years. Pupils should work out the total takings for each year, using the method introduced in the lesson.

1990	16	1995	12
1991	19	1996	42
1992	15	1997	18
1993	27	1998	17
1994	15	1999	49

5 mins

Plenary: whole class

Ask pupils to volunteer their answers for particular years. Begin with children who tackled the smaller numbers:

> In 1998 Sally made 17 piles. How much did she make that year?

Ask the pupils to explain how they worked out their answers. Record these on the board, using partitioning notation. Make sure to interpret the answers, to help children adopt good habits:

17 piles of 9 coins

$$17 \times 9 = (17 \times 10) - 17 = 170 - 17 = 153$$

In 1998 Sally made 153p or £1.53

> The answer isn't just 153, is it? What was the question?… Yes, "How much did Sally make in 1998?" So the answer is… Yes, she made 153p, which is… £1.53.

Stress that Sally's method of counting in nines may have pleased Sally but is not the most efficient. Talk about why counting in tens is so much easier.

Multiplying bigger numbers

Resources

multiplication squares

Language

multiply, multiplied by, divide, divided by
multiple, factor, product, divisible by
sequence

5 mins

Starter: whole class

Explain that today's lesson focuses on a strategy for making multiplying easier.

Write four numbers on the board: 21, 28, 35 and 42.

Look at these four numbers. What number sequence are they part of?… That's right, the 7 times table.

Repeat this kind of question several times with sets of numbers from different multiplication tables.

Write on the board:

2	4	6	8	10
3	6	9	12	15

What number series are these?… Yes, parts of the 2 and 3 times tables.

What do you get when we add the numbers in the columns?… Yes, the 5 times table.

Suppose we wanted to work out the 15 times table. Which two tables might we write out in this way?

If pupils do not come up with the idea, you could suggest the 10 and 5 times tables.

20 mins

Main activity: whole class and pairs

Now, suppose you just want to work out one multiplication fact, say, 13×7. How might you use what you have learnt to help you do that?

You will probably get the response: 10×7 added to 3×7.

And suppose you can't remember what 3×7 is? How might you work that out?

Objective

- use known facts and partitioning to multiply larger numbers mentally

Ask one or two similar questions, and discuss possible methods. Make sure you include some questions where division could be a useful strategy.

> Suppose you want to work out 7 × 15. What might you do?
> Yes, you could do 7 × 10 and 7 × 5. Suppose you can't remember what 7 × 5 makes. How might you work that out?
> Well, one easy way is to halve 7 × 10, which is half of 70.

Write up some problems on the board. Pupils can work in pairs to tackle them, and record their solutions in their own way, but tell them to make it clear how they worked out each one. If they 'just knew' the answer, they should say so. If they use their multiplication square, they should say so.

Problems might include:

17 lots of 4	the product of 13 and 6	13 × 8
14 times 8	17 × 5	5 × 18
13 multiplied by 6	23 × 6	11 × 9

Quickly go through the answers so that pupils can correct their own work.

Challenge

Children make up some more problems themselves. What are the hardest problems they can find the answers to?

5 mins

Plenary: whole class

Ask one pair to show on the board the method they used to solve one of the problems (for example, '17 lots of 4').

Then find another pair who tackled it differently, and get them to show their method on the board too.

Discuss the different methods these, and other pupils, used.

Make sure there is at least one solution that uses division. If necessary, provide it yourself.

Finish by stressing that working out a multiplication fact is sometimes easier if you break up the bigger number.

Multiplying and dividing by 10

Resources

Worksheet 1.G

Language

part, tenth, decimal point, decimal place
approximate, tens, units, tenths
hundreds, digit, divide, divided by
multiply, multiplied by, predict, record

5 mins

Starter: whole class

Choose a multiple of 4 – say 36. Ask three questions about it.

What is double 36?
What is half of 36?
What is 36 divided by 4?

Choose more multiples of 4 to ask similar questions about. Encourage the pupils to respond quickly.

20 mins

Main activity: whole class and pairs

Explain that today's lesson is about decimal numbers and dividing numbers by ten.

Ask the pupils:

Where have you seen decimal points?

The pupils are likely to mention calculator displays and price labels, among other things.

Ask for particular examples that pupils can write on the board. Encourage the pupils to read the numbers correctly (for example, reading 3.59 as 'three point five nine' and 0.5 as 'nought point five').

Explain that money is a special case. Emphasise that although a label such as £4.99 tells us about pounds and pence, we only mark the pound sign (£) and not the pence sign (p). In the same way, if we write 8.5 m, this tells us about metres and centimetres, although 'cm' is not marked.

In order to find out what else pupils may know about reading decimal numbers, write the numbers 12.7 and 12.34 on the blackboard and ask:

Does anyone know which is the larger number?
What does 12.7 mean?… That's right, it means one ten, two units and seven tenths.
What does 12.34 mean?… Yes, it means one ten, two units, three tenths and four hundredths.

Now ask:

What happens to a number when it is multiplied by 10?

Reinforce the idea that the digits move one place to the left. The notion of 'adding a nought' is incorrect and should be discouraged. You could illustrate this by talking about, say, 2.3×10. Here, of course, the answer is 23: no nought is added.

What happens to a number when it is divided by 10?

Reinforce the idea that the digits move one place to the right. Put some division examples on the board to discuss orally:

$120 \div 10 \qquad 3400 \div 10 \qquad 60 \div 10$

Now, what happens when you divide 15 by 10?

Record on the board any answers that the pupils provide. If necessary, demonstrate the division with 15 objects and 10 pupils (make sure you use objects that can be divided in half, such as sheets of scrap paper or biscuits).

How many does each have?
What could we do with the remaining 5? Yes, give each person half.

Remind them about $\frac{1}{2}$ being equivalent to $\frac{5}{10}$ and 0.5.

What happens if we divide 37 by 10? Or 241 by 10?

Discuss the pupils' responses. Pupils might suggest approximate answers; if not, suggest ways of finding an approximate answer.

Give out worksheet 1.G. Ask the pupils to work in pairs to complete the first four questions from each section. Early finishers can go back and complete as many as they can of the rest.

5 mins

Plenary: whole class

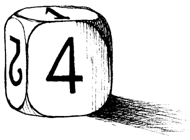
After a while, stop the pupils and read out the answers.

Finish by stressing that when a number is divided by 10, the digits move one place to the right.

Division with remainders

Resources

2–9 number cards for each group
1–6 dice for each group
counters



Language

round up, round down
division, divide
remainder

5 mins

Starter: whole class

Write on the board four digits, the four operation signs, and a two-digit target number. Give children a few minutes to make a number as close as possible to the target from the four digits (used once each) and the four operations. Ask one or two volunteers to demonstrate their calculations on the board.

3 5 7 9
+ − × ÷

target: 50

20 mins

Main activity: whole class and groups

Explain to the class that they are going to do more work on division but this time the answers won't necessarily be whole numbers. Write a division by 10, with no remainder, on the board, then another which does have a remainder.

$20 \div 10 = 2$

$23 \div 10 =$

> Imagine that you have 23 apples to share between 10 children.
> How many apples would each child get?… Yes, 2. And how many apples would you have left over?… Yes, 3 apples.

Remind them that in number sentences, the 'bit left over' is called a remainder and the answer should be written with this word in it.

$$23 \div 10 = 2 \text{ remainder } 3$$

Work through some other division problems with the children, this time inviting them to come up with word problems to match the equations.

> When might you divide 41 things by 4 in real life?… Rachel says 41 pens shared between 4 children. How many pens would they each get?… Yes 10, and there would be how many left over?… Yes, one pen.
> How shall we write this as a number sentence?

$41 \div 4 =$

$25 \div 3 =$

$39 \div 5 =$

$362 \div 100 =$

Objectives

- find remainders after division
- round up or down after division, depending on the context

Ask the class:

> What would you subtract from 41 in order to do an exact division?
> Yes, 1. What you need to subtract is the remainder.
>
> Suppose a class of children need to fit onto the tables at the Science Centre. Each table seats 6 children. If there are 34 children, how many tables will be needed? (rounding up)… Yes, there are five sixes in 34. Will five tables be enough?… No, because the remaining children need a table too.
>
> Suppose you want to buy pencils at the school fair. The pencils cost 9p each and you have 50p to spend. How many pencils can you buy? (rounding down)… Yes, five pencils. And how much money will you have left over?

In groups, pupils play the game 'Leftovers'. Each group needs a 1–6 dice and a pile of number cards 2–9. One player, the dice-handler, is in charge of the dice and another, the card-dealer, in charge of the 2–9 cards. The card-dealer gives one card to each player; the number on a player's card is their division number for that round.

The dice-handler now rolls the dice twice and reads out the dice numbers, in order, to make a two-digit number.

Each player must write down a division calculation consisting of the two-digit number divided by their division number and the answer with its remainder. (Pupils should help each other with their working if necessary – this will help to keep the group together.)

They then read their calculation to the rest of the group. The player with the largest remainder wins a counter. Shuffle the number cards and play again. The first one to collect three counters is the winner.

5 mins

Plenary: whole class

Talk with the class about the activity they have just completed.

> Were any of the divisions you did easier than others?… What was easy about them? Which were the more difficult ones?
>
> How did you work out the difficult ones?
>
> Did you know what the remainder would be before working out the sum?

Doubling two-digit numbers

Resources

none

Language

method, double, halve, times, times table
multiply, multiplied by, divided by
multiple, product, sequence, add

5 mins

Starter: whole class

Explain that today you are going to practise doubling.

Start off by getting the class to chant the 2 times table and the 4 times table.

As they do so, write the ‘answers’ on the board, in the form of two sequences, one under the other.

2	4	6	8	10	12	14	16	18	20
4	8	12	16	20	24	28	32	36	40

Many of you will know the eight times table, but for those of you who don’t yet, how can we get to the eight times table from the four times?…
Yes, just double it.

Get the class to work out the ‘answers’ to the eight times table by doubling each number in the second row, and then get the whole class to chant the table.

Do the same for the 16 times table.

2	4	6	8	10	12	14	16	18	20
4	8	12	16	20	24	28	32	36	40
8	16	24	32	40	48	56	64	72	80
16	32	48	64	80	96	112	128	144	160

20 mins

Main activity: whole class

Move on to doubling multiples of 10.

I’m going to say a number and I want you all to call out the number that is double it.

10… 30… 50… 200… 70…

Start doubling other two-digit numbers where no ‘carrying’ is involved.

Again, I want you all to call out the number that is double my number.

24… 31… 41… 15… 43…

Now move on to doubling numbers where ‘carrying’ is necessary. For this, ask some volunteer pupils to come to the front of the class to act as the class memory. To start with you will only need two.

Objective

- derive quickly doubles of two-digit numbers by doubling the most significant digit first

> When it comes to doubling numbers such as 96 or 47 you have more to remember than when you are doubling a number such as 42.
>
> That is because when you double the 'ones' digit you get a number that is more than ten, and you have to remember what it is while you double the 'tens' digit, and then add them together.
>
> Today I'm going to suggest we use some volunteers to help us remember.
>
> Let's start by doubling 47. Steve, which bit do you want to double?… The tens?
>
> OK… Yes, that's 80, so please will you remember 80.
>
> Now, Kirsty, you need to double the 7, which gives you… Yes, 14.
>
> Now, class, listen while Steve and Kirsty tell you their numbers, and I want you to add them mentally. When I say 'go', I want you all to call out the answer.
>
> Ready?… Go!

Now use this method to involve the class in doubling more two-digit numbers that involve carrying, and then move on to multiples of 10 that are in the hundreds. These may or may not involve carrying – for example, 470, 360 or 120.

> Now we are going to move on to some higher numbers, using some different volunteers if we need to.
>
> Double 360. Jacky, you double the 300 and Kim, you double the 60…

If pupils find it helpful, the volunteers can write their numbers on the board to provide a visual reminder.

If the pupils are confident, move on to doubling other three-digit numbers – for example, 356, 479 or 956. Use three volunteers, one for each digit that is being doubled.

In the last few minutes, put some two- and three-digit numbers on the board, and get the class to write down the numbers and their doubles, doing any necessary jottings at the side of the page.

5 mins

Plenary: whole class

Get pupils to mark their own work as you call out the answers.

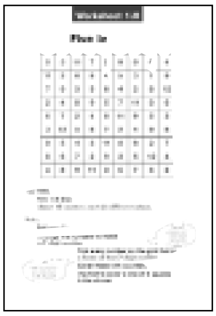
Finish by doing some repeated doubling with the class. Start with a single-digit number, and send it round the class, each person doubling that of the person before. See how high the class can go.

> 9, 18, 36, 72, 144…

When someone gives up, or gets it wrong, the next person starts a new number.

Finding pairs of factors

Resources
OHP
counters in two colours
squared paper
Worksheet 1.H: 'Five in a row'
two 1–6 dice



Language
inverse
array, row, column
divisible by, factor
calculation, operation
sign, symbol
equation, expression

5 mins

Starter: whole class

Spend a few minutes asking the class facts from the multiplication tables, adjusting the questions according to pupils' individual learning targets. Those who answer correctly score a point for their table or team.

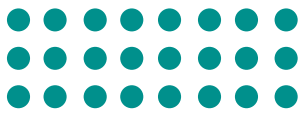
Every now and then stop and discuss how to work out any facts that they have forgotten.

> If you can't remember 8 times 7, how might you work it out?… Deepika says start with 4 times 7 and double it. Any other methods?

20 mins

Main activity: whole class and pairs

Prepare a 3 × 8 array of counters on the overhead projector.



Invite the class to suggest the two multiplications that describe the array, write these on the overhead projector, and ask the pupils to read them. Encourage a variety of expressions: '24 is 8 lots of 3', '24 is 3 lots of 8', '8 multiplied by 3 equals 24'.

$24 = 8 \times 3$
$24 = 3 \times 8$

$24 \div 3 = 8$
$24 \div 8 = 3$

Then get volunteers to help you write the corresponding division calculations. Again, encourage the use of different expressions: 'how many 3s in 24?', '24 shared between 3 is 8', '24 divided into 3 parts makes 8', '24 divided by 3 equals 8'.

Ask how the 24 counters could be rearranged in a different rectangle.

> Susie says we could make 4 rows of 6.

Make the new array with the 24 counters. This time ask the children, with their maths partners, to write the two multiplications and two divisions. Give them sufficient time to do this, then ask one of the children to read them out.

Objectives

- know by heart all multiplication facts up to 10 × 10
- find all the pairs of factors of any number up to 100
- understand and use division as the inverse of multiplication

Give the children a few more minutes, again with their maths partners, to work out other rectangular arrays that are possible for 24 counters. In discussion, establish that the counters could be rearranged:

— in a 2 × 12 array leading to these equations: 12 × 2 = 24, 2 × 12 = 24, 24 ÷ 12 = 2 and 24 ÷ 2 = 12

— as 1 row of 24, leading to these equations: 24 × 1 = 24, 1 × 24 = 24, 24 ÷ 24 = 1 and 24 ÷ 1 = 24

and so on, for other factors of 24.

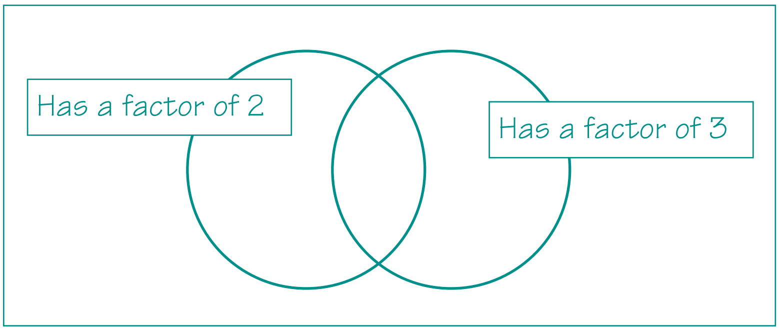
Remind the class that the pairs of numbers 8 and 3, 6 and 4, 12 and 2, and 24 and 1 are all factors of 24.

Now give pupils the game Worksheet 1.H: ‘Five in a row’ to play in pairs or small groups.

5 mins

Plenary: whole class

Bring the class together. Ask pupils to sort all the numbers from 1 to 36 in a Venn diagram with sets ‘Has a factor of 2’ and ‘Has a factor of 3’.



Approaches to division

Resources

1–10 spinner or 1–10 number cards

Language

share, divide
calculate
method, strategy
product

5 mins

Starter: whole class

Ask the children to draw 4 × 4 grids and write an 'answer' from the multiplication tables in each box.

Then spin a 1–10 spinner twice (or draw two of the 1–10 cards) and announce both numbers. Ask a volunteer to give the product. Any pupil who has written that number on their grid crosses it out.

The first pupil to cross out three numbers in a line is the winner.

~~10~~	24	36	12
~~8~~	~~4~~	~~9~~	15
16	21	~~20~~	45
~~3~~	18	48	50

20 mins

Main activity: whole class and groups

Explain to the class that in this lesson they will be encouraged to think about multiplication and division problems, focusing on what they mean rather than trying to solve them.

Ask the class to work in small groups.

> Now you are going to look at some division problems and work in groups to decide what they mean, and which is the easiest and which is the hardest.
>
> In order, decide how easy or hard they are, think about how you would do them, although you need not actually find the answers today.

Write these problems on the board.

$363 \div 3$

$6000 \div 6$

$34 \div 7$

$6 \div 12$

$68 \div 17$

$4 \div \frac{1}{2}$

How many 30 g servings can you get from a 500 g packet of cereal?

Talk about the problems with the class.

Objectives

- know by heart all multiplication facts up to 10 × 10
- understand and use division as the inverse of multiplication
- explain methods and reasoning

Which problems look easy to you? And which problems look hard?…

I want you to talk about these problems in your groups, then each of you copy them out and number them. Call the easiest one 'number 1', the next easiest 'number 2', and so on. The hardest one will be 'number 7'.

You don't have to agree with everyone else in your group.

Allow time for discussion. Move around the class and listen to the way the children interpret each problem. Points to discuss with the children include the following:

— 6000 ÷ 6 can appear difficult if it is interpreted as 'how many sixes in 6000?'; a simpler way is to see it as 6000 shared between 6; the answer will be the same either way

— if children read $4 \div \frac{1}{2}$ as '4 shared by half' they may find it difficult to understand; an easier way is to ask 'how many halves make 4?'

— if children just look at the calculation 6 ÷ 12 they may say "Easy, it's 2", confusing the problem with 12 ÷ 6; encourage them to read it out and explore its meaning: '6 divided by 12, six apples shared between twelve people'

5 mins

Plenary: whole class

Ask children to say which problem they thought was easiest and why. Ask for some indication of how they interpreted the problems, and how they would set about working out the answer.

Naomi, you say you think 6000 ÷ 6 is easiest. Can you explain your thinking? What did you say to yourself when you saw it?… You said 'How many is 6000 shared amongst 6 people'? And did that help you?…

What did other people say to themselves?

You might choose to record opinions on the board in some way. If so, make the point that no one's opinion is 'correct'. Everybody's opinion is valid. On a scale of 1 to 7 (1 = easiest; 7 = hardest), how hard did everyone find these calculations?

	Rating
363 ÷ 3	1 1 2 1
6000 ÷ 6	3 6 2 3 4 3 3
34 ÷ 7	1 1 2 1 2 1 1
6 ÷ 12	3 1 6 5 7 7 5 4 4
68 ÷ 17	2 1 3 3 2 3 3 3 3
$4 \div \frac{1}{2}$	7 7 5 7 6 7 6 7 7
How many 30 g servings can you get from a 500 g packet of cereal?	3 4 5 3 4 2 3

Finish by saying that sometimes number problems can be made easier by thinking about them in a different way.

More money problems

Resources

Worksheets 1.I and 1.J: ‘The school fair’

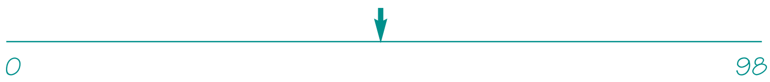
Language

half, quarter, fifth, tenth
remainder
pence, pounds
total

5 mins

Starter: whole class

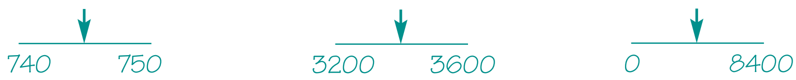
Draw an empty number line on the board and put a number at each end. Put an arrow halfway along the line and ask children to estimate what number goes there.



Invite children to suggest a method for working out what number belongs there.

> Zandra says if halfway really is 56 then we could double 56 as a check. What is double 56?

Give children other ‘halfway’ problems, such as:



20 mins

Main activity: whole class and pairs

Work together with the class to ‘invent’ a school fair. Quickly decide what stalls and attractions it has and how much these cost, writing these down on the board.

tombola	9p
face painting	5p
roll-a-penny	1p
tail on the donkey	3p
wet sponges	5p
cafe	various

Now take turns with the pupils to make up a number problem based on the fair. Invite individual pupils to work out answers to the problems, and use this to help you assess what they have learned during the week. Where appropriate, ask them to come and write on the board to work through a problem.

> Aaron, if 25 people enter the wet sponge game, how much money will that make?… What is the calculation you need to do there?
>
> Yes, Aaron, 25 times 5. And what is a quick way to do 25 × 5?
>
> Do you need to come and do it on the board, or can you do it in your head? The answer is 125 what?… Yes, 125 pennies. Come and write that in pounds.

Objectives

- use multiplication and division to solve word problems involving numbers in ‘real life’ and money
- explain methods and reasoning about numbers orally

Try to include in your questions problems that touch on the work of the previous few lessons: multiplying by 9; dividing multiples of 100 and 1000 by 10 and 100; division with remainders; multiplying two-digit numbers by single-digit numbers.

You will need photocopies of Worksheets 1.I and 1.J. Explain to the class that they are going to solve some problems about a school fair. They should work in pairs, solving each problem mentally or in writing. Remind them to check their work.

Give each pair one problem initially, replacing this with the other problem as appropriate. Aim for all children to try both problems eventually.

5 mins

Plenary: whole class

Discuss with the children how different pairs solved certain problems.

How did you work out the money Callum’s pile of coins was worth?
Did anyone try just dividing by 10?
Is there a quick rule for turning pence into pounds?… Yes, the decimal point moves two places to the left.
How did you find a fifth of Claudio’s savings?
What written methods did you use?

Finish by reminding them of some of the calculation strategies they have learned and used in the last two weeks.

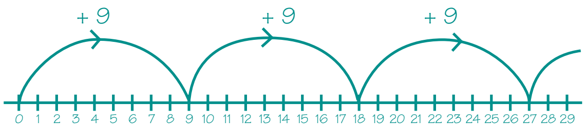
Other ideas

5-minute starters

Counting in nines

Add and subtract mentally a 'near multiple of 10'

Draw nine-jumps on a number line.



Ask the pupils questions about the pattern made: "Where does the jump land each time? Is it getting closer or further away from the tens? How much further?"

Compare the pattern to that for jumps of 10 which go 10, 20, 30, and so on. Help the pupils to see that whereas ten-jumps land on 40, the nine-jumps fall short by 4; similarly, the ten-jumps land on 50, but the nine-jumps fall short by 5, and so on. Can they explain why?

Repeat the work for jumps of 8, 11 and 12.

What did I do?

Find remainders after division

Understand division as the inverse of multiplication

Explain methods and reasoning

Do some mental divisions and tell pupils the results. "My answer was 7 remainder 3. I started with 38. What did I divide by?" or "My answer was 4 remainder 8. I divided by 9. What did I start with?"

Discuss with the pupils how they solved these problems. Can they explain their method to the others?

Variation

Pupils make up their own questions.

Exploring factors

Know by heart all multiplication facts up to 10 × 10 and derive quickly corresponding division facts

Give pupils a number (or let them choose one), and ask them to find as many pairs of factors as possible. Some pupils can work with lower numbers such as 40, 60 or 100 and others with numbers over 100.

<u>40</u>
2 and 20
4 and 10
5 and 8

<u>60</u>
2 and 30
4 and 15
3 and 20
5 and 12
6 and 10

Doubling

Derive quickly doubles of two-digit numbers

Talk with the pupils about how the numbers in an equation such as $3 \times 7 = 21$ can be doubled to give other equations, such as $6 \times 7 = 42$ and $3 \times 14 = 42$.

Doubling $6 \times 7 = 42$ gives $12 \times 7 = 84$ and $6 \times 14 = 84$.

Doubling $3 \times 14 = 42$ gives $6 \times 14 = 84$ and $3 \times 28 = 84$.

Now ask the pupils to suggest as many facts as they can that are obtained by doubling $3 \times 4 = 12$.

Finding equivalents

Know by heart all multiplication facts up to 10 × 10 and derive quickly corresponding division facts

Use known facts and partitioning to multiply large numbers mentally

Pupils write down a multiplication sum (but not the answer), then work out as many alternative ways of writing it as possible.

$14 \times 5 =$

$10 \times 5 + 4 \times 5 =$

$7 \times 2 \times 5 =$

$7 \times 10 =$

Products

Know by heart all multiplication facts up to 10 × 10

Understand division as the inverse of multiplication

The children choose any three of these numbers (no number may be chosen twice):

2, 3, 5, 8

The aim is to make as many different products as possible.

Chains

Recall addition and subtraction facts to 20

Know by heart multiplication facts for the two times table

Give the children a number from the 10–20 range. Decide together whether the number is odd or even. If it's even, divide it by 2. If it's odd, add 5. Keep going. Discuss what happens? Children investigate what is the longest chain they can make.

Mental methods

Know by heart all multiplication facts up to 10 × 10 and derive quickly corresponding division facts

Use known facts and place value to consolidate mental addition and subtraction

Explain methods and reasoning

Pupils take turns to ask the class a mental arithmetic question.

The aim is not to get a correct answer, but to suggest ways of doing that problem mentally.

Estimating

Explain methods and reasoning

Use known facts and place value to consolidate mental addition and subtraction

Children take turns to ask the class a question in mental arithmetic.

The aim is not to work it out exactly, but to estimate the answer and explain how the estimate was reached.

Prime numbers

Know by heart all multiplication facts up to 10 × 10 and derive quickly corresponding division facts

Find all the pairs of factors for any number up to 100

Give pupils a number between 100 and 999 and challenge them to find out whether it is a prime number or not. If not, what are its factors?

They must explain their working.

Factors

Know by heart all multiplication facts up to 10 × 10 and derive quickly corresponding division facts

Find all the pairs of factors for any number up to 100

Pupils work in pairs. Each pupil chooses five numbers from this set (a number can be chosen more than once):

2, 3, 5, 7, 9

They multiply the numbers together, and write down the resulting number. They give this – but not the list of numbers – to their partner, who has to work out all the factors of the number.

Challenge
Children choose more than five numbers, and don't say how many they chose.

Simplification
Children choose just three numbers.

Whole class activities

Next door numbers

Know by heart all multiplication facts up to 10 × 10 and derive quickly corresponding division facts

Find all the pairs of factors for any number up to 100

Pupils work in pairs. Each pupil chooses two adjacent numbers in secret and multiplies them together, then presents the product to their partner. Their partner works out what the original numbers were.

Challenges

Pupils choose two-digit numbers over 50.

Pupils choose any pair of two-digit numbers (not necessarily adjacent).

Simplification

Pupils choose numbers between 15 and 20.

Fill the blank

Know by heart all multiplication facts up to 10 × 10 and derive quickly corresponding division facts

Understand and use division as the inverse of multiplication

Present multiplication and division calculations containing blanks where digits should be – include some of the pupils' preferred methods as well as standard methods. Ask the pupils to fill in the missing numbers.

$$\begin{array}{r} 3\square \\ \underline{\times\ \ 4} \\ 120 \\ \underline{12} \\ 132 \end{array}$$

Afterwards, discuss how they worked each one out.

Variation

Give pupils a series of equations with one or more digits missing for them to fill in:

$48 \times \square = 48$

$7 \times 39 = 2\square\square$

$1\square \times 14 = 168$

$112 \times 6 = 6\square 2$

Using information

Know by heart all multiplication facts up to 10 × 10 and derive quickly corresponding division facts

Understand and use division as the inverse of multiplication

Find remainders after division

With the pupils' involvement, write on the board a few facts from the multiplication tables.

$3 \times 4 = 12$
$5 \times 7 = 35$
$6 \times 8 = 48$

Ask the pupils to help you derive some division facts from the multiplications, and write these on the board.

$12 \div 3 = 4$
$12 \div 4 = 3$
$35 \div 5 = 7$
$35 \div 7 = 5$
$48 \div 6 = 8$
$48 \div 8 = 6$

Now present some slightly more complex division calculations such as

14 ÷ 4, 34 ÷ 5, 50 ÷ 8

Ask the pupils how they can use the information on the board to find the answers. Discuss the way that the 'bit left over' can be written as a 'remainder' or calculated as a fraction or decimal:

14 ÷ 4 = 3 r2 or $3\frac{1}{2}$ or 3·5

How many days?

Know by heart all multiplication facts up to 10 × 10

Ask: "How many days have you been alive?" (No calculators.)

For pupils who want a bigger challenge, ask: "How many hours have you been alive?"

Loop

Know by heart all multiplication facts up to 10 × 10 and derive quickly corresponding division facts

Understand and use division as the inverse of multiplication

Draw the following diagram on the board and explain that it is a 'loop' of numbers, starting and ending with the same number and using only the operations of multiplication and division.

35	÷ 5	7	× 5	35

Circles always have numbers in them and boxes always have operations in them.

Either clear the first diagram or make a new one and start with '48' in the first circle and '÷ 4' in the first box. Ask the pupils what goes in the remaining box and circles.

Next time omit the first number and put '8' in the middle circle and '÷ 3' in the first box. Ask the pupils to explain how they know what to put in the other places.

Next time change the numbers and swap the positions of the division and multiplication operations.

13	× 3	39	÷ 3	13

Three hops to a hundred

Add and subtract mentally any pair of two-digit numbers

Explain methods and reasoning

Pupils start with any number under 100. They can do up to three operations to change that number to 100, but each operation can only use a single-digit number.
For example:

$48 \times 2 = 96$; $96 + 4 = 100$; $22 \times 5 = 110$;

$110 - 5 = 105$; $105 - 5 = 100$

Variations
Try four hops to a thousand. For example:

$12 \times 9 = 108$; $108 - 8 = 100$;

$100 \times 5 = 500$; $500 \times 2 = 1000$

Four in a row – a game for two players

Know by heart all multiplication facts up to 10 × 10 and derive quickly corresponding division facts

Each player needs some cubes in their own colour and a shared 100-grid. They take turns to spin two 1–10 spinners and multiply the numbers, then cover a square showing that number with a cube. The first person to get four cubes in a row is the winner.

Make a hundred – a game for four players

Use known facts and partitioning to multiply large numbers mentally

Players each toss a dice and they all write down the numbers they get. Players then use these four numbers, plus the operation of multiplication, to generate numbers.
For example, 2, 3, 4 and 4 could be used to make, among other arrangements:
$2 \times 3 \times 4 \times 4 = 96$; $24 \times 34 = 816$;
$243 \times 4 = 972$; $43 \times 42 = 1806$
The aim is to make the largest possible product.

Variation
Aim to make a number as close as possible to any hundreds number (100, 200, 300 and so on).