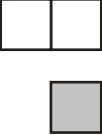
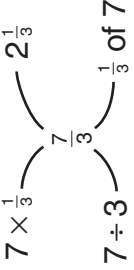


Unit plan

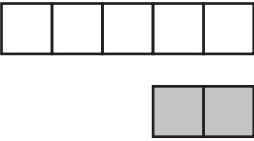
Phase 1 (about three lessons)

<p>Oral and mental starter</p> <p>Objectives</p> <ul style="list-style-type: none"> • Consolidate and extend mental methods of calculation to include decimals, fractions and percentages, accompanied where appropriate by suitable jottings. • Understand the relationship between ratio and proportion; use direct proportion in simple contexts. 	<p>Main teaching</p> <p>Objectives</p> <ul style="list-style-type: none"> • Use fraction notation to describe parts of shapes and to express a smaller whole number as a fraction of a larger one; identify equivalent fractions. • Calculate simple fractions of quantities (whole-number answers); multiply a fraction by an integer. • Use ratio notation. • Represent problems mathematically, making correct use of symbols, words and diagrams. 	<p>Key vocabulary</p> <p>operator, multiplier increase, decrease fraction, decimal fraction proper/improper fraction, mixed number numerator, denominator convert, equivalent ratio ($a : b$)</p> <hr/> <p>Resources</p> <ul style="list-style-type: none"> • 'Images of fractions': <ul style="list-style-type: none"> – either as an ITP* from the CD-ROM – or as OHTs (resource sheets FR1–FR4) • Linking blocks or alternative (e.g. Cuisenaire rods) • Large sheets of plain paper • Mini-whiteboards, calculators (for plenary only) <p>Supplementary notes (in this mini-pack)</p> <ul style="list-style-type: none"> • Key lesson (pp. 10–13) • Prompts for phase 1 (pp. 14–19) <p>*Interactive teaching programs (ITPs)</p> <p>ITPs are simple computer programs offering visual images and graphics to support teaching of specific mathematical topics. Instructions are given on the CD-ROM, if required, but most ITPs are easy to use after a few minutes of exploration.</p>
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Oral and mental starter	Main teaching	Notes	Plenary
<p>First lesson: see plan for extended starter, to review knowledge of fractions, decimals, percentages from KS2.</p> <p>Recap on KS2 language of ratio and proportion, including likely images used for solution. Draw on pupils' knowledge choosing pitch and terminology accordingly. For example: There are 5 toffees for every 2 chocolates in a box of sweets.</p> <p>Q If there are 15 toffees how many chocolates are there?</p> <p>Model the solution using a number line:</p> $\begin{array}{ccccccc} 5 & 10 & 15 & 20 & 25 \\ & & & & \\ \hline 2 & 4 & 6 & 8 & 10 \end{array}$ <p>Q How many sweets altogether?</p> <p>Q What proportion of the sweets are toffees?</p> <p>Q What proportion of the sweets are chocolates?</p> <p>Further contexts: Framework p. 4.</p>	<p>Fraction operators</p> <p>Framework supplement of examples, pp. 60, 64, 66, 68.</p> <p>Stage 1: Detailed plan for first lesson (prompts pp. 11–13) on developing images of thirds (use ITP <i>Images of fractions</i>, OHTs or sketches):</p> <ul style="list-style-type: none"> • multiples of one third, e.g. $7 \times \frac{1}{3}$; • thirds of whole numbers, e.g. $\frac{1}{3}$ of 7. <p>Establish results such as $\frac{7}{3}$ (seven thirds) = $7 \div 3$ (seven divided by three) and draw web diagram of expressions equivalent to $\frac{7}{3}$: $7 \times \frac{1}{3}$, $2 \frac{1}{3}$, $7 \div 3$, $\frac{1}{3}$ of 7 (see plenary column).</p> <p>Follow a similar sequence to develop images for quarters and fifths.</p> <p>Stage 2: Develop a sequence of images for $\frac{1}{3}$ of 7, $\frac{2}{3}$ of 7, ..., $\frac{5}{3}$ of 7. Establish that, for example:</p> <ul style="list-style-type: none"> • $\frac{5}{3}$ of 7 = $\frac{5}{3} \times 7$ ('of' means 'multiply'); • $\frac{5}{3}$ of 7 = $(7 \div 3) \times 5$ • the answer will be an increase in 7 because $\frac{5}{3} > 1$. <p>Develop similar sequences for quarters and fifths.</p> <p>Ratios and equivalence</p> <p>(Prompts p.18) Using linking blocks, pupils set up this arrangement:</p>  <p>Another way of expressing that the number of black blocks is $\frac{1}{2}$ of the number of white blocks is 'the ratio of the number of black blocks to the number of white is 1 to 2' (no. black : no. white = 1 : 2). Using more blocks in each 'strip', consider other fractions (ratios) equivalent to $\frac{1}{2}$ (1 : 2). Describe this relationship as 'one black for every two white'. Construct a diagram.</p> <p>Repeat for other unit fractions, such as $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$. For each, pupils build up a diagram or table of equivalent ratios, fractions and divisions.</p>	<p>Keep a record of pupils' contributions in the starter to use in final plenary of the unit.</p> <p>Extension/support</p> <p>This is not delineated here because all pupils should benefit from developing a shared image of fractions. Higher attaining pupils will secure their understanding by having to explain connections. For other pupils, some of these connections will be new.</p> <p>The unit focuses mainly on halves, thirds, quarters, fifths and associated ratios.</p> <p>The blocks illustrate use of fractions to compare any two quantities (not just parts of a whole). Pupils may not have met ratio notation before.</p>	<p>First lesson: Copy a web diagram (see detailed plan), such as:</p>  <p>Explore changes (e.g. replace $2\frac{1}{3}$ by $2\frac{2}{3}$), asking pupils to say how the other entries will have to change and explain links.</p> <p>Remind pupils of the relationship between fractions and division, e.g. $\frac{3}{8} = 3 \div 8$. Use calculators to convert $\frac{3}{8}$ to a decimal. Establish equivalence of $\frac{3}{8} \times 20$ and 0.375×20 and check with calculator. Try another example, e.g. $\frac{5}{16} \times 20$.</p> <p>Start with strips showing a ratio in unsimplified form, e.g. 6 : 18. Ask pupils what fraction this is equivalent to and why. Encourage ways of 'seeing' the result in the strip image. (See prompts p. 19.)</p>

Phase 2 (about three lessons)

<p>Oral and mental starter</p> <p>Objectives</p> <ul style="list-style-type: none"> • Simplify fractions and identify equivalent fractions. • Use ratio notation. • Understand percentage as the 'number of parts per 100'; recognise the equivalence of percentages, fractions and decimals. 	<p>Main teaching</p> <p>Objectives</p> <ul style="list-style-type: none"> • Use fraction notation to describe parts of shapes and to express a smaller whole number as a fraction of a larger one; simplify fractions and identify equivalent fractions. • Calculate simple fractions of quantities and measurements (whole-number answers); multiply a fraction by an integer. • Understand percentage as the 'number of parts per 100'; recognise the equivalence of percentages, fractions and decimals; calculate simple percentages. • Understand the relationship between ratio and proportion; use direct proportion in simple contexts; use ratio notation and divide a quantity into two parts in a given ratio; solve simple problems about ratio and proportion using informal strategies. • Consolidate and extend mental methods of calculation to include decimals, fractions and percentages, accompanied where appropriate by suitable jottings; solve simple word problems mentally. • Check a result by considering whether it is of the right order of magnitude and by working the problem backwards. • Represent problems mathematically, making correct use of symbols, words and diagrams. • Present and interpret solutions in the context of the original problem; explain and justify methods and conclusions, orally and in writing. 	<p>Key vocabulary</p> <p>operator, multiplier inverse increase, decrease fraction, decimal fraction, percentage (%) proper/improper fraction, mixed number numerator, denominator, reciprocal (optional) convert, equivalent ratio ($a : b$) proportion</p> <p>Resources</p> <ul style="list-style-type: none"> • Linking blocks or alternative (e.g. Cuisenaire rods); optional extra, ITP <i>Ratio strips</i> • Large sheets of plain paper • Whiteboards • Resource sheets FR5–FR8 (ratio strips) • Sets of problems (resource sheets cut into cards): FR9 problems related to ratios 2 : 5 and 3 : 4 FR10 problems related to ratios 4 : 5 and 2 : 3 FR11 mixed problems • Prepared posters or OHTs from first lesson (for final plenary) <p>Supplementary notes (in this mini-pack)</p> <ul style="list-style-type: none"> • Prompts for phase 2 (pp. 20–23)
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Oral and mental starter	Main teaching	Notes	Plenary
<p>Use whiteboards; keep a good pace.</p> <p>These fractions or ratios simplify to $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$ or $\frac{2}{3}$. Which is which?</p> <ul style="list-style-type: none"> • $\frac{24}{36}$, $\frac{36}{90}$, $\frac{36}{48}$ • 28 : 70, 45 : 60, 36 : 45 <p>Give me two numbers:</p> <ul style="list-style-type: none"> • first is 25% of second; • second is $2\frac{1}{2}$ times first; • second is 150% of first; • ratio of first to second is 2 : 5. <p>Defining percentages as hundredths and including examples >100%:</p> <ul style="list-style-type: none"> • use $1\% = \frac{1}{100}$ to convert percentages to fractions; • use 1 = 100% to convert fractions to percentages, e.g. $\frac{3}{4} = \frac{3}{4}$ of 100%; • use $1\% = 0.01$ to convert percentages to decimals and vice versa. 	<p>Multiplicative relationships</p> <p>Stage 1: Using linking blocks pupils construct two strips, each a different colour. Model the process of expressing:</p> <div style="display: flex; align-items: center; margin: 10px 0;">  <div style="margin-left: 20px;"> <ul style="list-style-type: none"> • no. of black blocks : no. of white blocks = 2 : 5, • no. of white blocks : no. of black blocks = 5 : 2; • no. of black = $\frac{2}{5} \times$ no. of white, no. of white = $\frac{5}{2} \times$ no. of black; • no. of black = $0.4 \times$ no. of white, no. of white = $2.5 \times$ no. of black; • no. of black = 40% of no. of white, no. of white = 250% of no. of black. </div> </div> <p>Stage 2: Pupils work in pairs, using linking blocks to complete a similar set of expressions for the ratio 3 : 4, perhaps recording on resource sheet FR6. Then discuss this in a mini-plenary.</p> <p>Stage 3: Give a set of problems on cards, all based around the ratios 2 : 5 and 3 : 4 and their inverses (see resource sheet FR9 for a suggested set). In groups of four, pupils sort the problems into two sets, according to the ratio with which they are associated. In pairs, the pupils then solve the problems. Finally, each pair checks one or two answers of the other pair.</p> <p>In the following lesson, consolidate and extend by repeating the above for a different pair of ratios, e.g. 4 : 5 and 2 : 3 (resource sheets FR7 and FR8). The introductory modelling can be kept brief, with more time devoted to the plenary. (See resource sheet FR10 for a set of problems.)</p> <p>Mixed problems</p> <p>Give a set of mixed problems for pupils to solve in pairs (see resource sheet FR11). The problems are the same types as before, but related to a mixture of ratios or fractions.</p> <p>Plan a mini-plenary to discuss solutions as before (see first note in plenary column), allowing time for the final plenary.</p> <p>Further contexts: Framework pp. 60, 66, 68, 72, 74, 78, 80, 98.</p>	<p>The process could be modelled using the ITP <i>Ratio strips</i> or an acetate version of resource sheet FR5 (also used by pupils for recording). Exploring the relationship both ways encourages flexible thinking and sows the seeds of ideas of inverses to be developed later.</p> <p>Support: If pupils are having difficulty solving problems, intervene to discuss one or two examples. In classes of mixed attainment, group pupils and distribute tasks so that all are able to contribute.</p> <p>Extension: Ask pupils to compose their own sets of problems related to a specific ratio.</p>	<p>Select two or three problems to discuss:</p> <ul style="list-style-type: none"> • how pupils identified links to the chosen ratio; • whether they found it useful to represent the problem with blocks or diagrams; • alternative methods of solution, e.g. converting a percentage to a fraction or decimal calculation; • whether the answer makes sense in terms of order of magnitude, units, etc. <p>Final plenary</p> <p>Return to the OHT or poster of ideas from the first lesson. Ask pupils:</p> <ul style="list-style-type: none"> • to add new entries, links or diagrams; • what new ideas, connections and applications have helped them make most progress; • what they need to do more of next time you visit the topic. <p>(See prompts p. 23.)</p>