Durham Maths Mysteries

"Putting a little mystery into mathematics"

November 2005

Durham LA

The Mysteries

Index of mathematical mysteries and details of the mathematics needed to solve each of them:

Ratio and Proportion

Fractions and percentages of quantities; equivalence of fractions and percentages; multiples of ratios; average of two percentages.

Directed Numbers

Addition and subtraction of directed numbers, odd and even numbers (integers).

Algebra

Addition and subtraction of linear algebraic terms of the form $ax + b$; expansion of $c(ax + b)$ and factorisation of $acx + bc$.

Properties of Shape

Lines of symmetry; centres of rotation; properties of quadrilaterals; 'regular' shapes.
A set of 15 shape cards are available to provide visual support.

Locus

Constructions including right angles, parallel lines, perpendicular bisector of a line; loci of points which are equidistant from either a fixed point or a fixed line.

Probability

Probability line 0-1; knowledge of probability of certainty; vocabulary: 'evens chance'; likelihood and chance.

LiL 'RATIO AND PROPORTION' MYSTERY

Who should get the maths prize?

You may wish to draw up a table to show each pupil's marks and percentages on each paper and their average percentage over both papers. Hence, put the pupils in rank order.

The maths exam consists of 2 papers.	Adam got $^3/_4$ of the marks on Paper 1 but only 30% on Paper 2 when he forgot his calculator.	Brian scored 10 more marks on Paper 1 than he did on Paper 2.
Susie's average percentage on the two papers was 65%.	Cathy scored 62 marks on Paper 1 and got 60 marks more than Adam on Paper 2.	Brian got $^2/_5$ of the marks for Paper 2.
The ratio of Debbie's marks on Paper 1 to her marks on Paper 2 was 5:4.	Susie's percentage on Paper 1 was 15% higher than Adam's on the same paper.	Debbie's mark on Paper 1 was 10 less than Brian's on Paper 1.
There are 100 marks available on Paper 1 and 150 on Paper 2.	Pupils are ranked by their average percentage on the two papers.	

EXTENSION

Would the order change if pupils were ranked on total marks scored?

LiL 'DIRECTED NUMBERS' MYSTERY

There is a 3 x 3 grid of integers with one number in each grid square.

Use the cards to decide which number lies in each grid square.

No number is bigger than 6 or smaller than -5.	There are two equal positive numbers.	There are two equal negative numbers.
The largest number is in the centre square.	The sum of the totals of the three rows is zero.	The sum of the totals of the three columns is zero.
The middle row adds up to the same total as the diagonal from top right to bottom left.	The top row adds up to the same total as the diagonal from top left to bottom right.	The difference between the largest and smallest numbers is 11.
There are an equal number of positive and negative numbers.	The difference between the number in the centre square and that in the bottom right is 8.	The right hand column contains two numbers the same and adds up to -3.
The two 3s do not lie in the same row or column.	The bottom row contains no odd numbers.	Numbers in the top row are all different and are all odd.
The smallest number occupies a corner square.	The smallest number is opposite one of the two equal negative numbers.	

LiL	'ALGEBRA' MYSTERY

There is a 3 x 3 grid square with one linear algebraic expression of the form $ax + b$ in each square.

Use the cards to decide which expression lies in which square.

The right hand column adds up to $5x + 1$.	The expression in the middle of the bottom row is twice that in the top left corner.	The sum of the diagonal top left to bottom right is 3 less than that on the other diagonal.
The bottom row adds up to $5x + 16$.	Two of the cards in the right hand column add up to $2x + 1$.	The sum of the middle row is 8 times that of the card in the right hand column of that row.
The sum of the expressions in the left hand column is 4 times the expression in the bottom right hand corner.	The cards in the top right and bottom left squares are the only cards which do not have two terms.	The sum of the middle column is 10 times that of the card in the right hand column of the middle row.
The difference between the top two cards in the left hand column is 6.	The sum of the top row is 3 times that of the left hand card in the middle row.	The sum of the middle column is $10x - 10$.
The expression on the middle card is $4x - 3$ more than that to its right.		

LiL 'PROPERTIES OF SHAPE' MYSTERY

There is a 3 x 3 grid with one shape drawn in each grid square.
Use the cards to decide which shape is in which square.
Is your answer unique?

The shape in the top left hand corner has 3 lines of symmetry.	There is a square directly above the hexagon.	The shape to the left of the small square has 2 lines of symmetry and 4 right angles.
Two shapes each have 4 lines of symmetry.	Each row and column contains 2 quadrilaterals.	Each of the shapes in the top right and bottom left hand corners has one line of symmetry.
5 of the quadrilaterals include at least one pair of parallel sides.	One shape has no straight sides and one centre of rotation.	4 of the shapes are regular with straight sides.
5 shapes have all sides equal in length.	One of the shapes has one line of symmetry and its diagonals cut at 90º.	4 quadrilaterals have diagonals which cut at 90º.
Shapes in the middle column contain a total of 14 lines of symmetry.	Shapes in the middle row and in the right hand column contain an infinity of lines of symmetry.	No shape has more than 6 vertices.

EXTENSION

(i) Create an additional card to make your solution unique.
(ii) Replace one of the cards with one of your own. Does your new problem have a solution? Is it unique?
(iii) Design your own 3 x 3 shape grid and a set of cards.

LiL 'LOCUS' MYSTERY

You have the plan of an area of land centred on a four sided field reputed once to have belonged to a notorious highwayman.
Use the cards to construct the diagram and hence solve the puzzle of where to dig for the treasure.

CD is the longest side.	The right angle in quadrilateral **ABCD** is opposite the longest side.	Point **E** is a quarter way along **DC**.
AF is parallel to **DC**.	Points **L** and **M** lie on the line through **A** and **F** and are each 4cm from point **E**.	**F** lies on **BC**.
PQ bisects **EF** at **R**.	**AB** is 1½ times **AD**.	**BC** is 9cm long.
AD is 3cm long.	Angle **BAD** is obtuse.	The line through **D** and **A** meets **PQ** at **T**.
Point **L** is closer to **A** than is point **M**.	The point where the treasure is buried lies within 8cm of point **B**.	The treasure is closer to **D** than to **C**.
The treasure lies inside quadrilateral **ABCD**.	The treasure is buried at a labelled point.	**CE** is 3" long.

EXTENSION

Investigate what happens as you allow the length of **BC** to vary.

LiL 'PROBABILITY' MYSTERY

Six friends enter a race. Use the following cards to determine who is most likely to win the race and with what probability. In what sequence would you expect the runners to finish the race?

C is twice as likely to win as **B**.	The probability that ***A*** wins is equal to the sum of the probabilities that **F** or ***C*** win.	Two runners have a better than evens chance of winning.
Two runners have an equal but not very good chance of winning.	The probability that ***C*** wins is half the combined probability that **D** or **E** win.	The probability that **D** wins is half that of each of two other runners.
The chance that ***C*** wins is less likely than two other runners.	Runners **B**, **F** and ***A*** have a combined probability equal to that of certainty.	The least likely winner has a probability 0.6 smaller than the most likely winner.
Runner ***A*** is three times more likely to win than runner **B**.	Runner **F** has a probability of winning that is $^1/_3$ that of runner ***A***.	Each runner's probability of winning is a multiple of 0.1.
Runners ***A*** and ***C*** have a combined probability of 1.	Only one runner has a chance of winning greater than $^2/_3$.	

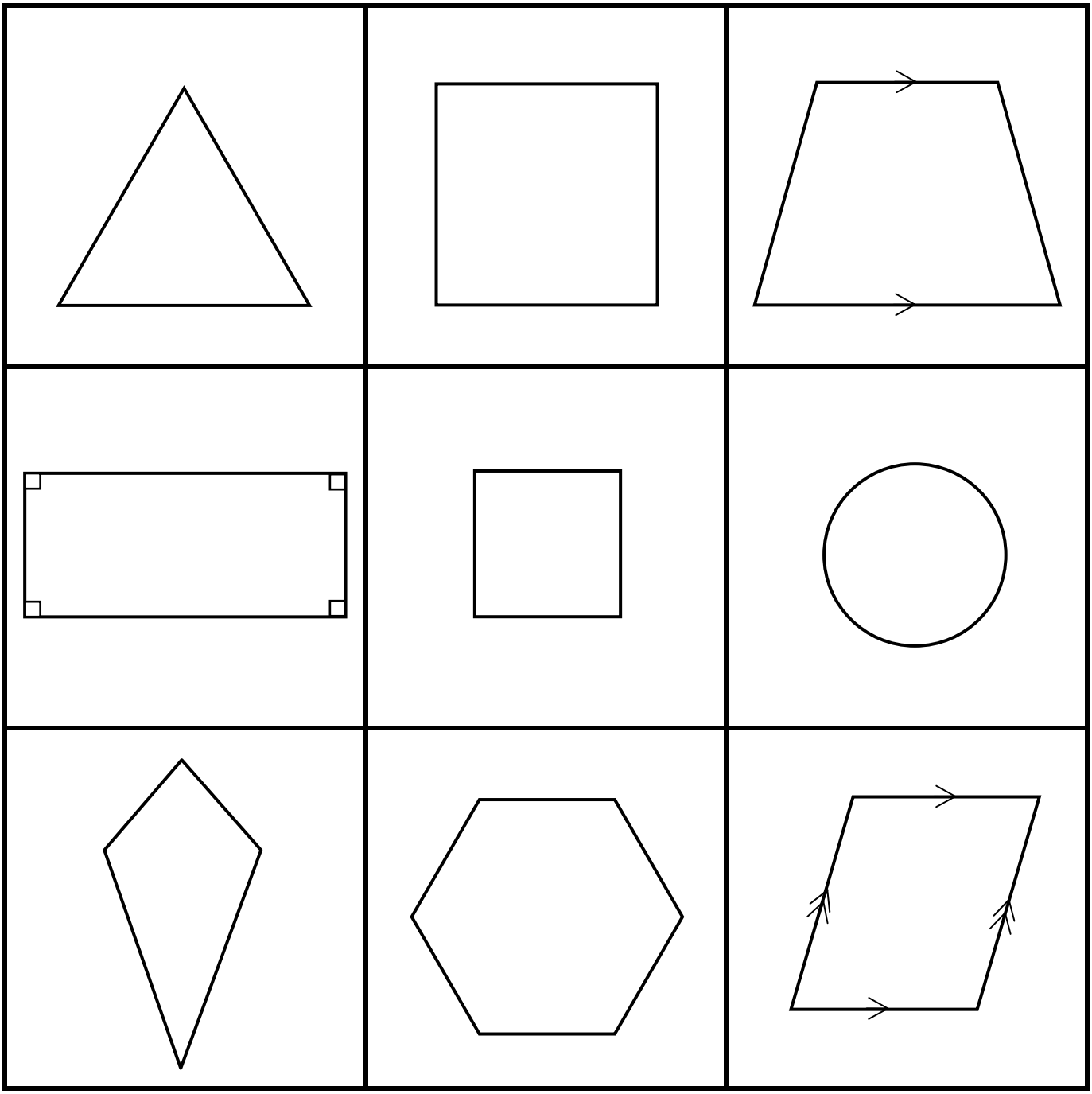
EXTENSION

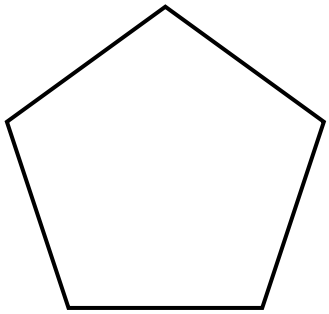
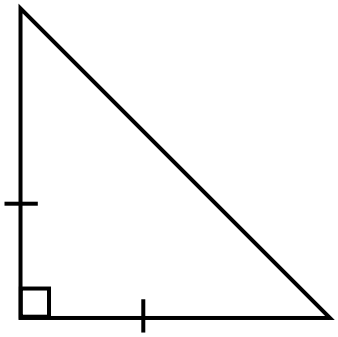
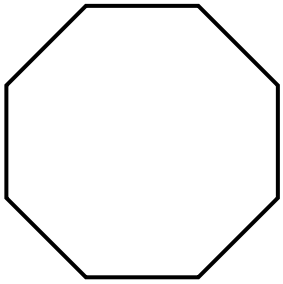
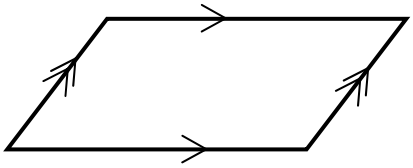
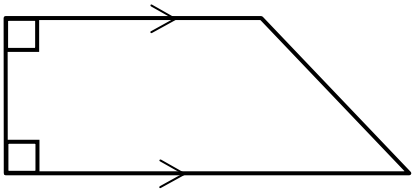
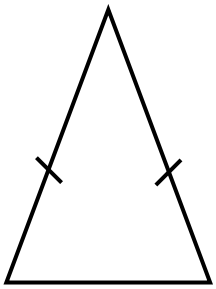
What is the smallest number of cards that you need to solve the problem? Which cards do you need?

-5	3	1
3	6	-2
0	-4	-2

$2x + 3$	$x - 12$	$3x$
$2x - 3$	$5x - 4$	$x - 1$
8	$4x + 6$	$x + 2$

$x - 10$	6	$5x - 13$



TEACHERS' NOTES

Ratio and Proportion

Pupil	*Paper 1*		*Paper 2*		*Average %*	*Rank by average %*	*Total mark*	*Rank by total number*
	mark	*%*	*mark*	*%*				
Adam	75	75	45	30	52.5%	4	120	4
Brian	70	70	60	40	55%	3	130	3
Cathy	62	62	105	70	66%	1	167	1
Debbie	60	60	48	32	46%	5	108	5
Susie	90	90	60	40	65%	2	150	2

Directed Numbers

Support could be to provide pupils with the set of integers involved ie:

-5 -4 -2 -2 0 1 3 3 6

Solution:

-5	3	1
3	6	-2
0	-4	-2

Algebra

Support could be to provide pupils with the set of 9 algebra cards (page 10/16) that form the solution or the set of 12 cards, which include some 'rogue' cards.

Properties of Shape

Support could be to provide pupils with the set of 9 shape cards (page 12/16) that form the solutions or the set of 15 cards, which include some 'rogue' cards.

Solution: see page 12/16 for one solution. Another solution is to swap the kite with the isosceles trapezium.

Locus

Demonstrate the range of possible solutions as **BC** varies by using a dynamic geometry package.

Probability

More cards are given than are necessary to find a solution - a smaller sufficient set might support some pupils.

Solution: **E** is most likely to win, with a probability of 0.7. Sequence is **E** (first), **A, C, B** or **F** in either order, **D** (last).