## Guidance for Summer Numeracy Schools

## Set 3

## To support session 3

## A three-part daily mathematics lesson with the whole class

The Summer Numeracy School guidance asks you to provide one complete mathematics lesson each day, in addition to the other direct teaching sessions.
The 17 units in this set are complete mathematics lessons with supporting information and resources. You will need to choose the ten most appropriate lessons for the pupils attending your summer school.
Each three-part lesson has a mental and oral starter, a main activity and a plenary. The objectives for the lessons focus on the key objectives in the Framework for Teaching Mathematics.
There are links made between some of the lessons. For example, some of the mental and oral starters are used to consolidate previous learning. Homework or out-of-class activities are used to follow up lessons and to prepare for a later lesson, and some lessons build on those taught before. You will need to bear these factors in mind when you are choosing lessons.

| Unit | Objectives |
| :---: | :---: |
| 3.1 | Place value and ordering <br> - order a given set of positive and negative integers, or decimals with up to two places <br> - use the vocabulary of comparing and ordering numbers, including greater than, less than, and the symbols >, < and = ; give a number lying between two others |
| 3.2 | Adding and subtracting 3-digit numbers <br> - recall two-digit pairs that total 100 <br> - add mentally any pair of two-digit numbers <br> - carry out column addition and subtraction of positive integers less than 10000 |
| 3.3 | Tenths and hundredths 1 <br> - use decimal notation for tenths and hundredths <br> - round a decimal to the nearest whole number |
| 3.4 | Tenths and hundredths 2 <br> - use decimal notation for tenths and hundredths <br> - round a decimal to the nearest whole number |
| 3.5 | Adding and subtracting decimals <br> - use known number facts and place value to consolidate mental addition and subtraction <br> - read and write numbers in figures and words <br> - carry out column addition and subtraction of numbers involving decimals |
| 3.6 | Multiplying and dividing by 10 and 100 <br> - know by heart all multiplication facts up to $10 \times 10$ and derive quickly corresponding division facts <br> - multiply and divide mentally whole numbers and decimals by 10 or 100 and explain the effect |
| 3.7 | Multiplication (TU x TU) <br> - know by heart all multiplication facts up to $10 \times 10$ and derive quickly corresponding division facts <br> - carry out multiplication of TU x TU |
| 3.8 | Multiplication (HTU x U) <br> - use, read and write standard metric units, including their abbreviations and relationships between them, eg $\mathrm{km}, \mathrm{m}, \mathrm{cm}$, and mm <br> - carry out multiplication of $\mathrm{HTU} \times \mathrm{U}$ and then numbers involving decimals |
| 3.9 | Division (HTU $\div$ U) <br> - know by heart multiplication facts to $10 \times 10$, and derive quickly corresponding division facts <br> - carry out division of HTU by U |


| Unit | Objectives |
| :---: | :---: |
| 3.10 | Division with remainders <br> - know by heart all multiplication and division facts to $10 \times 10$ and derive quickly corresponding division facts <br> - find whole number remainders, and begin to express a quotient as a fraction or a decimal |
| 3.11 | Word problems <br> - use all four operations to solve word problems involving numbers and quantities based on 'real life', money and measures (including time), explaining methods and reasoning |
| 3.12 | Finding fractions <br> - use decimal notation for tenths and hundredths <br> - relate fractions to division and to their decimal representation <br> - find simple fractions of numbers or quantities |
| 3.13 | Fractions and decimals <br> - multiply and divide mentally whole numbers and decimals by 10 and 100 and explain the effect <br> - recognise the equivalence between the fraction and decimal forms of one half, one quarter, three quarters... and tenths and hundredths |
| 3.14 | Percentages 1 <br> - multiply and divide mentally whole numbers and decimals by 10 and 100 and explain the effect <br> - understand percentage as the number of parts in every 100, and find simple percentages of whole number quantities |
| 3.15 | Percentages 2 <br> - recognise the equivalence between the decimal and fraction forms of one half, one quarter, three quarters... and tenths and hundredths <br> - express simple fractions as decimals and vice versa |
| 3.16 | Percentages 3 <br> - understand percentage as the number of parts in every 100 and find simple percentages of whole number quantities |
| 3.17 | More fractions <br> - find simple fractions of numbers and quantities |

## Resources

Some photocopiable resource materials for each lesson are included with this guidance. In some cases the material is from commercially published sources. All such extracts are used with the agreement of the publishers and are acknowledged. The Summer Numeracy School will need to provide others. These are all listed in the chart below. (It is assumed that there will be access to a blackboard or flipchart for every lesson.)

| Units | Resources provided with the lesson, from which multiple copies may need to be made, or an OHT | Resources to be provided by the summer school |
| :---: | :---: | :---: |
| 3.1 | - Sheet 3.A: 'Point it out' grid <br> - Sheet 3.B: 'More or less' board <br> - Sheet 3.C1: 'Be nasty’ board <br> - Sheet 3.C2: 'Be nasty' cards | - partitioning cards for individual pupils <br> - 0-1000 number line <br> - two sets of 0-9 number cards per pair <br> - OHP (optional) |
| 3.2 | - Sheet 3.D: 'Number board 1' | - OHP |
| 3.3 | none | none |
| 3.4 | - Sheet 3.E: Shop till receipt <br> - Sheet 3.N: 'What's next?' cards | - large display cards for the activity 'Stop!' |
| 3.5 | - Sheet 3.E: Shop till receipt <br> - Sheet 3.N: 'What's next?' cards | - OHP <br> - partitioning cards, one set per pupil <br> - 'Tables timer speed test' <br> - calculators |
| 3.6 | - Sheet 3.A: 'Point it out' grid <br> - Sheet 3.F: 'Find the pairs' grid | - 'Tables timer speed test' <br> - two grids of numbers to multiply and divide by 10 and 100 <br> - calculators <br> - OHP |
| 3.7 | none | - counting stick <br> - sticky labels numbered 0,8,16... 80 <br> - multiplication grid (optional) |
| 3.8 | - Sheet 3.G: 'Number board 2' | - OHP |
| 3.9 | none | - demonstration number line 0-500 |
| 3.10 | - Sheet 3.H: ‘Gozinto’ board | - OHP <br> - two OHP markers in different colours <br> - two large 1-6 dice <br> - calculators |


| Units | Resources provided with the lesson, from which multiple copies may need to be made, or an OHT | Resources to be provided by the summer school |
| :---: | :---: | :---: |
| 3.11 | - Sheet 3.1: ‘Problem solving' | - OHP <br> - vocabulary flash cards <br> - ten word problems of varying difficulty <br> - calculators |
| 3.12 | - Sheet 3.J1: ‘Loot’ board <br> - Sheets 3.J2 and 3.J3: 'Loot' cards | - counting stick and sticky labels <br> - whole cooked (vegetarian) pizza and knife <br> - 20 sweets <br> - plenty of coins (optional) |
| 3.13 | - Sheet 3.K: ‘Three in a line’ game | - flash cards for the game 'Disappearing Dan' <br> - OHP <br> - OHT of a blank 100-grid <br> - calculators <br> - two 1-6 dice per pair of pupils <br> - centimetre squared paper |
| 3.14 | none | - flash cards for the game 'Disappearing Daisy' <br> - sets of percentage discount cards <br> - list of items for discount |
| 3.15 | - Sheet 3.L: ‘Dominoes’, (Set A and Set B) | - card strip with paper clip for each pupil <br> - sticky tack or masking tape |
| 3.16 | none | - at least five packages of, for example, toothpaste with 'special offers' stated as percentages - '10\% off', '15\% extra free' and so on (some real examples are essential but if necessary you can photocopy these to make enough for the class) |
| 3.17 | - Sheet 3.M: ‘Fraction problem’ | - OHP |

## Place value and ordering

## Resources

OHP (optional)
partitioning cards for individual pupils
Sheet 3.A: ‘Point it out' grid on an OHT or wall chart

0-1000 number line
Sheet 3.B: 'More or less' board and
Sheet 3.C1: 'Be nasty' board for each pupil set of 'Be nasty' cards (Sheet 3.C2) and
two sets of 0-9 number cards for each pair

## Language

place, place value, units, tens, hundreds thousands, tenth, hundredth, thousandth decimal point, equals, greater than, less than, largest, smallest, sum, difference product


## Starter: whole class

Write three different numbers on the board, such as 7, 9, 15. Ask pupils to make statements about a chosen number to assess how confident they are with numbers and their familiarity with vocabulary like 'multiple', 'product'...

John says 9 is a square number. Is he right? What else could you say about 9 ? Yes, it's 1 less than 10... it's 4 more than 5... it's half of 18. Myra thinks 15 is a multiple of 3 . Is it a multiple of any other number?

Now give the class three numbers to hold in their heads in order, such as 3,5 , and 9 . Ask them:

Tell me the sum of the first and last number... the difference between the first and second number... the product of the middle number and the smallest number.

What is the largest number you can make with all three digits? With just two of the digits?

## 40 mins

## Main activity: whole class and pairs

Show pupils the 'Point it out' grid on an overhead projector or as large wall chart.
Ask:

What do you notice? Are the numbers arranged in a special way?
Can you explain any patterns you notice?

Indicate the 700, 40, and 5. Demonstrate how these numbers can be put together with partitioning cards to make a three-digit number: 745. Use the cards to check that pupils can give the value of each digit (seven hundred, forty, five, not seven hundreds, four tens, five).

Indicate other three-digit numbers for pupils to make and read using their own partitioning cards. Get pupils to mark the position of each three-digit number on a large 0-1000 number line. You could extend this to simple decimals to give you an opportunity to assess pupils' understanding of decimals for future work.

- use the vocabulary of comparing and ordering numbers, including greater than, less than, and the symbols >, < and = ; give a number lying between two others

Check pupils' understanding of the vocabulary 'largest', 'smallest', 'greater than', 'less than'. Show and explain the use of the > and < symbols, then draw this grid on the board or OHP.


Challenge the class to play a game against you. With a set of digit cards face down, a pupil selects one at random and indicates where it should go on the grid. This is repeated by three other pupils. If the completed sentence is correct, the class get a point. Then you have a go. Repeat for as many rounds as seems appropriate. Extend into three-digit numbers or decimals.

Now explain how to play 'More or less' and 'Be nasty'. Focus on the language used on the cards for 'Be nasty'. Ask pupils to work in pairs and choose one of the games to play.

## Challenge

Pupils can make another set of target cards for the 'Be nasty' game.

## Plenary: whole class

Check pupils' understanding by asking them questions such as:
Tell me a number between 56 and 62.
Tell me a number greater than 207 and less than 210.

Now check their understanding of the > and < symbols by asking them to put numbers in place in a statement such as:


Finish by asking them about the games that they played.
Did the games depend on skill? What skills were involved? Was there an element of luck involved?

Was the first digit card you drew from the pack a deciding factor in winning or losing?
Was a high digit card necessarily a better card?
Have your skills at ordering and comparing numbers improved?
What different target cards could we use for the 'Be nasty' game?

For homework, pupils can take one of the games home to play with their families.

## Adding and subtracting 3-digit numbers

## Resources

OHP
Sheet 3.D: 'Number board 1' on an OHT, or one between each pair

## Language

digit, approximately, operation, addition subtraction, equals, inverse

## Starter: whole class

| 56 | 20 | 7 | 60 | 51 |
| :---: | :---: | :---: | :---: | :---: |
| 33 | 12 | 78 | 93 | 30 |
| 88 | 63 | 41 | 24 | 75 |
| 15 | 49 | 76 | 50 | 25 |
| 19 | 40 | 37 | 85 | 35 |

Use Number board 1 to identify:
Two numbers with a sum of 100
Three numbers with a sum of 100
Four numbers with a sum of 100

Record pupils' suggestions on the board. Ask the class:
What strategies did you use to choose your numbers?
How can we be sure we have not had that combination of numbers before?

## Main activity: whole class and pairs

Introduce 'cloud calculations'. Give one simple addition and one simple subtraction calculation with the operation and one digit obscured by a 'cloud'. For example:


Ask the class:

How do you know which operation it is?
Which digits are missing?
Rose thinks the missing digit is a 4 because 40 add 20 equals 60 .

Introduce vertical calculations that involve 'carrying' or 'exchanging' with just one digit obscured, for example $5 \square 3+282=805$ and $5 \square 3-282=241$. If children have difficulty due to misconceptions with standard methods, model expanded methods.

For example,


Now provide a series of 'cloud calculations' (addition and subtraction, with varying degrees of difficulty, for example, one, two or three clouds, obscured operation signs).

Ask pupils to work in pairs to order the calculations by degree of difficulty. Then ask them to complete the calculations, either in standard written form or expanded form whichever is most appropriate for their stage of development.
For pupils who finish early, ask them to devise their own challenging cloud calculations, which they must be able to solve and explain for themselves.
$\qquad$

## Plenary: whole class

Provide a different type of example - larger numbers or numbers with different numbers of digits. You may be able to use some of pupils' own challenges.

Ask pupils to demonstrate and explain how to do each calculation.
Show them how to use inverse operations to check the answers, using subtraction to check addition and vice versa.

For homework, take one addition and one subtraction example from this lesson's work. Check the answers by carrying out the inverse operations.

## Tenths and hundredths 1

## Resources

none

## Language

part, tenth, hundredth, thousandth decimal, decimal number, decimal point decimal place, fraction

## Starter: whole class

Begin with some counting activities in jumps of 0.1 then 0.2 , then 0.3 , starting from zero. Practise rounding decimals to the nearest whole number.

## 40 mins

## Main activity: whole class and individuals

Explain that today's lesson is about drawing number lines that show the decimal numbers 'hidden between' the whole numbers.

Draw a segment of number line on the board, with markers only at 0 and 10.


Point to a place about halfway
between the two and ask:

## What number am I pointing to?

Draw in the markers for the whole numbers and point to where the 6 would go. Ask:


What number goes here?

Write it in, then indicate a point somewhere between the 5 and 6 positions and ask:

## What number goes here?

Discuss with the class the idea that, when the line is marked in intervals of one, it is impossible to be accurate with parts of a number less than one. This makes it hard to say exactly what number value you are pointing at between the 5 and the 6 .

Now I am going to zoom in on that part of the line, and magnify it.

Draw another segment of number line on the board, showing 6 and 7 , with nine markers between. Draw a blob above one of the
 unnumbered markers.

What does this line show? ... Yes, the numbers between 6 and 7.
What is each interval (the space between two markers) worth? ... Yes, tenths.
So what is the number where I've drawn the blob? ... Yes, six and two tenths, or six point two.
Count along the line with me: six, six point one, six point two,..

Invite one of the pupils to come to the front of the class and write in
 the decimal numbers that belong at each marker. Explain that the 6 and 7 markers should ideally be written as 6.0 and 7.0 , because this line is paying attention to the values of the digits in the tenths place, so it is important to show that 6 and 7 have no digits there.

These intervals are all worth one tenth, or 0.1. If we zoom in again, and look at the bit between the 6 and 6.1 through a magnifying glass, what might the next set of intervals be worth? ... Yes, hundredths, or 0.01.

Draw another segment of number line on the board, showing 6.0 and 6.1 with nine
 markers between. Get the class to call out what number belongs at each marker as you write them in.
Point to several numbers on the line in turn and ask:

Can you say that number in fraction terms? ... Yes, it's six and no tenths and five hundredths.

Pupils should now copy the original 0-10 number line and write in the numbers, then choose one section of it to magnify into tenths, and construct the new decimal line.

Once they have done this, they should choose a section of the tenths line to magnify, and construct a decimal line showing hundredths.

## Challenge

Choose a section of the hundredths line to magnify, and construct a decimal line showing thousandths.

## Plenary: whole class

Write a variety of numbers on the board, such as:

$$
\begin{array}{llll}
15.37 & 5.96 & 23.17 & 124.58
\end{array}
$$

Ask the pupils to read one of the numbers, using decimal language.
Point to particular digits and ask for their value.
Point to the numbers on the board one at a time, and ask the class to say together the number that is a tenth more, a tenth less, a hundredth more or a hundredth less. Stress that there are tenths and hundredths numbers between each whole number. For homework, get pupils to copy about ten decimal numbers from the board, and to put these in order.


## Tenths and hundrediths 2

## Resources

Sheet 3.E: Shop till receipt, for each pair large display cards for the activity 'Stop!' (as shown below)
about eight sets of 'What's next?' cards, made from Sheet 3.N

Language
decimal point, tenth, hundredth

## Starter: whole class

Play the game 'Stop!' You will need to nominate a pupil to call, "Stop," and have ready some display cards showing $1,10,100,1000$, perhaps three of each.

| 1 | 10 |
| :--- | :--- |

Shuffle the cards and ask a pupil to stand at the front and hold them. The pupil should hold up the first card so that everyone can see it. Start the class counting in multiples of this first number (for example 10, 20, 30, 40...) until the designated pupil shouts, "Stop". The pupil holding the cards then displays the next number, say 1000, and the class continues to count on (for example 1040, 2040, 3040...). Carry on until all the cards have been used.

You can extend this activity by replacing the 1000 card with a card for 0.1. You can also give the pupil who calls, "Stop," a four-digit target to reach, such as 584.7.

## Main activity: whole class and groups

Write on the board the three decimals $0.5,0.25$ and 0.75 . Ask the class:

What does this digit represent? [Point to each digit in turn.]
Which is the largest number? How do you know?
Which is the smallest number? How do you know?

Now give each pair of pupils a copy of a till receipt (Sheet 3.E).
Ask them to order the amounts from the lowest price to the highest price.
After a few minutes, get one or two pairs to report back their answer and how they went about it.

Now introduce the game 'What's next?'. First discuss and agree the order of the set of cards with the pupils, making it clear that all the cards lie in the range $0-10$. Then demonstrate the game by playing with the whole class.

## Rules

- Shuffle the cards and place them in a pile in the middle of the pupils.
- Turn over the first card and ask a pupil to predict whether the next card is going to be 'higher' or 'lower'.
- The same pupil continues until his/her prediction is incorrect. The play then passes to the next player.

Encourage pupils to justify their predictions.
Sasha, you think that the next card will probably be higher because 3.5 is closer to 0 than to 10. Do the rest of you agree?

Now let pupils play the same game in groups of 3 or 4 . Ask them to keep a record of the total number of predictions for each player and to try to beat their previous best.

## 10 mins

## Plenary: whole class

Ask the pupils questions such as:
What was a 'good' card to turn over? Why?
How did you work out whether the new card was higher or lower?
When you are ordering a set of numbers with up to two decimal places, what is a good starting point? How do you continue?

Ensure that pupils know they need to start by identifying the set of numbers with the lowest whole number, and will then need to look at the first decimal place.
For homework, ask the class to collect some examples of measurements given as decimals, for example, labelling on products. Remind them to record the relevant unit of measure.

## Adding and subtracting decimals

## Resources

OHP
partitioning cards, one set for each pupil
several sets of 'What's next?' cards made from Sheet 3.N
Sheet 3.E: Shop till receipt on an OHT
‘Tables timer speed test' (as described below)
calculators

## Language

addition, subtraction, difference, decimal
decimal place, inverse

## Starter: whole class

Make sure pupils are familiar with the partitioning cards by asking a series of 'show me' questions.

Show me five hundred and thirty-nine.
Show me ninety... four hundred and seven... two hundred and sixty...
Show me the largest three-digit number you can make, with each digit different.
Show me the smallest three-digit number you can make with each digit different.
Jocelyn thinks it's 123, Mary thinks it's 120, and Vanessa thinks it's 102.
What do you think now, Jocelyn?

Now ask pupils to use their partitioning cards to respond to 'quick fire' questions using $+10,+100,-10,-100$, then multiples of 10 and 100 .

Now ask questions like:

What do I need to add to 3 to make 10?... To 60 to make $100 ?$
To 350 to make 1000?

Ask the class to explain their strategies and how they calculated their answers.

## Main activity: whole class and pairs

Tell the class that today's lesson should help them to add and subtract decimals. Show the till receipt (Sheet 3.E) on the OHP. Ask the class to check the change given by adding the amount spent to the change given. Discuss what the 'multisaver' means and how shops offer discounts such as two for the price of one.
Demonstrate how to add two amounts of money, relating to coins as and when necessary, pointing out that decimal points line up under each other.

- use known number facts and place value to consolidate mental addition and subtraction
- read and write numbers in figures and words
- carry out column addition and subtraction of numbers involving decimals

Then ask:

What if I only had $£ 12$ to spend and had to leave something behind?
Yes, I could leave the peaches (£2.99) and subtract that from the total ( $£ 14.78$ ).

Demonstrate the calculation, talking it through slowly.
What are the three most expensive items you could buy if you had just £2?

Encourage pupils to discuss this problem in pairs before taking answers.
Now arrange the class in pairs of similar ability. Ask them to use a selection of about ten cards from the 'What's next?' activity as described in Unit 3.4. They should:

Choose two or three numbers with up to three digits and add them using column addition.

Choose two numbers with up to three digits for which it would be appropriate to do a vertical calculation and find the difference using column subtraction.

Provide calculators for pupils to check their answers. For those who finish quickly, ask some 'What if...' questions.

What if you used numbers with more digits?
What if you used clouds to obscure some digits - could your partner do the calculation?

## Plenary: whole class

Write two 'cloud' calculations on the board (one addition, one subtraction). Ask for explanations and demonstrations on how to solve them.


For homework, give pupils a 'Tables timer speed test' to do - ten questions to test recall of multiplication and division facts, based on a table that they find particularly tricky. They should time themselves when they are doing the test and repeat it at least twice to see if they can improve their time. They should record their best time and bring it to school tomorrow.
Tell them that either tomorrow or later in the week they will have another opportunity to see if they can beat their best time on the same test.

## Multiplying and dividing by 10 and 100

## Resources

OHP
grids of numbers to multiply and divide by 10 and 100 (as shown below), for each pair calculators
Sheet 3.A: 'Point It Out' grid on an OHT
Sheet 3.F: 'Find the Pairs' grid, one for each pupil
'Tables timer speed test' (as described below)

## Language

multiply, multiple, multiplied by, divide divided by, divided into, divisible by inverse, calculate

## 10 mins

## Starter: whole class

Warm up as if in an aerobics session. Stand with legs slightly apart, facing the instructor at the front. The class should chant the table rhythmically (for example, the five times table) and perform the actions.

## Tables aerobics

$1 \times 5=5$ raise right shoulder level with ear - keep it there
$2 \times 5=10 \quad$ raise left shoulder level with ear - keep it there
$3 \times 5=15 \quad$ lower right shoulder
$4 \times 5=20 \quad$ lower left shoulder
$5 \times 5=25 \quad$ cross arms in front of body
$6 \times 5=30 \quad$ stretch right arm round past left ear until pointing at the back of the room - keep it there
$7 \times 5=35 \quad$ same but with left arm
$8 \times 5=40$ bring right arm back round past ear to point at the front of room
$9 \times 5=45 \quad$ same but with left arm
$10 \times 5=50 \quad$ jump in the air with both arms stretched upwards

Now sit down and do a 'Tables timer speed test' - ten quick recall multiplication and division questions, based on a table the pupils find hard. Encourage pupils to keep a note of their times and to try to beat these next time.

## Main activity: whole class and individuals

Tell pupils that today they are going to become good at multiplying and dividing numbers by 10 and 100. Give each pair the two grids below and a calculator. Pupils should use the calculator to complete the first three rows of each grid.

## Objectives

- know by heart all multiplication facts up to $10 \times 10$ and derive quickly corresponding division facts

|  | $\times 10$ | $\times 100$ |
| ---: | :--- | ---: |
| 5 |  |  |
| 20 |  |  |
| 110 |  |  |
| 99 |  |  |
| 1010 |  |  |
| 900 |  |  |


|  | $\div 10$ | $\div 100$ |
| ---: | :---: | :---: |
| 500 |  |  |
| 2000 |  |  |
| 10000 |  |  |
| 300 |  |  |
| 800 |  |  |
| 3500 |  |  |

Now ask the class:

What do you notice about the digits?

Deal with the response, "You add a zero," and demonstrate with some examples.
Now use the 'Point it out' grid (Sheet 3.A) on an OHP and go through the last three rows on each grid. Ask:

What happens to the number when you move up one space (that is, multiply by 10)? Two spaces?
What about dividing by 10 and moving down? Dividing by $100 ?$

Set some more questions so pupils can use the 'Point it out' grid to find answers, still leaving it displayed on the OHP.

Now switch off the OHP and ask pupils to visualise the grid. Give them a few more calculations to do together as a whole class. Then set some calculations for pupils to do individually. Let them set their own target for the number that they can complete in ten minutes.

## Challenge

Ask pupils to consider what happens when you multiply and divide by 1000.
Extend to multiplying decimals with 1 then 2 decimal places by 10 or 100, and dividing whole numbers by 10 or 100 .

## Plenary: whole class

Ask the class:
Can anyone explain how to multiply by $20 ? .$. By $50 ? \ldots$ By $80 ?$

For homework, give each pupil sheet 3.F: ‘Find the pairs' - grids of paired numbers plus some red herrings! The aim is to pair numbers where one is 10 times or 100 times the other.

## Multiplication (TU x TU)

## Resources

sticky labels numbered 0, 8, 16... 80
counting stick
multiplication grid (optional)

## Language

multiply, multiplication, multiplied by product, digit, tens, units, partition estimate, calculate

## Starter: whole class

Count together from zero in 10s, then 20s, then 25 s . Count back again to zero. Now do some counting stick activities.


Use the stick to build up the 8 times-table - placing numbers in position on the stick. Focus on the mid-point, doubles and using known facts. Get pupils to count in multiples of 8 , using the numbers on the stick to help. Gradually remove some of the numbers and repeat, including counting both forwards and backwards. Finally remove all the numbers and repeat. Can pupils identify the multiple when you point to the mid-point, the fifth division, the third division?

## Main activity: whole class and individuals

Set a challenge.

How shall we calculate $28 \times 14$ ?
What size would you expect the answer to be?
Can you estimate the answer? How?

Now model $15 \times 30=450$. Show and explain that this is the same as $(15 \times 3) \times 10$.

Would you expect the answer to $28 \times 14$ to be greater than or less than 450 ?
Why?

Model the grid method for $28 \times 14$, partitioning each of the numbers


Demonstrate how to use the grid to get the answer to $28 \times 14$. Involve different pupils in completing the grid. At appropriate times, ask them how they worked out their answers.

Now model a second example, such as $46 \times 19$. Estimate an answer first.
Set some multiplications for pupils to calculate using the grid method. Choose digits that they will be confident at multiplying, drawing on their knowledge of multiplication tables. Consider having a multiplication grid available to check or aid work.

## Challenge

Using the digits $1,3,5$ and 7 , make up some multiplication questions.
Find the biggest product.

## Plenary: whole class

Ask the class questions such as:
Why does partitioning or breaking the number into tens and units help to make multiplication easier?
Will the method always work?
Could we use this method to calculate $927 \times 8$ ? What would the grid look like? Could we use it for $927 \times 18$ ? What would the grid look like?

For homework, set this problem:
Find some multiplication problems of the form $A B \times C D$. The answer must lie between 300 and 500 . How many problems like this can you find?

## Multiplication (HTU x U)

## Resources

OHP
Sheet 3.G: 'Number board 2' on an OHT, or one for each pair

## Language

times, product, multiply, multiplied by multiplication, estimate

## Starter: whole class

Use Number board 2.

| $\frac{1}{2} \mathrm{~cm}$ | 500 m | $\frac{3}{4} \mathrm{~km}$ | 0.3 km | 20 mm |
| :---: | :---: | :---: | :---: | :---: |
| 1500 m | $\frac{1}{10} \mathrm{~km}$ | 0.25 km | 5 mm | 95 cm |
| 0.05 m | 300 m | 2 cm | 20 cm | 55 cm |
| 450 cm | 0.5 km | 1.05 m | 25 cm | 40 cm |
| 750 m | 1.45 m | 100 m | $\frac{2}{5} \mathrm{~m}$ | $\frac{1}{5} \mathrm{~m}$ |

Ask the class questions such as:

What are these measures of? Yes, length or distance.
What do the abbreviations stand for?
Who can tell us how many millimetres in 1 centimetre? centimetres in 1 metre? metres in 1 kilometre? millimetres in 1 metre?
Can you see measures that are written as decimals? As a fraction?
Look at the measure in the first column and first row on the grid. Can you find a measure that is the same? Tell everyone why.
Can you spot other pairs?
What do I need to add to 1.05 m to make it up to 2 m ?
Look at length in the fourth column and fourth row. How many millimetres is that?
Which length is the same as half a kilometre?

Let pupils challenge each other with similar questions. Then remove the number board from the OHP and ask questions based on other measures.

## Main activity: whole class and pairs

Refer to the plenary in the previous lesson on multiplication. This looked at a possible grid for $927 \times 8$.

What did the grid for $927 \times 8$ look like?
Jennie, come and explain the calculation to us on the board.

Focus on how the number has been partitioned to break the calculation up into stages and make the problem easier to solve.

Now tell them that they are going to look at a different paper and pencil method, that still involves partitioning. Demonstrate how to calculate $427 \times 8$. Estimate first to locate answer between 3200 and 4000. Now explain the calculation:

| 427 |  |
| :--- | ---: |
| $\times 88$ |  |
| $7 \times 8$ | 56 |
| $20 \times 8$ | 160 |
| $400 \times 8$ | $\underline{3200}$ |
|  | $\underline{3416}$ |

Write and explain each part of calculation carefully, especially the need to place numbers in the correct columns. Ask:

Is the answer reasonable?

Model a second example, such as $363 \times 9$. Involve the pupils in estimating, scribing and explaining. Then show how this method can be refined by demonstrating short multiplication, explaining each step as you go.

$$
\begin{array}{r}
363 \\
\times \quad 9 \\
\hline 3267 \\
\hline 52
\end{array}
$$

Now model a further example, this time involving the pupils in scribing and explaining each stage. Use questioning to get pupils to identify the similarities in the two methods and to identify the important points they need to remember to carry out short multiplication. Ask the class:

```
Which method do you think is quicker?
Which method is easier to use?
```

Now set some similar questions for the class to do. Have some multiplication grids available in case some pupils need them. Encourage pupils to try both methods for written recording, and to estimate the answer first.

## Challenge

Discuss how to calculate $4.7 \times 3$ and $6.2 \times 7$.

## Plenary: whole class

Go through answers to the calculations. Ask individual pupils to talk through any they got wrong in order to identify and correct any misconceptions.

Let one or two pupils explain their ideas for the decimal calculations. Show pupils the extended written method for $4.7 \times 3$ and then the standard written method.

For homework set two or three multiplications for pupils to do at home. Higher attainers can be given some involving decimals.

## Division (HTU $\div$ U)

## Resources

demonstration number line 0-500

## Language

divisible by method, strategy, inverse repeated addition, repeated subtraction

## Starter: whole class

Start by counting on and back to 100 and beyond in jumps of various sizes.
Count with me in nines: $9,18,27,36 \ldots 90,99,108,117 \ldots$
Now count back from 180 in nines: 180, 171, 162, 153...
Count in twelves to 120: 12, 24, $36 \ldots 108,120$. And back: 120, 108, $96 \ldots$

Talk about the clues that can help the pupils know what number comes next in a sequence, demonstrating on the number line.

Counting in nines is easy because you can quickly add 10 and take off 1. When counting in twelves you can quickly add 10 then 2.

Now chant the 9 times table forwards and backwards. Then ask a few questions, such as, "What is 36 divided by 9 ?"

## Main activity: whole class and individuals

Tell the class that this lesson will look at the way division can be seen as repeated subtraction and that you will be developing an efficient method for division of three-digit numbers by one-digit numbers. Start by posing a problem.

A baker has made 360 cakes to pack in boxes of 8 . How many boxes are needed?
Write up the division on the board.

$$
360 \div 8=
$$

We are going to use repeated subtraction to help divide three-digit numbers.
We can think of 360 divided by 8 as 'how many 8 s in 360 ?' and we could find the answer by repeatedly subtracting eights. This would take a long time so instead we can work with 'chunks' of eights such as 8 , ten times 8 , five times 8 , or twenty times 8.
First let's estimate roughly what the answer might be. Ermer, can you estimate the answer?... Ermer suggests it will be a bit less than 50 because five 8 s are 40 , so fifty 8 s are 400.

Show the class how to use repeated subtraction to solve the problem on the board, involving the pupils at each stage by asking them to suggest what to do next. Use two different methods:

- subtracting 80 repeatedly (that is, ten lots of 8 at a time)
- subtracting 160 (that is, 20 lots of 8 at a time)

Tell higher attaining pupils that you would expect them to do the quicker method, subtracting the biggest possible chunks.

## Objectives

- know by heart multiplication facts to $10 \times 10$ and derive quickly corresponding division facts

| $360 \div 8$ |
| :---: |
| $\frac{360}{\frac{-80}{280}}$ $\frac{-80}{200}$ $(10$ lots of 8$)$ $\frac{-80}{120}$ $\frac{-80}{40}$ $(10$ lots of 8$)$ $\frac{-40}{0}$ $(10$ lots of 8$)$ $(5$ lots of 8$)$ |
| $4 \times 10$ lots of 8 and 5 lots of 8 $\longrightarrow 45$ |



Ask pupils to suggest some scenarios that involve division. For example:
There are 154 pupils who want to make up teams of seven players. How many teams can they make?
There are 444 Brownies at a camp. Tents hold eight Brownies. How many tents are there?

Now set some examples for pupils to try on their own. Tell them if they can do any examples mentally, to do so, but to use repeated subtraction for the others.

Suggest that they write down useful 'chunks' in advance, such as 10 times, 5 times (by halving the 10 times), 20 times (by doubling) and so on.

```
351\div9
360\div8
693\div3
168\div8
168\div7
168\div12
```


## Simplification

Challenge

Some pupils will feel happier with smaller steps and a longer calculation. They should work at the level they understand but discuss with them the efficiency of different chunks.

Set problems involving higher numbers such as $256 \div 6,1452 \div 5,482 \div 15$. Also set problems with remainders.

## Plenary: whole class

Go over one of the calculations with the class, inviting one of the children to demonstrate the method of repeated subtraction on the board. Compare short and long methods.
Ask the class questions such as:
Were any problems more difficult than others? Why? What if the numbers were larger?

For homework, ask pupils to think of three practical examples when division is needed to solve the problem, and the answer must be a whole number.

## Division with remainders

## Resources

OHP
two OHP markers in different colours
Sheet 3.H: ‘Gozinto’ board on an OHT
copy of the 'Gozinto' board for each pupil
two large 1-6 dice
calculators

Language
divide, division, quotient, remainder fraction, decimal


## Starter: whole class

Write $7 \times 12=84$ on the board. Ask the class:
What else can you state if you know this fact?

Draw out three other related facts: $12 \times 7=84,84 \div 7=12,84 \div 12=7$. Some pupils may suggest facts that they can derive by doubling, such as $14 \times 12=168$, or $7 \times 24=168$. Others may suggest a fact that can be derived by adding.

$$
\text { If } 7 \times 12=84 \text {, then } 8 \times 12 \text { must be } 12 \text { more, so } 8 \times 12=96 \text {. }
$$

Draw out the other multiplication and divisions from these facts as well. Stress that if you know a multiplication fact then you know two division facts.
Now have a class game of 'Gozinto'. Split the class into two teams. Use the large dice and roll pairs of numbers. Colour each team's suggestions in different colours.


## Main activity: whole class and small groups

Explain that sometimes when we divide there is a whole number answer and sometimes there is a remainder. Today's lesson is about dealing with remainders.

Begin with a problem. Ask pupils to talk in pairs about ways to get an answer.
Four children have 11 bars of chocolate to share equally between them.
How much chocolate will each of them receive?

Model three different ways of finding a solution.

1. Using repeated subtraction or chunking to get a whole number answer and remainder.
2. As a fraction: $\frac{11}{4}=2 \frac{3}{4}$
3. As a division, using a calculator: $11 \div 4=2.75$.

Ask the class:
Do all the methods give the same answer?
What do you notice about the last two answers?
What is the most appropriate way to express the answer?

Now use a second example to illustrate the three methods.

Six adults have won $£ 57$ on the lottery.
How much money did each of them win?

Ask pupils to look carefully at each answer in the context of the problem.

```
What would the adults do with the £3 reminder?
What does £9\frac{3}{6}}\mathrm{ pounds mean?
How should we interpret 9.5 on the calculator?
```

Work through a third problem, focusing particularly on fractions and their decimal equivalents and interpreting the calculator display.

Now set the class a range of division problems to work on in groups of three, where one uses a calculator to find a decimal answer, the second represents the answer as a fraction and the third uses chunking or repeated subtraction to find the whole number remainder. Pupils should take turns to try the different methods. Each time they should decide as a group which solution is the most appropriate.

Challenge
Set problems where the decimal outcome is not one that the pupils would immediately recognise, for example division by 7 or 11 .

## 10 mins

## Plenary: whole class

Ask the class questions such as:

Did you use your knowledge of tables to help solve these problems?
Which method do you prefer? Why?

For homework, give each pupil a copy of the 'Gozinto' playing board to play at home. Explain that if they do not have two dice at home, they can use one suit of a set of playing cards, with the Ace and King removed; the Jack counts as 11 and the Queen as 12. One player fans out the cards so that the other cannot see them, and the other draws a card. The number is equivalent to the total score on the two dice.

## Resources

OHP
vocabulary flash cards
ten word problems of varying
difficulty (see below)
Sheet 3.I: ‘Problem solving' on an OHT
a copy of 'Problem solving' for each pupil
calculators

## Language

add, plus, total, altogether, sum how many more/less, how much subtract, minus, difference between, left share equally

## Starter: whole class

Choose a flash card, such as:

## altogether

Tell the class you will now make up a problem using this word.
There are 30 pupils in Class 1, 27 pupils in Class 2 and 34 pupils in Class 3.
How many pupils altogether are there in the three classes?

A pupil who can say what operation is required wins the flash card. This pupil now takes another flash card and poses a problem containing that word. Continue with the game. After a while, give a challenge.

Who can pose a problem that needs more than one operation?

## Main activity: whole class and small groups

Give the class an example of a simple word problem.

I bought a new coat for $£ 59.99$, a pair of socks for $£ 3.45$ and a hat for $£ 11.50$.
How much did I spend in total?

Introduce Sheet 3.I: 'Problem solving' on the OHP. Go through the problem using the stages as shown on the sheet. Obtain agreement from everyone at each stage by getting individual pupils to explain their thinking.

Max, you think we should first add $£ 59.99$ and $£ 11.50$.
Why is that?

## Objective

- use all four operations to solve word problems involving numbers and quantities based

Give another simple example, but with a different operation. Give the class a short time to discuss with a partner how to solve the problem. Choose someone to demonstrate their method to the others.

Now give the class a set of problems (no more than ten) of varying levels of difficulty. Read them through with the whole class. Then ask the pupils in pairs to grade them in order of difficulty. Bring the class together and discuss their decisions and reasoning. Ask:

> If you used a calculator, would that make the problem easier?
> How would you enter the numbers?

Ask pupils to solve the problems, using the 'Problem solving' sheet to support them.

## Challenge <br> Make up some problems for your partner to solve.

## Plenary: whole class

Go quickly through all the answers to the problems. Choose one or two of the more difficult problems and discuss strategies for solving them, using pupils to demonstrate.

Justine, can you show us how you tackled the problem of packing the shrimps in the tubs?

For homework, set a harder problem.

There are 24 children in a class.
Three quarters of them have pets.
8 children have dogs.
3 children have a hamster.
How many children have other kinds of pets?

## Finding fractions

## Resources

counting stick and sticky labels
whole cooked (vegetarian) pizza and knife 20 sweets
Sheet 3.J1: ‘Loot’ board for each group pack of 'Loot' cards (double-sided) made from Sheets 3.J2 and 3.J3, for each group plenty of coins (optional)

Language
fraction, decimal numerator, denominator, equivalent one whole, half, quarter, tenth, hundredth

## Starter: whole class

Begin with some counting stick activities.


Say that one end of the stick is 7 and the other end is 8 . Identify the middle point, then other specified points on the ten-point scale. Encourage pupils to express some answers in both decimal and fraction form.

Count along the stick, forwards and backwards, in fraction jumps of one tenth, then decimal jumps of 0.1.

Repeat with a different pair of consecutive numbers at each end. Now say:

One end of the stick is 4 , and the other end is 6 .
What is the middle point?
And this point?

Continue until each of the points has been identified. Then count along the stick, as before, in jumps of one fifth and jumps of 0.2.

## 40 mins

## Main activity: whole class and small groups

Take the whole cooked pizza and the 20 sweets.


- use decimal notation for tenths and hundredths
- relate fractions to division and to their decimal representation
- find simple fractions of numbers or quantities

Demonstrate finding half of the pizza, then half of the sweets. Physically divide the sweets into two groups of 10, but discuss also how to find the answer without using the sweets.

Use the key vocabulary listed above as you explain and demonstrate the equivalence between two halves and one whole.

$$
\frac{2}{2}=\text { one whole }=\frac{1}{2}+\frac{1}{2}
$$

Now demonstrate the equivalence of four quarters and one whole.

$$
\frac{4}{4}=\text { one whole }=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}
$$

Now involve the class in finding $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}$ and $\frac{1}{10}$ of the pizza and the sweets. Introduce the 'Loot' game for pupils to play in groups of three or four. Players can either draw coins from the 'bank' or keep a record of the amount that they win or lose in their notebooks. Tell them that they should be prepared to answer questions about the game in the plenary.

## Challenge

Use a wider range of fractions in the 'Loot' game. There will need to be an agreed rule for coping with 'unequal' shares, such as what to do with $\frac{2}{3}$ of 20p.

## Plenary: whole class

Ask the class questions such as:

What was a good fraction with 'win' to turn up? Why?
What was a good fraction with 'lose' to turn up? Why?
Robert, how did you work out half of 56 p?
Nasreen, how did you work out three quarters of 60p?
Winston, how would you work out one tenth of 140 sweets?
What about three tenths?

For homework, give each pupil a copy of the 'Loot' game rules and a pack of cards to take home to play with their families or friends.

Finish the lesson by eating the apparatus!

## Fractions and decimals

## Resources

flash cards for the game 'Disappearing Dan' (as described below)
OHP
OHT of a blank 100-grid
calculators
two 1-6 dice for each pair
Sheet 3.K: ‘Three in a line’ for each pair
centimetre squared paper

Language
part, equal part, decimal, decimal point fraction, one tenth, one hundredth equivalent

## Starter: whole class

Play the game 'Disappearing Dan'. Draw a large shape of a man on the board. Write four-, five- and six-digit numbers randomly on parts of the body, including joints like the ankle, knee, wrist, elbow, shoulder. For example, head: 3450, left foot: 65700 , and so on. Explain to the class that you are going to use flash cards:

| $\times 10$ | $\times 100$ |
| :--- | :--- |
| 10 | $\square 100$ |

These will be the operations that they have to perform on Dan. If they give the correct answer, the associated body part is rubbed out - hence 'Disappearing Dan'! Scores can be included. Team 1 takes a flash card. Team 2 chooses the body part number for Team 1 to operate on. If a player from Team 1 answers correctly, Team 1 scores points equal to the answer. For example, the left foot (65 700) divided correctly by 10 gives a team 6570 points. Team 2 now chooses a card.

## Main activity: whole class and small groups

Tell pupils that today's lesson is about the equivalence between some fractions and decimals. Remind the class that one half means 1 divided by 2.

Use a large paper square, or an OHT, to represent one whole.


- multiply and divide mentally whole numbers and decimals by 10 and 100 and explain the effect
- recognise the equivalence between the fraction and decimal forms of one half, one quarter, three quarters... and tenths and hundredths

Divide the square into ten vertical sections and label each 0.1. Use these to demonstrate the equivalence between $\frac{1}{2}, \frac{5}{10}$ and 0.5 . Confirm using a calculator, keying in:


Similarly, demonstrate the equivalence between tenths and their decimal fractions. Use questioning to get pupils to understand that each small square represents $\frac{1}{100}$. Now demonstrate that one quarter of the whole square is equivalent to two tenths (two vertical strips) plus five hundredths (five small squares), which is represented by 0.25 . Ask pupils to confirm on their calculators that $1 \div 4=0.25$.


Demonstrate in a similar way that $\frac{3}{4}=0.75$.
Use a counting stick to point to 0.25 and 0.75 . Use questioning to make sure that pupils understand the equivalence with $\frac{1}{4}$ and $\frac{3}{4}$.

Now explain how to play the game 'Three in a line' (Sheet 3.K). Before asking them to play the game in pairs, ask:

What strategies are you likely to use?

Challenge

Instead of using dice, pupils nominate two whole numbers to make the numerator and denominator for their fraction.

## Plenary: whole class

Ask the class questions such as:
Did you use any of your knowledge of fractions?
Did you find any equivalent fractions? What were they?
Was this a game that depended on strategy?
Did anyone try the alternative game of choosing your own numbers for the fraction? Was this game better? Why? What made it more challenging?

For homework, give pupils a sheet of centimetre squared paper to play 'Three in a line' with their families.

## Resources

flash cards for the game 'Disappearing Daisy' (as described below) sets of percentage discount cards for the main activity, one set for each pair list of items for discount, either on OHT or one for each pair

## Language

per cent, percentage, proportion hundredth, equivalent, fraction, decimal reduce, discount

## Starter: whole class

Play ‘Disappearing Daisy’ (see Unit 3.13). Draw a large person on the board. Label the body parts with small whole number quantities. Use flash cards such as:
$\left.\begin{array}{|l|}\hline \begin{array}{c}\text { one } \\ \text { half }\end{array} \\ \hline \begin{array}{c}\text { three } \\ \text { quarters }\end{array} \\ \hline \begin{array}{c}\text { one } \\ \text { tenth }\end{array} \\ \hline\end{array} \begin{array}{|c|}\text { one } \\ \text { hundredth }\end{array}\right]$
and other simple fractions. Team 1 takes a flash card. The aim is to find that fraction of the appropriate body part. Team 2 chooses the body part number for Team 1. If a player from Team 1 answers correctly, then Team 1 scores points equal to the answer. Team 2 now takes a flash card, and so on.

## Main activity: whole class and small groups

Remind the class of the meaning of the terms per cent and percentage.
Question the class to make sure that they know that $100 \%$ represents the whole.

```
\(45 \%\) of the people on a bus are female.
```

What percentage are males?
$70 \%$ of the pupils in a class have a packed lunch and 4\% go home.
What percentage have a school dinner?

Show how $50 \%$ can be written as $\frac{50}{100}$. Use questioning to get pupils to reduce this to its equivalent fraction: $\frac{1}{2}$. Repeat with $10 \%$, $1 \%$ and $25 \%$, scribing equivalent fractions against percentages. Ask questions about 50\%.

I want to take $50 \%$ of you to the cinema tonight.
How many of you will that be?
Paul, how did you work it out?

Link this back to earlier work on finding fractions of quantities, and the relationship to division. Use 25\% of different quantities to consolidate the link.
Extend to $10 \%$ and dividing by 10 , involving the class in explaining calculations to respond to a range of questions and situations.

Now demonstrate how they can use the skills of finding 50\%, $25 \%$ and $10 \%$ to help solve other percentage problems. For example,

A computer game costing $£ 48$ is reduced by $20 \%$ in a sale. $10 \%$ of $£ 48$ is $£ 4.80$.

So $20 \%$ of $£ 48$ will be $£ 4.80 \times 2=£ 9.60$.

Give further problems. Invite pupils to write their solutions on the board and explain them to the class. Include finding $5 \%, 30 \%, 75 \%$ and $80 \%$.

Now give an activity for pupils to work on in pairs. Show them a list of items:

| Jacket | $£ 60$ | Jeans | $£ 35$ |
| :--- | :--- | :--- | :--- |
| Trainers | $£ 94$ | Video recorder | $£ 200$ |
| Personal stereo | $£ 80$ | Roller blades | $£ 88$ |

and a set of percentage discount cards.

| $10 \%$ | $20 \%$ |
| :--- | :--- |

Ask the class:

## Which percentage discount cards would give the greatest saving? Why?

Explain that they are going to practise their skills by playing a game in pairs. Stress that they do not have to find the new prices of items, but work out the discount.

## Rules

- Player 1 chooses an item from the list, then turns over a discount card from the stack.
- Each player calculates the amount of the discount or saving, using jottings to help if they wish.
- If Player 2 agrees with the calculation, then Player 1 records that amount on a score chart.
- At the end of the game, each player finds his or her total savings.
- The winner is the player who has made the greatest saving.

Challenge

Include 5\%, 15\% and 17.5\% on the discount cards.

## Plenary: whole class

Ask the class questions such as:

How did you calculate $80 \%$ ? $75 \%$ ? 20\%?
Did anyone do it a different way?
Can we use the discount to work out the new price? How?
What would $10 \%$ be as a decimal? And $80 \%$ ? And 20\%?

For homework, ask pupils to find examples of percentages in newspapers, magazines, catalogues, or on packets, and bring them in for a later lesson on percentages (Unit 3.16).

## Percentages 2

## Resources

card strip with paper clip for each pupil one set of Set A Dominoes, made from Sheet 3.L: 'Dominoes'
Set B Dominoes, also made from
Sheet 3.L: 'Dominoes', one set for every pair
sticky tack or masking tape

## Language

fraction, decimal, hundredth, tenth
half, quarter, three quarters,
per cent, percentage, equivalent

## Starter: whole class

Ask some quick-fire questions finding fractions and decimals of numbers and quantities.

What is one quarter of 60 ? Three quarters of 60 ?
What is one half of 300 grams?... Of 2 miles?... Of 5 litres?
What is 0.1 of $60 \mathrm{~km} ? \ldots$. Of 1 metre?... Of 25 ml ? How did you work it out?

Give each pupil a card strip with a paper-clip to slide along it. (See photograph on cover of summer school folder). Ask pupils to slide the paper clip to an estimated position.

## Show me 0.1 of the strip.

Now show me $25 \%$.

## Main activity: whole class and pairs

Tell the class that today they are going to be thinking about the equivalent values of percentages, decimals and fractions.
Write ' $25 \%$ ' on the board and ask the class to say what they understand it to mean.
Write $\frac{25}{100}$ on the board too.
Ask a pupil to come to the board and write the same quantity in another way.

What other ways could we write this fraction?... Yes, $\frac{5}{20}$ or $\frac{1}{4}$. And does anyone know the decimal equivalent?

Do the same for one or two other percentages, such as $40 \%$ and $75 \%$.
Distribute the dominoes from Set A so that each group of three or four pupils has one domino. Ask each group to copy their domino onto a sheet of A4 paper, making the numbers as clear and legible as possible.

Now ask someone to stick their domino copy on the board with sticky tack or masking tape.

Who has a domino that would fit at either end of that one?

Ask those pupils to attach their dominoes in the correct place.

## Are these correct?

Continue like this until the dominoes have formed a complete ring, getting the class to check each new domino that is placed, and to explain why they think it is right or wrong.


Percentages can also be expressed as decimals, so 25 per cent can be written as $25 \%$ or 0.25 [writing $25 \%$, and 0.25 on the board].
The actual digits are the same in the percentage and decimal equivalents; there are a ' 2 ' and ' 5 ' in both.
Can anyone see why they are the same?... Yes, 0.25 means $\frac{25}{100}$; and so does $25 \%$.

Distribute the dominoes from Set B so that each pair of pupils has a complete set. Ask each pair to complete a new domino ring. When they have finished, they should write down all the matched pairs that they used.

When they have finished their work they should invent a domino set of their own, using percentages, fractions and decimals - and words, if they choose.

## Plenary: whole class

Pick up any domino and copy on the board one of the numbers (a percentage, or fraction or decimal).

What are some alternative ways of writing that number?

Spend the rest of the lesson on revision. Ask the class to provide you with some percentage values and their fraction and decimal equivalents, and write these on the board in three columns. Tell the class to pay attention to which further facts can be deduced

| percentage | fraction | decimal |
| :--- | :---: | :---: |
| $100 \%$ | $\frac{1}{1}$ | 1.0 |
| $50 \%$ | $\frac{1}{2}$ | 0.5 |
| $10 \%$ | $\frac{1}{10}$ | 0.1 | from 'easy' ones they already know.

If $10 \%$ is the same as a $\frac{1}{10}$ and 0.1 , what is equivalent to $5 \% ? \ldots$ And $2 \frac{1}{2} \%$ ?

For homework, ask pupils to continue with the domino set they have started inventing.

## Percentages 3

## Resources

at least five packages of, for example, toothpaste with 'special offers' stated as percentages - '10\% off', '15\% extra free' and so on (some real examples are essential but if necessary you can photocopy these to make enough for the class)

## Language

part, whole, per cent, percentage increase, decrease, discount

## Starter: whole class

Play 'Percentage snap'. Write five simple percentages on the board, with their decimal and fraction equivalents. Divide the class into three teams: the Percentages, the Fractions and the

| $20 \%$ | $\frac{1}{5}$ | 0.2 |
| :--- | :--- | :--- |
| $25 \%$ | $\frac{1}{4}$ | 0.25 |
| $50 \%$ | $\frac{1}{2}$ | 0.5 |
| $90 \%$ | $\frac{1}{9}$ | 0.9 |
| $75 \%$ | $\frac{3}{4}$ | 0.75 | children hold up their team's number. If two of the numbers

$75 \% \quad \frac{3}{4} \quad 0.75$ are equivalent, whichever team first says 'Snap' wins a point.

## Main activity: whole class and pairs

Tell the class that in this lesson they are going to do some work on special offers such as ' $5 \%$ off'. Remind them that 'per cent' means 'for every hundred' or 'out of a hundred'.

Then, in preparation for the next activity, pose some problems on ' $10 \%$ extra'.
For example:

A class had exactly 30 pupils in it. Then they were told to expect an extra $10 \%$ joining from another school. How many pupils were joining?
What was the new number of pupils in the class?

Now look at the packages that have offers stated in terms of percentages.
Choose one to look at closely, for example: ' $10 \%$ extra: 55 ml for the price of 50 ml '.
The way the package is coloured, they seem to be saying all this is extra. That must be a good quarter of the box.
But how much extra does the label say they are really giving you? Yes, just 10\%!

What volume is the [product] normally, according to this package?... And what volume are they offering you here?

So how much extra volume are you getting?
What is $10 \%$ of the normal volume? ... Yes, 5 ml .
So if they are giving you an extra 5 ml , that is an extra $10 \%$.
The statement is correct.

Write some of the pupils' statements up on the board.

| $10 \%$ extra free | 330 g for the price of 300 g (toothpaste) |
| :--- | :--- |
| $5 \%$ extra | 110 g for every 100 g (antiseptic cream) |
| $10 \%$ extra | 66 g for the price of 60 g (toothpaste) |
| 13\% extra | 226 g for the price of 200 g (shaving cream) |
| 18\% extra free | 212 ml for the price of 180 ml (mouthwash) |

Ask the class to work in pairs and for each product decide:
— the original quantity/weight

- the extra quantity/weight as judged by information on the packet
- the percentage extra as claimed on the package
— whether the extra percentage claimed matches the extra quantity/weight they have worked out.


## Challenge

Invent some more claims, and slip in one or two false ones. Can their partner spot the deliberate mistakes?

## Plenary: whole class

Ask pairs to show on the board the methods they used to solve the problems.
Discuss the different methods used in the class.

How did you work out the $13 \%$ problem?
What quantity did you have to find $13 \%$ of?... Yes, 200 g .
So you found $10 \%$ and then $1 \%$ and added three lots of $1 \%$ to the $10 \%$.
Did anyone do it differently?
What was the extra quantity of toothpaste?
Did the extra percentage claimed on the box match the extra weight?

Help the class work out any problematic ones from the board.
Finish up with a session of mental work. Set a few similar problems for homework.

```
What is 10% of £30?
What is 5% of 80? What is 10% ... and 20%?
What is 25% of 44? And what is 12\frac{1}{2}% of 44?
What is 10% of 243? 20%?
```

Stress that you can always work out percentages by dividing by 100 to find $1 \%$.

## More fractions

## Resources

Language
OHP
Sheet 3.M: ‘Fraction problem’ on an OHT
equivalent, numerator, denominator
reduce, cancel

## 10 mins

## Starter: whole class

Spend some time with the pupils finding given fractions of an amount of money such as $£ 10$.

What is half of $£ 10$ ?... If half of $£ 10$ is $£ 5$, then what is a quarter?... Yes, $£ 2.50$.
And three quarters?

Write up the answers, then do the same with other amounts of money which can be halved and quartered easily.

Move on to finding a third and two thirds of amounts which divide easily in three.

```
What is a third of £12?... Yes, £4, and so two thirds is... Yes, £8.
```

Invite individual pupils to explain how they worked them out.

## Main activity: whole class then individuals

Present the class with a scenario such as this. Invite the pupils to look at Plan A first, and discuss how much each child would get under this plan.

```
There are three children in a family, Alice, Bryn and Chloe.
Their grandmother gives them £240 to share between
them. She has devised several plans for sharing it.
Plan A
Alice gets }\frac{5}{10}\mathrm{ of the money, Bryn }\frac{3}{10}\mathrm{ , Chloe }\frac{2}{10
Plan B
Alice gets }\frac{1}{4}\mathrm{ of the money, Bryn }\frac{5}{12}\mathrm{ , Chloe }\frac{1}{3
```

Their grandmother gives them $£ 240$ to share between them. She has devised several plans for sharing it. Plan A
Alice gets $\frac{5}{10}$ of the money, Bryn $\frac{3}{10}$, Chloe $\frac{2}{10}$ Plan B
Alice gets $\frac{1}{4}$ of the money, Bryn $\frac{5}{12}$, Chloe $\frac{1}{3}$

Who gets the most money with Plan A?
Who gets the least?... Yes, Chloe. How do you know?... Yes, two tenths is less than either five tenths or three tenths.
How would you work out how much each child gets?... Yes, work out one tenth, and then multiply.

Use the scenario to devise some simple calculations involving addition and subtraction, and write these on the board.

As a fraction, how much more does Alice get than Chloe?

How much more does Alice get than Bryn? Can you express that as a fraction?...Yes, $\frac{2}{10}$. Who knows another way of writing $\frac{2}{10}$ ?

How much more does Alice get than Chloe?
$\frac{5}{10}-\frac{2}{10}=\frac{3}{10}$
How much more does Alice
get than Bryn?
$\frac{5}{10}-\frac{3}{10}=\frac{2}{10}=\frac{1}{5}$

Now look at Plan B and discuss what the three children would get under this.

It isn't so easy to see who gets the most this time, is it?... The fractions are not all the same kind; there are quarters, twelfths and thirds, whereas with Plan A all the fractions are tenths.
What can you tell by looking at these fractions?...
Yes, Alice gets less than Chloe because $\frac{1}{4}$ is smaller than $\frac{1}{3}$.
How could you work out who gets the most, and by how much?

The pupils may well suggest finding actual amounts Alice gets $£ 60$ and so on. Accept this method, and also suggest that they look at the fractions and try to convert them to the same 'kind', that is, twelfths. Show this on the board.

Now present Sheet 3.M: ‘Fraction problem' as an

```
How much more does
Bryn get than Alice?
5
= 2
= \frac{1}{6}
```

overhead transparency, and give them about 10 minutes to answer the questions.

Simplification Alice gets $\frac{1}{8}$ of the money, Bryn $\frac{2}{8}$ and Chloe $\frac{5}{8}$. Who gets the most? Who gets the least? What is the difference between these two amounts? What fraction of the whole amount is that?

Alice gets $\frac{1}{4}$ of the money, Bryn $\frac{10}{24}$ and Chloe $\frac{1}{3}$. Who gets the most? Who gets the least? What is the difference between these two amounts? What fraction of the whole amount is that? Write this as the difference between two fractions.

## Plenary: whole class

Ask some of the pupils who did the worksheet to share their findings with the rest of the class. Let one of them write their workings and answers on the board.

Finish with some mental work based on fractions of an hour:

```
Alice gets }\frac{1}{2}\mathrm{ of the money, Bryn }\frac{1}{8}\mathrm{ and Chloe }\frac{3}{8
Alice gets the most }\frac{1}{2}£12
Bryn gets the least }\frac{1}{8}£3
The difference between these two amounts is
\frac{1}{2}-\frac{1}{8}
\frac{1}{2}-\frac{1}{8}=\frac{4}{8}-\frac{1}{8}=\frac{3}{8}=£90
£00 is \frac{3}{8}}\mathrm{ of the whole amount
```

How many minutes are there in half an hour?
What is half of $60 ?$
What is a quarter of $60 ?$
How many minutes are there in three quarters of an hour?

Set a few similar problems for homework.

## Acknowledgements

Thanks are due to these publishers for allowing us to reproduce the following commercially available material.

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- 'More or less' board from A Feel for Number
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