The National Strategies
Secondary

Secondary mathematics algebra study units

Unit 8: Generalising from patterns and sequences

department for
children, schools and families

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Description

This unit is for individual teachers or groups of teachers in secondary schools who are considering their teaching of algebra. It examines some ways of using patterns and sequences to help pupils to generalise.

Other activities that could be combined with this one, either to create a longer session or to work through in a sequence over time, are:

- Unit 9: Linking sequences, functions and graphs
- Unit 10: Classroom approaches to algebra.

Study time

About 40 minutes

Resources

Each teacher or pair of teachers working together will need:

- a personal notepad
- copies of **Resources 8a**, **8b** and **8c** (which can be found at the end of this unit)
- a copy of the algebra pages from *The Mathematics overview and learning objectives* PDF, which you can download in A3 or A4 from the Framework for secondary mathematics at **www.standards.dcsf.gov.uk/nationalstrategies**. Search for the title: 'Mathematics learning objectives'.

Generalising from patterns and sequences

1. You are probably familiar with the definitions of a sequence, a function, and a graph. By the end of Key Stage 3, pupils need to be aware of and understand these definitions. Check your understanding against the definitions on **Resource 8a: Definitions**.
2. Pupils have been developing their ideas about pattern in number throughout Key Stages 1 and 2. One aspect of this work relates to number properties and sequences, such as:
   - one more than a multiple of 3
   - numbers based on spatial patterns, such as square numbers and triangular numbers
   - Fibonacci numbers.

   In the problems on **Resource 8b: Fibonacci chains**, the number sequences have similar properties to the Fibonacci sequence – that is, each term is the sum of the previous two terms. Solve the problems and make a few notes on a method that can be used to solve them.
3. While the answer to the first problem on **Resource 8b** is easy to spot, you probably used algebraic methods to solve the other problems.

   All the sequences can be generalised in this form:

   $a,\ b,\ a+b,\ a+2b,\ 2a+3b,\ 3a+5b,\ 5a+8b,\ \ldots,$

   where a is the first term and b is the second term. This allows the following equations to be formulated and solved.

$6 + 3b = 18$	3	4	7	11	18		
$5.66 + 3b = 25.91$	2.83	6.75	9.58	16.33	25.91		
$12 + 5b = 36$	4	4.8	8.8	13.6	22.4	36	
$30 + 8b = 4$	6	–3.25	2.75	–0.5	2.25	1.75	4

The four examples show that changing the first and last terms in sequences of this type alters the level of difficulty, so that an activity for pupils that is based on this task can readily be made harder or easier. For example, the first problem on **Resource 8b** would be suitable for pupils working confidently at level 4, whereas the other three problems are more suitable for pupils working at level 5 or level 6.

4. In the Fibonacci sequence problems, it was possible to express each of the sequences in a general form by using letters to stand for numbers.

One way of introducing pupils to algebraic generalisation is to ask them to extend number patterns. Answer the questions on **Resource 8c: generalising**, which are typical of the problems that can be given to pupils working at level 5.

5. In the first problem on **Resource 8c**, you have probably described the nth line in the pattern as:

$$(n - 1)(n + 1) = n^2 - 1$$

or as:

$$n(n + 2) = (n + 1)^2 - 1$$

The pattern can be extended backwards to explore multiplication of negative numbers, since any integer, positive or negative, can be substituted for n.

$1 \times 3 = 2^2 - 1$
$0 \times 2 = 1^2 - 1$
$(-1) \times 1 = 0^2 - 1$
$(-2) \times 0 = (-1)^2 - 1$
$(-3) \times (-1) = (-2)^2 - 1$

- What happens if fractional or decimal values are substituted for n?
  Is it still the case that $(n - 1)(n + 1) = n^2 - 1$?

An equation such as $n^2 - 1 = (n - 1)(n + 1)$ that holds true for all possible values of the variables is called
an identity.

The second problem on **Resource 8c** has a connection with the first problem, in that each of the four numbers 899, 3599, 10 403, 359 999 is 1 less than a perfect square. It can therefore be expressed in the form $n^2 - 1$, which factorises as $(n - 1)(n + 1)$. This helps to find the solutions:

$899 = 30^2 - 1 = 29 \times 31$
$3599 = 60^2 - 1 = 59 \times 61$
$10\,403 = 102^2 - 1 = 101 \times 103$
$359\,999 = 600^2 - 1 = 599 \times 601$

- What happens with other values? Check that:

  $(5.816)^2 - 1 = (5.816 - 1) \times (5.816 + 1) = 4.816 \times 6.816$

  and that:

  $(-15.216) \times (-13.216) + 1 = (-14.216)^2$

6. Giving pupils opportunities to explore generalisations like these helps them to develop an understanding of the power of algebra.

   Now consider these questions.

   - In which year groups do pupils in your school learn to generate sequences, use term-to-term and position-to-term rules, and find the nth term of a sequence?
   - How does this compare with the revised mathematics learning objectives produced by the National Strategies?
   - What other opportunities to generalise do pupils have in their learning of number and algebra? In which year groups?
   - Are there ways of extending these opportunities?

7. To round off, reflect on these two questions.

   - What have you learned?
   - What action will you take as a result?

   Jot down any points that you want to follow up and any modifications that you will make to your planning or teaching.

   If you are studying alone, jot down any points that you want to discuss with your head of department or colleagues.

8. You may wish to follow up this unit by reading and using lesson A8 from:

   - *Improving learning in mathematics: Mostly algebra* (sessions A1–A14).

   The materials, developed through national trials, provide interactive and lively resources for teaching and learning mathematics. They can be downloaded from the Learning and Skills Improvement Service (LSIS) Excellence Gateway website at **tlp.excellencegateway.org.uk/teachingandlearning/downloads/default.aspx#math_learning_PDFbinder**

Resource 8a: definitions

Sequences

A **sequence** is an ordered succession of terms formed according to a rule. There can be a finite or infinite number of terms.

The sequences most commonly considered in mathematics in Key Stages 3 and 4 have:

- an identifiable mathematical relationship between the value of a term and its position in the sequence; and (or)
- an identifiable mathematical rule for generating the next term in the sequence from one or more existing terms.

Examples

The squares of the integers: 1, 4, 9, 16, 25, …
The Fibonacci sequence: 1, 1, 2, 3, 5, 8, …

Functions

A **function** is a rule that associates each term of one set of numbers with a unique term in a second set. The relationship can be written in different ways.

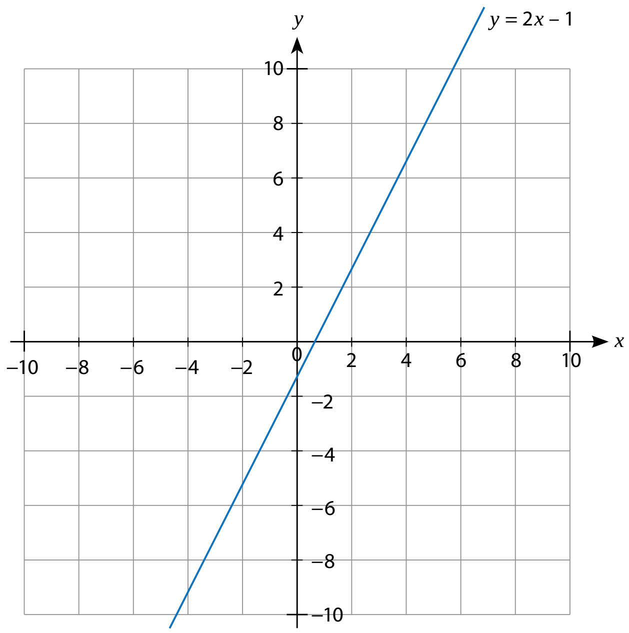
Examples

$x \rightarrow 2x - 1$
$y = 2x - 1$

Graphs

A **graph** of a function is a diagram that represents the relationship between two variables or sets of numbers.

Example



Resource 8b: Fibonacci chains

All these number chains have similar properties to the Fibonacci sequence – that is, each term is the sum of the previous two.

Find the missing terms.

3	…	…	…	18

2.83	…	…	…	25.91

4	…	…	…	…	36

6	…	…	…	…	…	4

Use this space for any working that you want to do.

Explain how to solve problems like these.

Resource 8c: Generalising

1. Consider this pattern:

   $1 \times 3 = 2^2 - 1$

   $2 \times 4 = 3^2 - 1$

   $3 \times 5 = 4^2 - 1$

   $4 \times 6 = 5^2 - 1$

   a. What will the next two lines be?

   b. What will the 10th line be?

   c. What will the 100th line be?

   d. If I wanted to know what a particular row will be, say the nth row, how could you tell me?

2. Find a pair of factors of:

   a. 899

   b. 3599

   c. 10 403

   d. 359 999

3. Make up two similar questions.

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