The National Strategies
Secondary

Secondary mathematics algebra study units

Unit 3: Constructing expressions and equations

department for
children, schools and families

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First published in 2010

Ref: 00138-2010PDF-EN-04

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Description

This unit is for individual teachers or groups of teachers in secondary schools who are considering their teaching of algebra. It discusses some helpful ways of teaching pupils to construct algebraic expressions, equations, and formulae, starting from numerical examples and building up to algebraic examples.

Other units that could be combined with this one, either to create a longer session, or to work through in a sequence over time, are:

- Unit 4: Rearranging equations and formulae
- Unit 10: Classroom approaches to algebra.

Study time

About 40 minutes

Resources

Each teacher or pair of teachers working together will need:

- a personal notepad
- copies of **Resources 3a** and **3b** (which can be found at the end of this unit)
- a copy of the algebra pages from *The Mathematics overview and learning objectives* PDF, which you can download in A3 or A4 from the Framework for secondary mathematics at **www.standards.dcsf.gov.uk/nationalstrategies**. Search for the title: 'Mathematics learning objectives'.

Constructing expressions and equations

1. The process of constructing expressions or equations is an essential first stage in using algebra to model situations.

   When pupils solve puzzles such as 'arithmagons' or 'pyramids' they form and solve equations to model the structure of the puzzle.

   Pupils also construct equations when they analyse real-life situations and develop and use formulae to represent the relationships between the variables involved.

   Both contexts provide opportunities for pupils to develop functional and other key process skills and to gain a sense of the power of algebra in solving problems. For example, they must be able to analyse the situation, select and explain variables from the context, and decide how best to represent the relationships between the variables. They will also use other higher-order thinking skills: interpreting, visualising, hypothesising, inferring, deducing, justifying and so on.

   Both puzzles and real-life scenarios can be selected or adapted to a level of difficulty suited to a particular class or year group. They can then be used to help pupils to construct expressions, equations, and formulae over a sequence of lessons.

2. When you are teaching pupils to form equations, expressions, and formulae, it is important to build on the knowledge and understanding that they already have of the rules of arithmetic.

   Give them opportunities to verbalise, starting with numerical examples. For example, when you ask a short question such as: 'What must I add to sixteen to make twenty-four?', encourage a response in the form of a complete statement: 'Sixteen plus eight equals twenty-four', and record the same equation, using symbols: $16 + 8 = 24$.

The progression into the use of algebra and algebraic notation could be:

sixteen add eight equals twenty-four (word statement)

$16 + 8 = 24$ (numerical statement)

$16 + \square = 24$ (using a symbol to represent the unknown)

$16 + n = 24$ (using a letter to stand for the unknown).

The next stage is to set into words some problems that require equations to be constructed, and ask pupils to consider and verbalise how they find the solution.

Jot down on your notepad two or three examples of word problems that pupils can solve, using one or two steps, without the use of formal algebraic methods.

Now jot down two or three more examples of word problems that can be solved more easily by formulating a linear equation in one variable, or a pair of simultaneous linear equations in two variables.

If you are working with colleagues, share your ideas with the group.

3. Here are some examples of simple problems that pupils can solve informally, explaining their methods to their peers.

- John has £5. He spends £2.47. How much change does he get?
- Cinema tickets for four adults cost £17.20.
  What is the cost for one adult?
- Add 3 to my number and then multiply the result by 5. The answer is 35.
  What is my number?

When they are presented with a sufficiently complex problem, then the need to model the problem using equations becomes clearer. For example:

- Zac is four years younger than Tom.
  In three years' time the sum of their ages will be 28.
  How old is Zac now?
- A rectangle is twice as long as it is wide.
  Its perimeter is 48 cm.
  What is the length of the rectangle?
- I think of a number, square it and then multiply the result by 2.
  I subtract 9 times the number and get the answer 5.
  What is my number?

4. A similar approach of starting with words and particular numbers, then using symbols, can be adopted when you ask pupils to express a functional relationship.

For example, you could start by describing a scenario in words.

Every metre of fabric costs £5.

The next step is to ask pupils to express this in numbers, using particular examples.

The cost of 4 metres of fabric is £$(5 \times 4) =$ £20.
The cost of 7 metres of fabric is £$(5 \times 7) =$ £35.

The final step is to get them to generalise from the particular examples and to use letter symbols to represent the variables.

The cost £C of x metres of fabric is given by $C = 5x$.

Where possible, ask pupils to construct formulae that model real-life situations. Then ask them to use the formulae by substituting given values for one of the variables to find unknown values of the other variable.

Jot down on your notepad some examples of real-life situations that can be represented by a linear function. The examples can come from other subjects, such as science, or from everyday life. For each example, write the equation or formula that represents the function, specifying what each variable represents.

5. Here are some examples of real-life situations that can be represented by a linear function.
   - When you go on holiday to New Zealand, you get 2 New Zealand dollars for every £1.
   - To change miles to kilometres, divide by 5 then multiply by 8.
   - Every time we added another 20 grams, the length of the elastic increased by 6 centimetres.
   - Roasting a turkey takes 20 minutes plus 35 minutes for each kilogram.
   - The cost of hiring a taxi is a fixed cost of £2 plus 80p per mile.
   - The monthly cost of local calls on a mobile phone is £7 per month plus 8p per call.
   - The length in centimetres of the metal strip used to make a horseshoe is twice the width of the horse's hoof plus 5 centimetres.
   - To change a temperature in degrees Celsius to degrees Fahrenheit, multiply by 1.8 and then add 32.

6. Sometimes an area of the mathematics curriculum provides a context for constructing a formula.

   Look at the example in **Resource 3a: Constructing formulae 1**.

   Having arrived at several different versions of a formula, for example, for the perimeter of a rectangle, pupils could then simplify each version to show that they are all equivalent. They could also discuss which version they would use as a formula, and why.

   Now work through the questions in **Resource 3b: Constructing formulae 2**.

   In question 1, by simplifying the expressions by multiplying out the brackets and collecting like terms, pupils could verify that they are all equivalent to $9n + 4$ and that they are different expressions for the total number of matchsticks. They can construct a formula relating the total number of matchsticks, T, to the number, n, of matchsticks in a row of the rectangle.

   $T = 9n + 4$

   They are now in a position to work out that a grid that is 4 matchsticks wide and that uses 130 matches altogether must be 14 matches long.

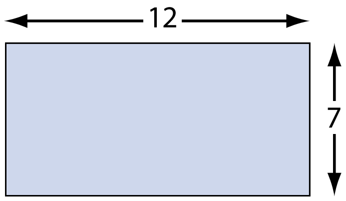
   In question 2, the number of circles in the nth shape can be expressed as $3n – 1$, while the number of squares is $2n + 1$, giving a total of $5n$. The best approach in the classroom would be as before: describing particular examples in words and numbers, and generalising from there. Substitution of 1, 2, 3 and 4 into the generalised formula will confirm that it works for the first four terms. The formula can then be used to find, say, the shape number for the shape that has a total of 175 squares and circles.

7. You may wish to follow up this unit by reading and using lessons A2 and A3 from the series *Improving learning in mathematics: Mostly algebra* (sessions A1–A14), which you can download from the Learning and Skills Improvement Service (LSIS) Excellence Gateway website at **tlp.excellencegateway.org.uk/teachingandlearning/downloads/default.aspx#math_learning_PDFbinder**

Resource 3a: Constructing formulae 1

Example

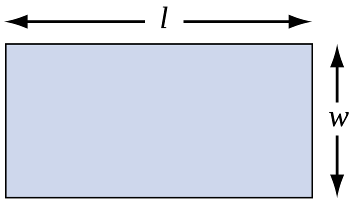
Here is an example of constructing a formula from an area of the mathematics curriculum, using the concept of perimeter. The approach remains the same: start with words and particular numbers, then generalise and express in symbols.



Ask pupils to express the perimeter of this rectangle in as many ways as they can.

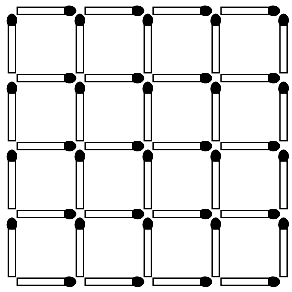
Perimeter = $12 + 7 + 12 + 7$	walking round the shape
Perimeter = $12 + 12 + 7 + 7$	pairing up lengths and widths
Perimeter = $12 \times 2 + 7 \times 2$	length repeated plus width repeated
Perimeter = $2 \times 12 + 2 \times 7$	two lots of length plus two lots of width
Perimeter = $(12 + 7) \times 2$	length plus width … repeated
Perimeter = $2 \times (12 + 7)$	two lots of … length plus width

Now generalise:



Perimeter = $l + w + l + w$	walking round the shape
Perimeter = $l + l + w + w$	pairing up lengths and widths
Perimeter = $l \times 2 + w \times 2$	length repeated plus width repeated
Perimeter = $2 \times l + 2 \times w$	two lots of length plus two lots of width
Perimeter = $(l + w) \times 2$	length plus width … repeated
Perimeter = $2 \times (l + w)$	two lots of … length plus width

Resource 3b: Constructing formulae 2



1a. Consider a 4 by 4 grid of squares from matchsticks.
How many matchsticks are needed?
Count systematically.
Find four different ways of counting.

b. How many matchsticks are needed for a 4 by n rectangular grid?
Find four different expressions.

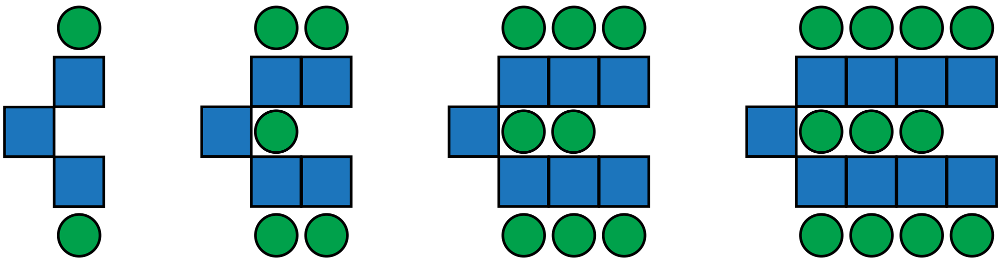
c. Check by substituting $n = 4$ that your expressions are correct for a 4 by 4 grid.

d. Write a formula connecting the total number of matchsticks T with the number n along the length of the rectangle.

e. A rectangular grid of matchsticks is 4 matchsticks wide and uses 130 matches altogether. How long is the grid?

f. How many matchsticks are needed for an m by n rectangular grid?

2a. Here is a sequence of shapes made from squares and circles.



shape number (n)	**1**	**2**	**3**	**4**

The sequence continues in the same way.

Write a formula for the total number of shapes in shape number n.

3. In the classroom, how would you suggest that pupils approach problem 2?

Audience: Local authority staff, National Strategies consultants, secondary mathematics subject leaders, secondary mathematics teachers
Date of issue: 03-2010
Ref: **00138-2010PDF-EN-04**

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