The National Strategies
Secondary

Secondary mathematics algebra study units

Unit 2: Transforming expressions and equations

department for
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Description

This unit is for individual teachers or groups of teachers in secondary schools who are considering their teaching of algebra. It discusses ways of helping pupils to form equivalent expressions through 'clouding the picture' activities. It also considers two of the teaching principles that help pupils to use and apply algebra with confidence:

- providing opportunities for pupils to express generality
- asking pupils to 'find as many ways as you can'.

Other units that could be combined with this one, either to create a longer session or to work through in a sequence over time are:

- Unit 5: Collecting like terms
- Unit 7: Using algebraic reasoning.

Study time

About 40 minutes

Resources

Each teacher or pair of teachers working together will need:

- a personal notepad
- copies of **Resources 2a**, **2b**, **2c** and **2d** at the end of this unit
- a copy of the algebra pages from *The Mathematics overview and learning objectives* PDF, which you can download in A3 or A4 from the Framework for secondary mathematics at **www.standards.dcsf.gov.uk/nationalstrategies**. Search for the title: 'Mathematics learning objectives'.

It would be helpful to have available a copy of:

- *Teaching Mental Mathematics from Level 5: Algebra*

which you can download from **www.standards.dcsf.gov.uk/nationalstrategies**; search using the ref 00692-2009PDF-EN-01.

Transforming expressions and equations

1. It is usual for teachers to model the simplification of an expression of like and unlike terms, and then to give pupils an exercise with similar questions of increasing complexity. A less common approach is to present pupils with an algebraic expression and ask them to find as many different ways to write it as possible. This activity supports an interactive teaching approach and can help you to identify pupils' misconceptions.

   Look at the 'clouding the picture' diagrams on **Resource 2a: Equivalent expressions**. The aim in each case is to rewrite the central expression in as many different ways as possible by continuing a process along the same or a different branch, then to generalise by explaining succinctly what is happening along each branch.

   Consider how you might use these prompts and suggestions if pupils were undertaking the activity in the classroom.

   - Choose two branches. What is the same and what is different about them?

- Can you start a new branch that does something different to change the expression?
- Evaluate each expression by substituting a value for each variable, for example, $x = 1$, $a = 2$.
- What can you say about the expressions in any pair of boxes? [They are equivalent.]

You can prompt pupils to be as creative as possible, for example, introducing brackets or fractional terms, or a second variable, differentiating the level of challenge for different pupils. For example, pupils who are ready to 'complete the square' could start by working in this way.

Recording pupils' results during lessons on a flipchart or OHT helps them to share their developments and generalisations.

2. The transformation of equations can be approached in a similar way. Traditionally, pupils are taught rules for solving equations such as 'change the side, change the sign', or 'do the same to both sides'. If they try to use these rules without understanding them and why they work, pupils often make errors such as:

$4x = 20 \Rightarrow x = \frac{20}{-4}$ (change the side, change the sign)

$3(x - 2) = 6 \Rightarrow x - 2 = 3$ (take 3 from both sides)

'Doing the same to both sides' is the more meaningful method, but there are two difficulties:

- knowing how to change both sides of an equation so that equality is preserved
- knowing which operations lead towards the desired goal.

Building equations is easier than solving them, because it postpones the second difficulty and so is an easier place from which to start.

Look at the diagrams on **Resource 2b: Transforming equations** as you read the text below. The central box contains the equation $x = 3$. Along each branch of the tree, build an equation, step by step, using any of the four operations, +, –, ×, ÷ and whole numbers from 1 to 10.

3. When you work with pupils in the classroom you could do a similar activity on an OHT or whiteboard. As pupils suggest each operation, you could supply the notation and explain it carefully. For example, you could explain that we use brackets to show that a whole expression is being multiplied and that we use the fraction bar rather than the usual division symbol (÷).

Ask pupils to work in pairs with another example, being as creative as they can. Then get them to swap with another pair and analyse each others' work to see what is the same, and what is different, about their approaches.

Encourage pupils to verbalise what they are doing to maintain the balance of their equations, and to check that they are generating equivalent equations by substitution of the original value of the variable.

4. From here, the next step is to generate equivalent equations by changing the rule before each stage, as in the first diagram in **Resource 2c: Changing the rule at each step**.

If you were doing this in the classroom, you could ask pupils to check that the original value of x still satisfies the final equation.

$$3\left(\frac{3+5}{4} - 1\right) = 3\left(\frac{8}{4} - 1\right) = 3(2 - 1) = 3$$

Now hide all the steps except the final equation and ask pupils to recall each operation in sequence, using prompts such as the following.

- This equation tells the story of 'x'. What happened to x first? How can you tell by looking only at the equation?
- What then?
- What then?
- What was the last thing that happened?

In this way, you can show pupils that the final equation 'tells the story' of the operations used. You could follow this by asking them these questions.

- Suppose you had started with this equation and you wanted to find the value of x. How could you do this?
- How can you undo what we have just done?

Gradually get pupils to unpick each step in reverse order. As they do this, uncover the preceding equations one by one and write the corresponding operation to the right of each equation (with upward arrows), as in the second diagram on **Resource 2c**.

You will probably need to work through more examples like this with the whole class, until they get the idea. It is worth changing the letter used each time, just to make the point that there is nothing special about the letter x.

Follow up by asking pairs of pupils to create equations, starting from a simple equation such as $a = 6$. The pairs then give one of their equations to another pair to find the original value of the variable by working in reverse order.

5. To provide further practice in the creation of equations you could ask pupils to use their mini-whiteboards to write down algebraic equations that correspond to some 'think of a number' problems. For example, you might say:

   Think of a number, call it n
   Double it
   Add 4
   Divide your answer by 7
   Multiply your answer by 2
   The result is 4
   Show me the equation

   And pupils might respond: $2\left(\frac{2n + 4}{7}\right) = 4$

6. You could also set pupils the task of generating simultaneous equations by using a chosen solution such as $a = 2$, $b = 3$ in the central box of a tree structure. Take a few minutes to sketch a tree structure in your notepad to show how they would do this.

   You could then ask pupils to choose two of the equations from the ends of branches to present to another pupil, who must then find the original solution. This can be done by transforming each equation, by changing the rule at each step in order to match the coefficients of one of the variables. Again, sketch out a possible process in your notepad.

Now look at one way of generating then solving a pair of simultaneous equations on **Resource 2d: Simultaneous equations**.

To sum up, the 'clouding the picture' approach helps pupils to:

- generate equivalent expressions and equations, including a simplified form
- substitute values into equations and formulae
- consider different approaches, for example, where another pupil has represented the problem or approached its solution in a different way
- communicate solutions
- gain confidence and develop increasing fluency in manipulating expressions into equivalent forms without being rule-bound.

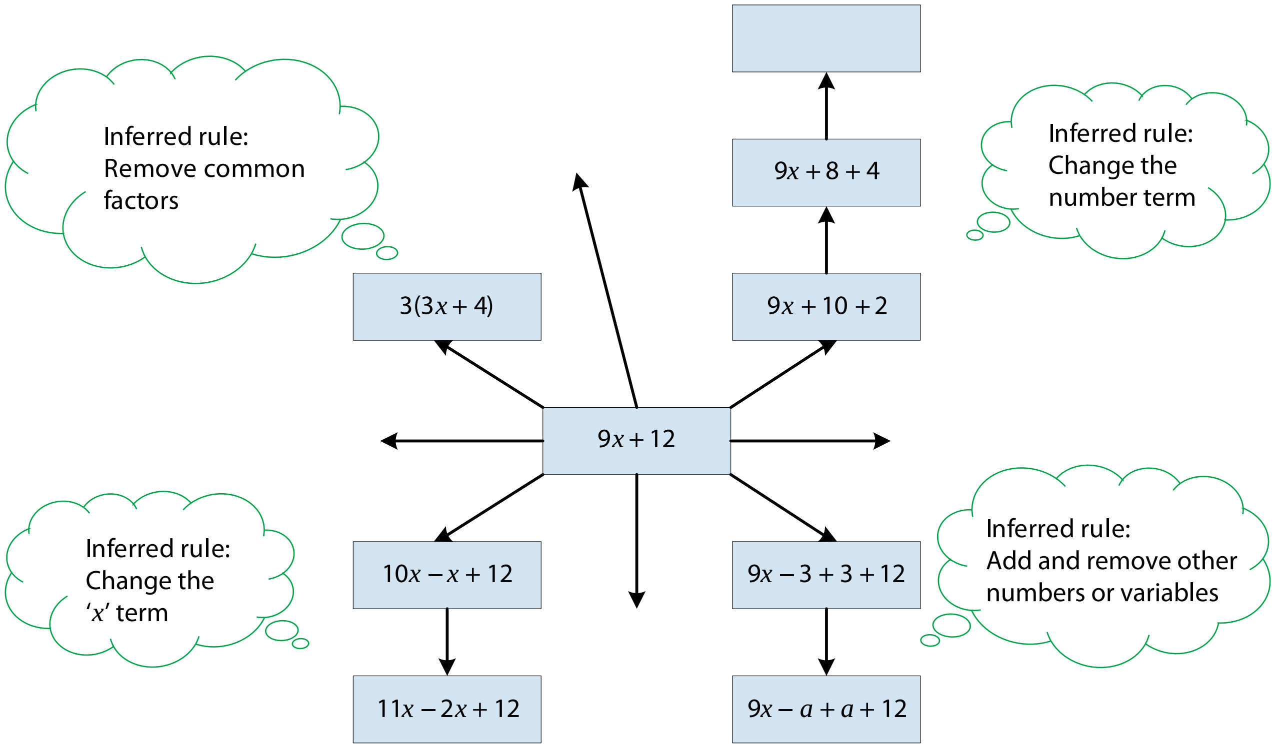
Now reflect on these questions.

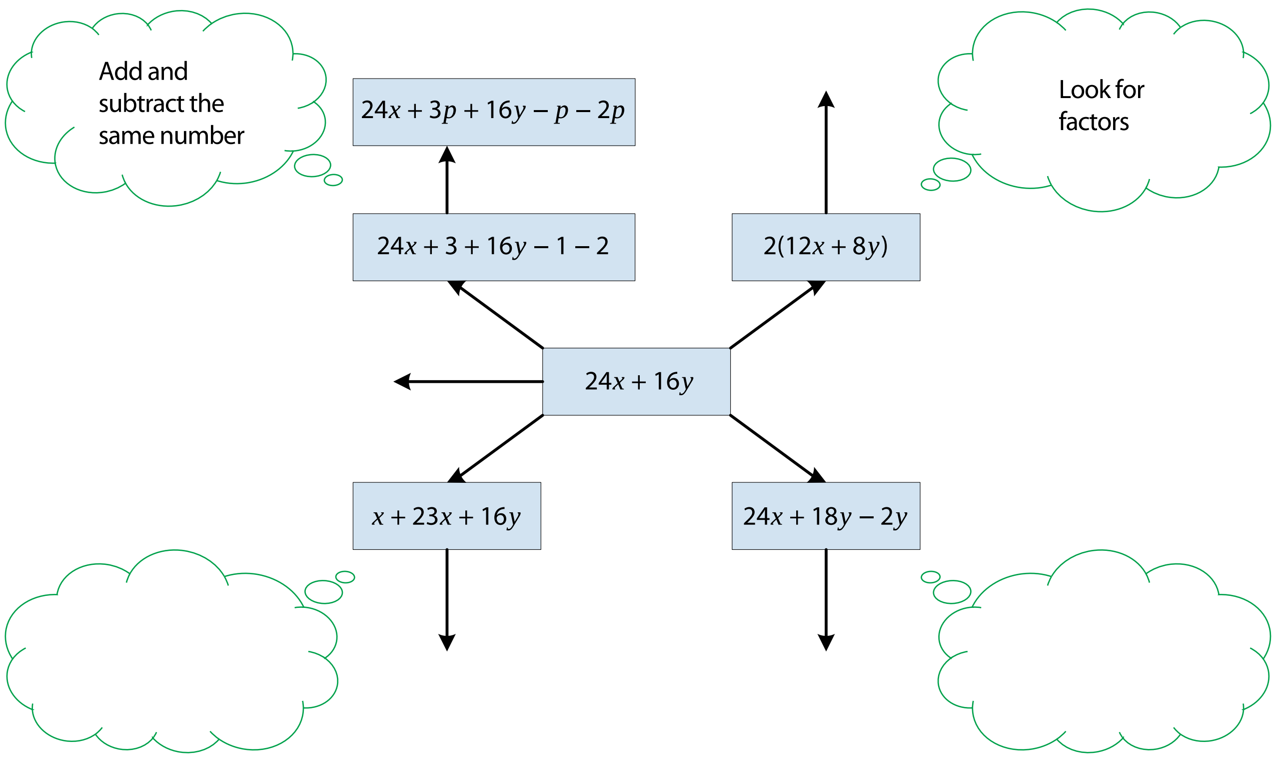
- Where could 'clouding the picture' activities for creating and solving equations fit into your mathematics lessons for the classes that you teach? To help, you may wish to refer to your copy of the algebra strand of *The Mathematics overview and learning objectives* (see Resources).
- What action, if any, do you need to take?

You can find more examples of 'clouding the picture' tree structures for pupils to complete on pages 22–24 of *Teaching Mental Mathematics from Level 5: Algebra* (see Resources).

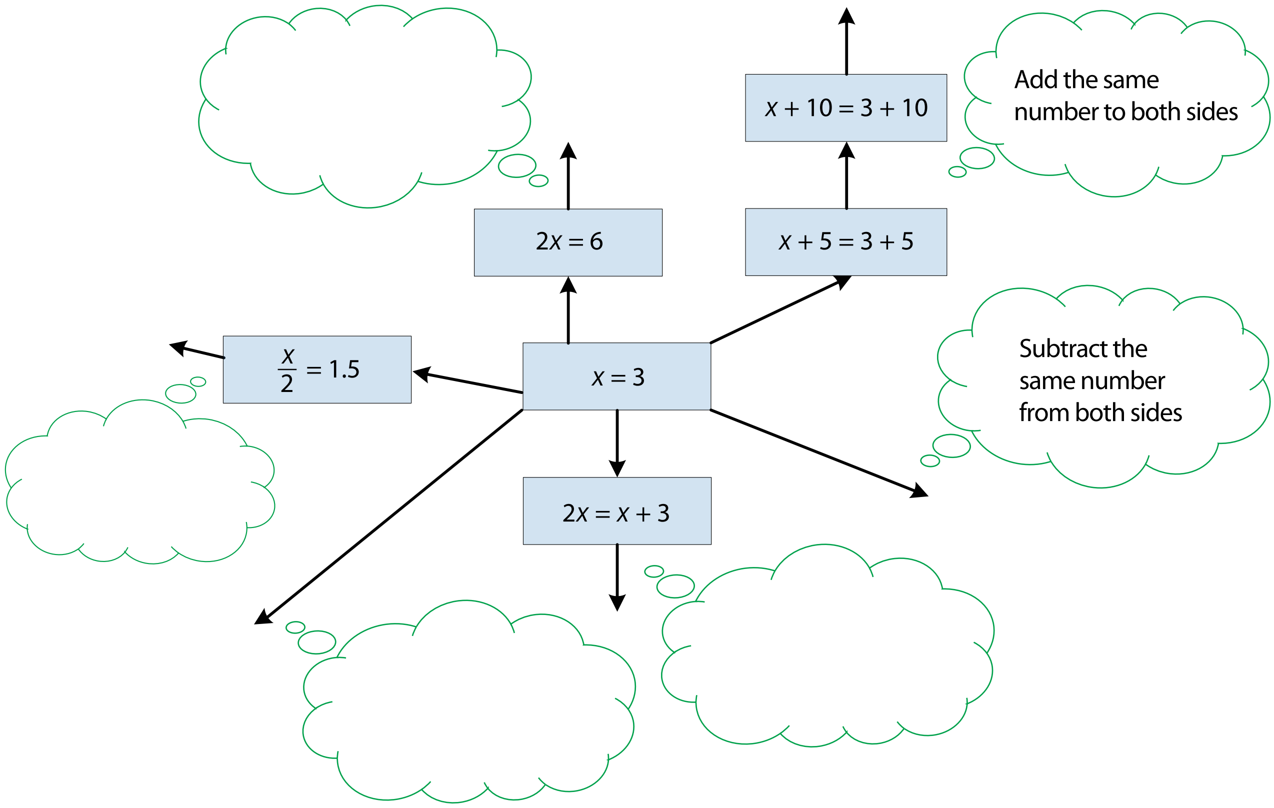
Finally, jot down any points to follow up and, if you are studying alone, any points that you want to discuss with your head of department or other colleagues.

Resource 2a : Equivalent expressions





Resource 2b: Transforming equations



Resource 2c: Changing the rule at each step

Diagram 1

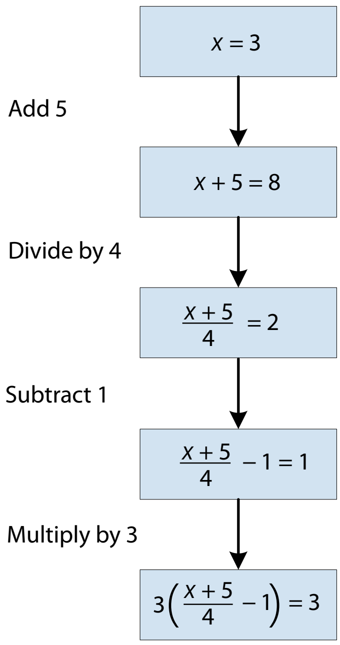
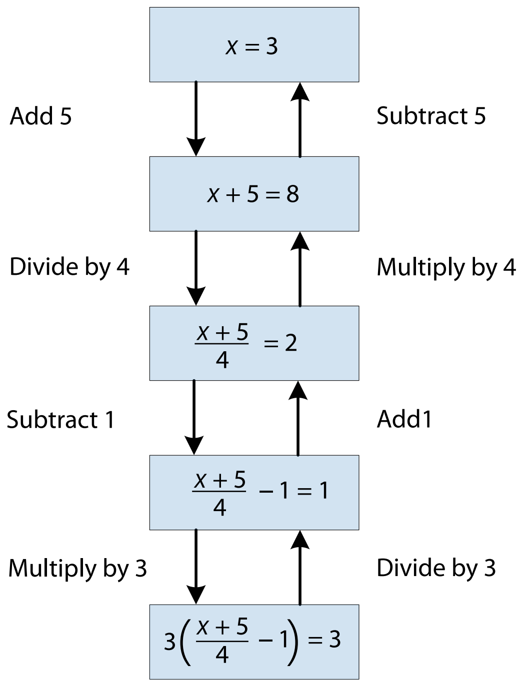
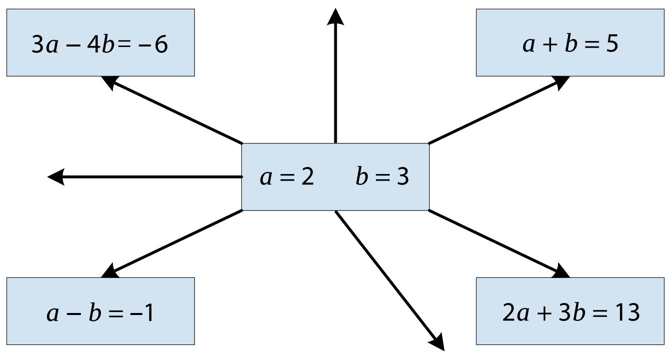


Diagram 2

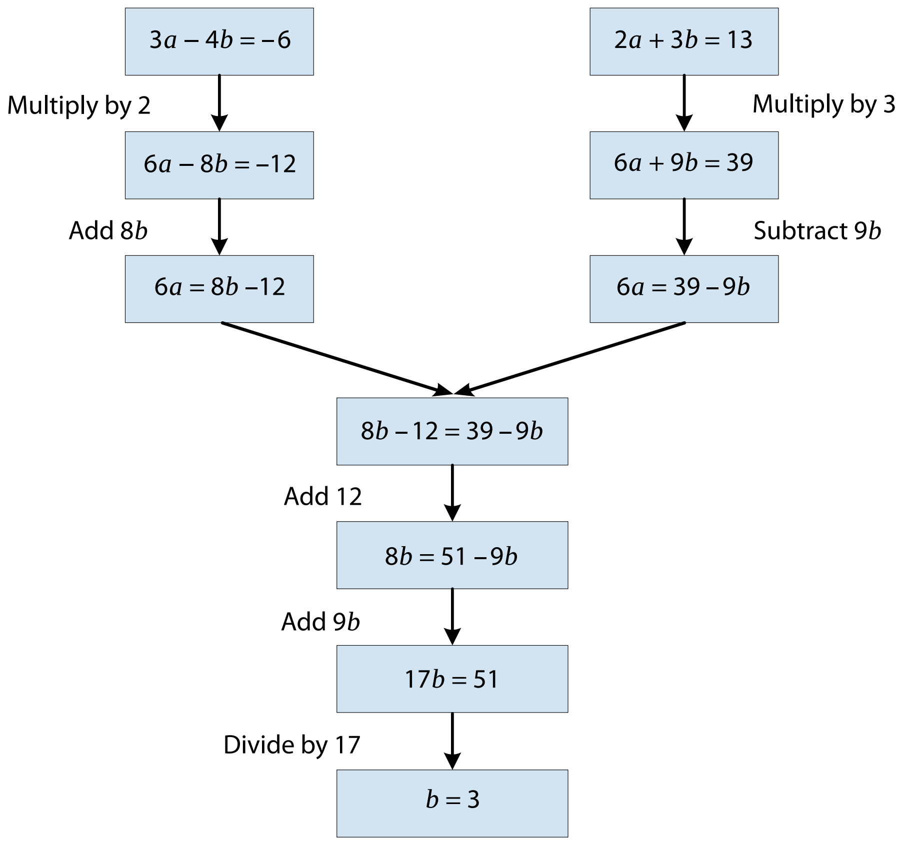


Resource 2d: Simultaneous equations

Generating simultaneous equations



Solving simultaneous equations



Substituting $b = 3$ in $2a + 3b = 13$ gives $2a + 9 = 13$, i.e. $2a = 4$ or $a = 2$

Solution $a = 2$, $b = 3$

Check by substituting $a = 2$, $b = 3$ in $3a - 4b = -6$, which gives $6 - 12 = -6$

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