The National Strategies
Secondary

Secondary mathematics algebra study units

Unit 1: Using a grid method to multiply expressions

department for
children, schools and families

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Description

This unit is for individual teachers or groups of teachers in secondary schools who are considering their teaching of algebra. It discusses ways of using a grid method to help pupils to multiply a single term over brackets and to find the product of two linear expressions.

Other units which could be combined with this one, either to create a longer session, or to work through in a sequence over time, are:

- Unit 5: Collecting like terms
- Unit 10: Classroom approaches to algebra.

Study time

About 40 minutes

Resources

Each teacher or pair of teachers working together will need:

- a personal notepad
- copies of **Resources 1a**, **1b**, **1c** and **1d** (which can be found at the end of this unit)
- a copy of the algebra pages from *The Mathematics overview and learning objectives* PDF, which you can download in A3 or A4 from the Framework for secondary mathematics at **www.standards.dcsf.gov.uk/nationalstrategies**. Search for the title: 'Mathematics learning objectives'.

Using a grid method to multiply expressions

1. The commutative, associative and distributive laws underpin strategies for calculation and, later on, algebraic ideas.

   In the grid method for the multiplication of numbers, which uses the distributive law, each number is partitioned. Each part of the first number is multiplied by each part of the second number, and the products are added to find their total.

   For example:

$$\begin{aligned} 37 \times 24 &= (30 + 7) \times (20 + 4) \\ &= (30 \times 20) + (30 \times 4) + (7 \times 20) + (7 \times 4) \\ &= 600 + 120 + 140 + 28 \\ &= 888 \end{aligned}$$

×	20	4	
30	600	120	720
7	140	28	168
			888

   The grid method for multiplying numbers can be generalised to introduce and develop the multiplication of a single algebraic term over brackets, or the expansion of a pair of brackets. The result is an expression in a simplified equivalent form, with the equals sign taking on the meaning 'is the same as'.

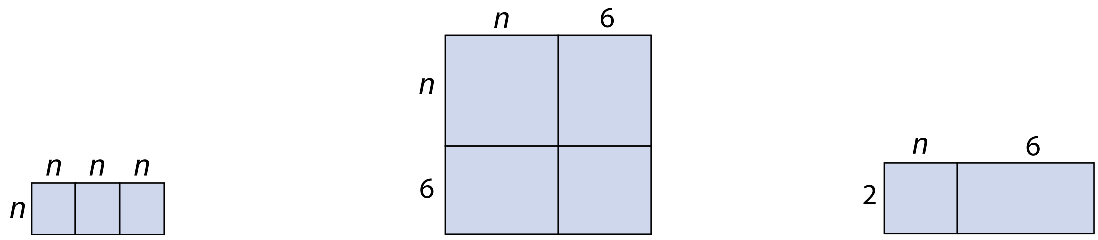
   The accuracy of the result can be checked by substituting, say, integer values for the variables into the original expression and the product when multiplied out.

2. Before they begin to multiply algebraic expressions, it is helpful if pupils have explored rectangular areas by partitioning the sides of the rectangles in various ways.

   Look at the example on **Resource 1a: Rectangular areas**, then work through the two problems below it.

   The two problems, which are based on 'working backwards', are useful introductions when pupils come to factorising linear and, later, quadratic expressions.

   Work with areas need not be constrained to examples with numbers. For example, you could create sets of cards for pupils to match, writing expressions such as $2n + 12$ or $3n^2$ or $(n + 6)^2$ on one set of cards, and drawing diagrams such as those below on the other set of cards.



   Now try the problems on **Resource 1b: More work with areas**.

3. A simple multiplication grid can be used to introduce multiplying an expression in brackets by a single term.
   Read **Resource 1c: Multiplying an expression in brackets by a single term**, and complete the questions in the factorising section.

   Pupils can find the method of factorising shown on **Resource 1c** easier than other methods, as it uses multiplication, albeit in reverse, rather than division.

4. A similar approach can be used to help pupils to find the product of two linear expressions and to prepare them for factorising quadratic expressions. Read **Resource 1d: Finding the product of two linear expressions**, and complete the questions in the factorising section.

   Examples **a**, **b** and **c** in **Resource 1d** demonstrate how the grid structure can support pupils' understanding of factorisation of quadratic expressions and, in particular, how the middle term needs to be split.

5. The advantages of using the grid method to multiply a pair of brackets are that:
   - the method links to a successful one used by pupils when they multiply numbers
   - links are developed between multiplication and division, and expansion and factorisation.

   Now consider the two questions below. You may wish to refer to your copy of the algebra strand of *The Mathematics overview and learning objectives* (see Resources).

   - Where could activities based on a multiplication grid, as in **Resources 1a**, **1b**, and **1c**, fit into mathematics lessons in Years 7 to 11? Pick out the learning objectives that could be supported in this way.
   - Do the mathematics textbooks that you currently use support the same approach as recommended in this unit?

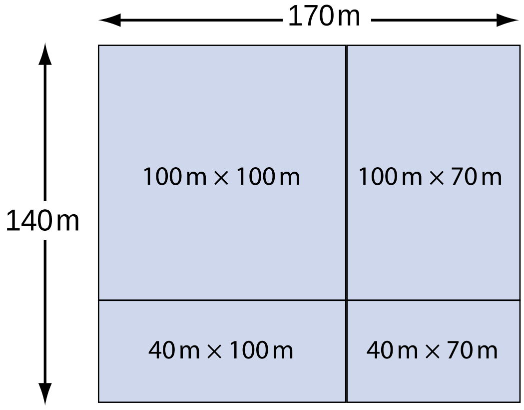
   If you are working with colleagues, first discuss the questions, in pairs or small groups, then in the whole group. If you are working alone, think about the questions and make notes on the answers in your notepad.

   To finish, consider and jot down any action that you need to take and, if you are working alone, any points that you want to discuss with your head of department or other colleagues.

Resource 1a: Rectangular areas

Example

Find the area of a large rectangular field by partitioning it into smaller areas.



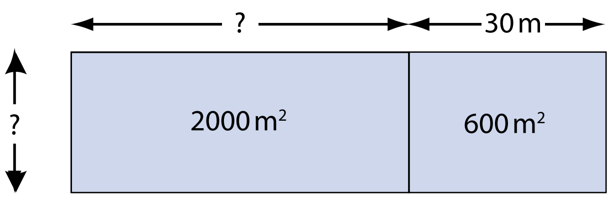
×	100	70	
100	10 000	7 000	17 000
40	4 000	2 800	6 800
			23 800

- What is the total area of the large field?
- Find three other ways to partition the sides of the large field.
  Is the total area of the large field always the same?

Rectangular field problems can be made easier or, like the ones below, more challenging.

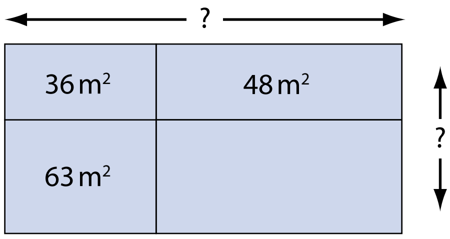
Problems

1. Find the unknown distances.



2. A gardener divides a large rectangular vegetable patch into four smaller rectangular vegetable patches.

   Three of the smaller patches have areas as shown on the diagram.



   What are the length and width of the large vegetable patch?

Resource 1b: More work with areas

Draw an area to match each expression.
For each area, write a different expression that gives the same area.

1.

a. $3(a + 5)$	**c.** $(b + 5)^2$
b. $(4n)^2$	**d.** $\frac{m + 8}{2}$

2. What rules have you found for rearranging expressions?

3. Now draw diagrams that show these expressions.

a. $2(p - 3)$	**c.** $y^2 - 16$
b. $(x + 5)(x - 5)$	**d.** $n^2 - 8n + 16$

Resource 1c: Multiplying an expression in brackets by a single term

Use a simple multiplication grid to introduce multiplying a bracket by a single term.

For example:

$\times$	10	4	
7	70	28	98

$$\begin{aligned} 7 \times 14 &= 7(10 + 4) \\ &= 7 \times 10 + 7 \times 4 \\ &= 70 + 28 \\ &= 98 \end{aligned}$$

- Will it work if 14 is partitioned in a different way?

$\times$	8	6	
7	56	42	98

$$\begin{aligned} 7 \times 14 &= 7(8 + 6) \\ &= 7 \times 8 + 7 \times 6 \\ &= 56 + 42 \\ &= 98 \end{aligned}$$

- Can you find other ways of doing it? Which are easy? Which are difficult? Which is the best one to use? Why?

Now generalise:

$\times$	x	4	
7	$7x$	28	$7x + 28$

$$7(x + 4) = 7x + 28$$

The result is the expression $7(x + 4)$ in the equivalent simplified form $7x + 28$.

- Is it true when $x = 10$? When $x = 8$? What must you do to check?

Now generalise further:

$\times$	b	c	
a	ab	ac	$ab + ac$

$$\begin{aligned} a(b + c) &= a \times b + a \times c \\ &= ab + ac \end{aligned}$$

- Is it true when $a = 7$, $b = 10$ and $c = 4$? Is it true for other values of a, b and c?

Now try **factorising**, or working the problem backwards, starting again with numbers, as in examples **a** and **b**, then generalising, as in examples **c** and **d**.

a. Find the missing numbers

$\times$	100	?
?	600	48

b. What could the missing numbers be?

$\times$	?	?
?	600	48

Are there other possibilities?

c. $6a + 8 = \bigcirc(3a + \square)$.
Find the missing terms $\bigcirc$ and $\square$.

$\times$	$3a$	$\square$
$\bigcirc$	$6a$	8

d. $8n + 24 = \bigcirc(\diamond + \square)$.
What could $\bigcirc$, $\diamond$ and $\square$ be?

$\times$	$\diamond$	$\square$
$\bigcirc$	$8n$	24

Are there other possibilities?

Resource 1d: Product of two linear expressions

Begin by considering how a numerical product such as 23 × 25 is calculated.

×	20	3	
20	400	60	460
5	100	15	115
			575

$$
\begin{aligned}
23 \times 25 &= (20 + 3)(20 + 5)\\
&= 400 + 100 + 60 + 15\\
&= 575
\end{aligned}
$$

Now consider an algebraic product.

×	a	3
a	a^2	$3a$
5	$5a$	15

$$
\begin{aligned}
(a + 3)(a + 5) &= a^2 + 5a + 3a + 15\\
&= a^2 + 8a + 15
\end{aligned}
$$

- How could you check that this is correct? [Substitute a value for a.]
- How could you use this algebra to calculate the value of 15 × 17?
  What value would you need to substitute for a?
  What other numerical products could you calculate, using this algebra?

The next step is to extend, in a similar way, to products such as $(2a + 6b)(8a - 5b)$.

Prepare for **factorising** by working the problem backwards, starting again with numbers, as in example **a**, then generalising, as in examples **b**, **c** and **d**.

a. Find the missing numbers.

×	100	?
100	?	?
?	2000	1000

In questions **b**, **c** and **d**, find the missing terms ○ and □, and fill in the empty boxes.

b. $3x^2 + 10x - 8 = (\bigcirc - 2)(x + \square)$

×	○	−2
x	$3x^2$	
□		−8

c. $10x^2 + 9xy + 2y^2 = (\bigcirc + y)(5x + \square)$

$\times$	$\bigcirc$	y
$5x$	$10x^2$	
$\square$		$2y^2$

d. $a^2 + 4ab - 12b^2 = (a + \bigcirc)(a + \square)$

$\times$	a	$\bigcirc$
a	a^2	
$\square$		$-12b^2$

Audience: Local authority staff, National Strategies consultants, secondary mathematics subject leaders, secondary mathematics teachers
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