The National Strategies
Secondary

Secondary mathematics algebra study units

Unit 9: Linking sequences, functions and graphs

department for
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Description

This unit is for individual teachers or groups of teachers in secondary schools who are considering their teaching of algebra. It discusses some stimulating activities to help pupils to link sequences, functions, and graphs.

Other activities that could be combined with this one, either to create a longer session or to work through in a sequence over time, are:

- Unit 8: Generalising from sequences and patterns
- Unit 10: Classroom approaches to algebra.

Study time

About 40 minutes

Resources

Each teacher or pair of teachers working together will need:

- a personal notepad
- copies of **Resources 9a**, **9b** and **9c** (which can be found at the end of this unit)
- a copy of the algebra pages from *The Mathematics overview and learning objectives* PDF, which you can download in A3 or A4 from the Framework for secondary mathematics at www.standards.dcsf.gov.uk/nationalstrategies. Search for the title: 'Mathematics learning objectives'.

Linking sequences, functions and graphs

1. Connections between sequences, functions, and graphs are sometimes given insufficient emphasis in mathematics lessons.

   Pupils are often introduced to functions through 'number machines' or 'function machines'. In this first example the rule is given, along with some input numbers. Pupils soon learn to work backwards intuitively from the output numbers.

   in → [multiply by 4 then add 1] → out

in	1	7	200	2.5			–2
out					121	2	

   In Year 7 pupils are expected to begin to use algebraic notation.

   $n \longrightarrow 4n + 1$ or $y = 4x + 1$

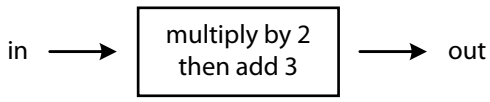
   The mapping can be separated out into two single operation machines.

   x → [$\times 4$] → [$+ 1$] → y

   By Year 8, pupils should be able to transform this, using inverse operations.

   x ← [$\div 4$] ← [$- 1$] ← y or: $x = \frac{y - 1}{4}$

This links closely to the second form of machine, which is of the 'What went in?' type.



in							
out	7	25	3	55	863	3.1	2

The function for this machine is $y = 2x + 3$ and its inverse is $x = \frac{1}{2}(y - 3)$.

Both types of machine, once introduced, can provide useful number practice in an oral and mental activity. The examples show how operations on different numbers can be targeted for practice, and so questions can be matched to the stage of development of individual pupils.

A third type of number machine has input and corresponding output numbers and the function has to be found.

in	3	4	6	10	1.7		−5
out	9	13	21	37		0	

In this machine, the function is $y = 4x - 3$ and $x = \frac{1}{4}(y + 3)$.

- What number is unchanged by this machine? Or, to put this question another way, for what number are the input and output numbers the same? What are the 'stay the same' numbers for the first two types of number machines?

A possible investigation for pupils is to explore 'stay the same' numbers for different number machines.

2. Another context in which pupils will meet mappings such as these is in work on number sequences. Work on number sequences will have started already in Key Stage 2. For reference you may find it useful to look at the questions in **Resource 9a: Typical questions**. These are similar to the kind of work done on sequences in Years 5 and 6 and questions in the Key Stage 2 tests. Consider for a moment how you build on this work in Years 7 and 8.

Now look at the number sequence in **Resource 9b: Squares in a cross**, and answer the accompanying questions.

3. The function related to *Squares in a cross* is represented by the equation $y = 4x + 1$.

The large numbers in the table are beyond pupils' capacity to draw and count. This encourages them to move from particular cases to the general. Pupils should first be asked to find the rule in words, to provide a step towards the algebra, and then to suggest a way of checking its accuracy.

Two levels of generalisation emerge from these types of spatial pattern. For pupils, it is easier to spot links between successive terms, for example, 'It goes up in fours', than to relate a term to its position in the sequence.

It helps pupils if you encourage them to justify and explain why a rule or relationship works in the context of the situation, relating back to the diagrams and not just to the pattern of numbers. One way of doing this is to ask them to calculate particular terms in the sequence. For example, in *Squares in a cross*, the 10th cross needs $10 \times 4 + 1$, or 41 squares, and the 100th cross needs $100 \times 4 + 1 = 401$ squares. From the particular examples, they are able to see that, in general, $y = 4x + 1$.

4. With number sequences based on spatial patterns like these, the variables have whole-number values only. In a true algebraic relationship, the variables can take any values on a continuous scale. Graphs of number patterns should really be a set of separate points, but to look at the algebraic relationship we usually join the points as though they represent a continuous function.

Look at **Resource 9c: Graphs of linear functions**. One of the graphs represents the function $y = 4x + 1$. Identify which graph it is and then find the equations of the other graphs.

5. Pupils in Year 7 are expected to draw graphs such as those on **Resource 9b**. In Year 8, pupils are introduced to ideas of gradient and intercept.

Compare your answers to the questions on **Resource 9b** with those below.

A $x + y = 4$

B $y = 4x + 1$

C $y = 4x - 3$

D $y = \frac{1}{2}x + 3$

E $y = 2x + 3$

The point at which the input value x for a function equals the output value y (the 'stay the same' number for the function) is the point where $y = x$. The point at which the graph of $y = x$ intersects with the graph of a function gives the 'stay the same' number for that function.

Linking elements of algebra together is part of the algebraic reasoning that needs to be developed throughout Key Stage 3. Pupils need to gain insight into the power and purpose of algebra as well as learning algebraic techniques.

6. Understanding the links between sequences, functions, and graphs is a cornerstone of mathematics in Key Stages 3 and 4. Teaching has to help pupils to appreciate that algebra allows them to represent and explore general relationships, and that this is more powerful than looking only at specific cases.

Pupils need opportunities to use their algebraic skills in problem solving to:

- increase their awareness of when and how algebra can be useful
- improve their knowledge of algebraic conventions
- deepen their understanding of algebraic rules
- practise their use of algebraic techniques

but most importantly they need opportunities to:

- see how algebra can provide insights into the underlying situation that the algebra is modelling.

7. Now think about these questions, or discuss them with colleagues in your group.

- What opportunities do you currently provide for linking work on sequences, functions, and graphs in mathematics lessons? Give some examples.
- What other opportunities could you provide for linking work on sequences, functions, and graphs? You may wish to refer to the revised objectives for mathematics in Years 7 to 11 produced by the Secondary Strategy (2010).

Note these examples on your notepad, together with points that you need to follow up and (or) discuss with colleagues.

8. You may wish to follow this unit by reading and using lesson A1 from:

- *Improving learning in mathematics: Mostly algebra* (sessions A1–A14).

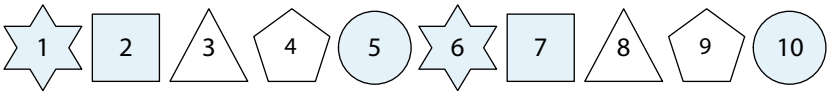
The materials, developed through national trials, provide interactive and lively resources for teaching and learning mathematics. They can be downloaded from the Learning and Skills Improvement Service (LSIS) Excellence Gateway website at **tlp.excellencegateway.org.uk/teachingandlearning/downloads/default.aspx#math_learning_PDFbinder**

Resource 9a: Typical questions

1. **Level 4**

Here is a repeating pattern of shapes.

Each shape is numbered.



The pattern continues in the same way.

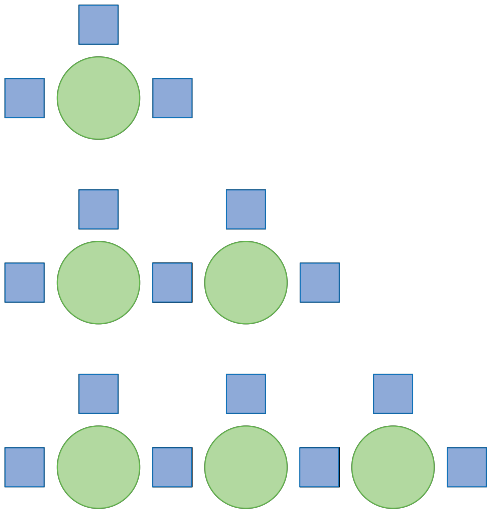
Write the numbers of the next two squares in the pattern.

Complete this sentence.

Shape number 45 will be a star because ...

2. **Level 5**

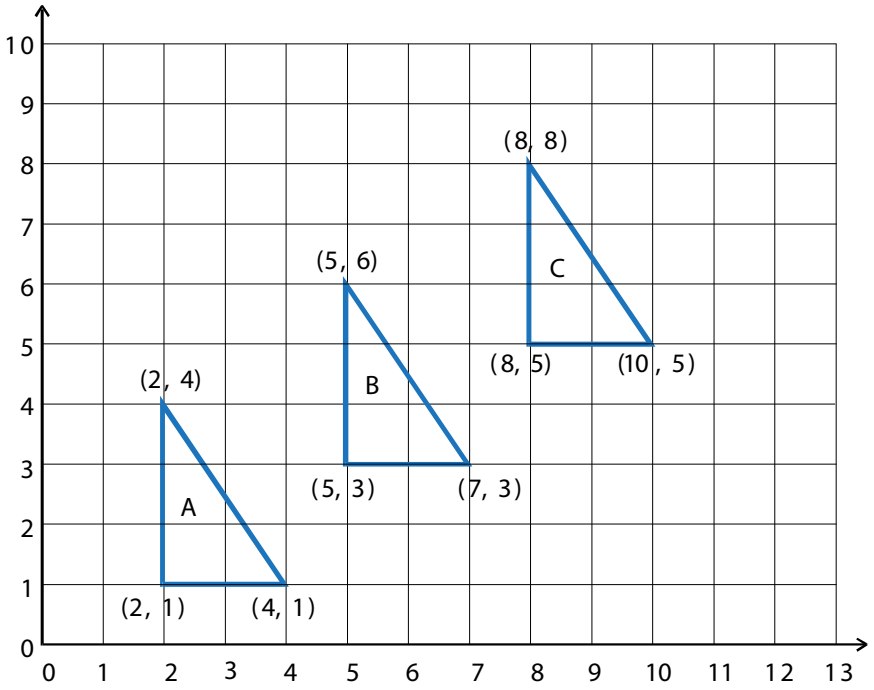
Here is a sequence of patterns made from squares and circles.



Number of circles	Number of squares
1	3
2	5
3	7

The sequence continues in the same way.
Calculate how many circles there will be in the pattern that has 25 squares.

3. Level 4

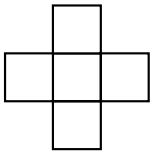


Write the coordinates of the next triangle in the sequence.

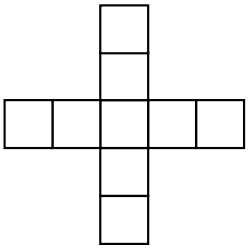
Resource 9b: Squares in a cross

Fill in the missing values in the table by studying the patterns.

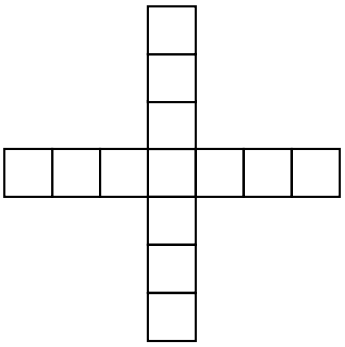
Pattern 1



Pattern 2



Pattern 3



Pattern number	x	1	2	3	4	5	10	50		200
Number of squares	y								1001	

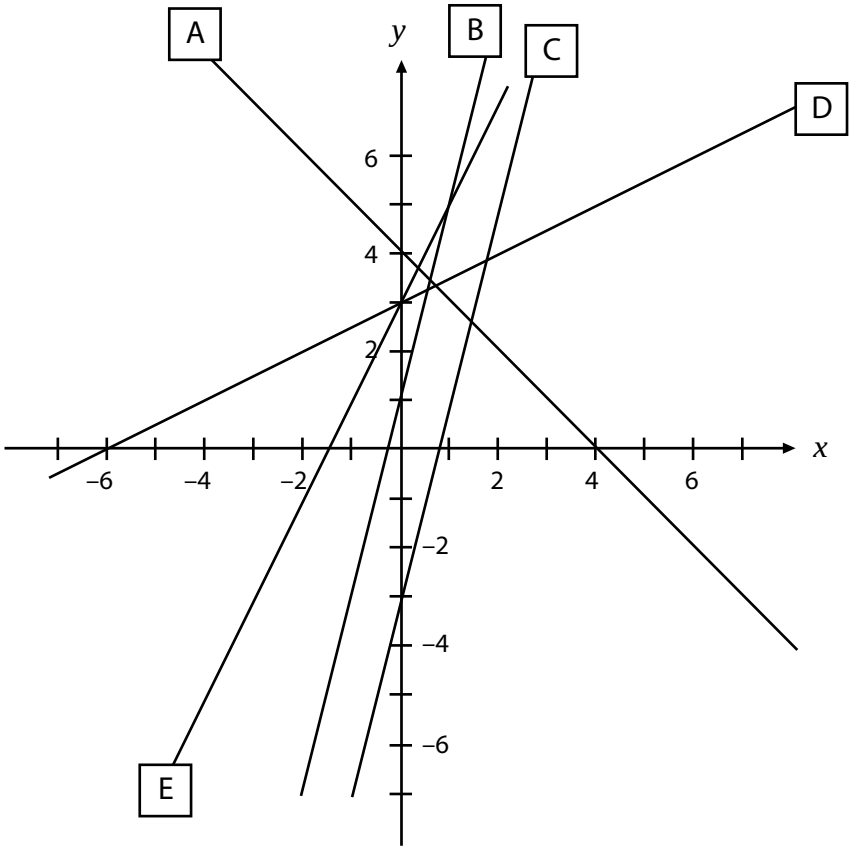
What is the rule that links the number of squares (y) in any cross to its position in the sequence (x)?

Explain why this is the rule by referring to the properties of the shapes.

Resource 9c: Graphs of linear functions

1. Which line represents the function $y = 4x + 1$?

A B C D E



2. Find the equations of the other lines.

Line	Equation
A	
B	
C	
D	
E	

3. What other graph would you need to draw so that the point of intersection with each line would be the 'stay the same' number for that function?

Audience: Local authority staff, National Strategies consultants, secondary mathematics subject leaders, secondary mathematics teachers
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